Douglas College Physics 1104 Summer 2021

# Douglas College Physics 1104
# Summer 2021

*DEPARTMENT OF PHYSICS AND ASTRONOMY AT DOUGLAS COLLEGE AND OPENSTAX*



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  OpenStax, College Physics. OpenStax CNX. Aug 22, 2016
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# Introduction to Open Textbooks at Douglas College

## Territorial Acknowledgment

We recognize and acknowledge the QayQayt (Ki-Kite) and the Kwikwetlem First Nation, as well as all Coast Salish Peoples, on whose traditional and unceded territories we live, we learn, we play, and we do our work.



*The welcome Pole at Douglas College. She faces up river and her arms are raised in welcome. Photo credit: Jennifer Kirkey CC0*

## Open Textbooks

Open textbooks became a cause of mine, Jennifer Kirkey, as I found more and more of my students could not afford the "required" textbook. BCcampus organized an Open Textbook Summit in 2015 and the keynote speaker, Rajiv Jhangiani from Kwantlen Polytechnic University was so inspiring that I became a BCcampus Faculty Fellow as an Advocate for Open Textbooks in 2016. I do wish to

thank BCcampus for their work with open educational resources such as this textbook, and for the opportunities they have given me. You can learn more about them by visiting https://bccampus.ca/

You can find a list of open textbooks at the BCcampus webpage at https://open.bccampus.ca/find-open-textbooks/

Being able to modify a textbook so that it better matched our courses was the next logical step. This book is the start of the process.




*Student Bulletin Board Covered with Books for Sale Posters at Douglas College Photo Credit Jennifer Kirkey 2016 CC-BY*

Here is a picture I took at the start of the semester in September 2016. This bulletin board is located outside Douglas College's bookstore. Students who clearly did not need their old textbooks, or who needed the money to buy new ones, put up posters in an attempt to sell their old books. I do hope that you find this textbook useful and worth keeping. Please tell us what worked for you in this

book, and what did not. We have the legal ability to change the book and your feedback will help us.

**Who Am I and How Did I get here?**

Looking up at the sky as a child, I was fascinated by the stars. The local library supplied a book on the constellations, and that led to a book explaining why the stars were different colours, and that led to a book on physics. I kept asking questions and that led me to a Bachelor of Science degree in Physics from Trent University in Peterborough, Ontario, Canada and then to a Master's degree in Physics from Simon Fraser University in Burnaby, British Columbia, Canada.

I have been working at Douglas College in New Westminster, British Columbia, Canada for more than twenty-five years. I teach a first year course in astronomy to liberal arts majors and physics to people whose background is no physics in high school to those who want to be engineers. I also teach in a post-graduate program for Elementary School Teachers.

I volunteer with our local science centre, Science World at Telus World of Science in Vancouver, B.C., doing outreach visits to elementary and secondary schools, as well as being active with the local branch of the Royal Astronomical Society of Canada.




*Jennifer Kirkey 2016 Photo credit: Jennifer Kirkey CC-BY*

# Preface to Open Stax College Physics

This textbook is based on the Open Stax College Physics textbook, which can can be downloaded for free at: https://openstax.org/details/college-physics

## About OpenStax

OpenStax is a non-profit organization committed to improving student access to quality learning materials. Our free textbooks are developed and peer-reviewed by educators to ensure they are readable, accurate, and meet the scope and sequence requirements of modern college courses. Unlike traditional textbooks, OpenStax resources live online and are owned by the community of educators using them. Through our partnerships with companies and foundations committed to reducing costs for students, OpenStax is working to improve access to higher education for all. OpenStax is an initiative of Rice University and is made possible through the generous support of several philanthropic foundations.

## About This Book

Welcome to *College Physics*, an OpenStax resource created with

several goals in mind: accessibility, affordability, customization, and student engagement—all while encouraging learners toward high levels of learning. Instructors and students alike will find that this textbook offers a strong foundation in introductory physics, with algebra as a prerequisite. It is available for free online and in low-cost print and e-book editions.

To broaden access and encourage community curation, College Physics is "open source" licensed under a Creative Commons Attribution (CC-BY) license. Everyone is invited to submit examples, emerging research, and other feedback to enhance and strengthen the material and keep it current and relevant for today's students. You can make suggestions by contacting us at info@openstaxcollege.org.

# To the Student

This book is written for you. It is based on the teaching and research experience of numerous physicists and influenced by a strong recollection of their own struggles as students. After reading this book, we hope you see that physics is visible everywhere. Applications range from driving a car to launching a rocket, from a skater whirling on ice to a neutron star spinning in space, and from taking your temperature to taking a chest X-ray.

## General Approach

College Physics is organized such that topics are introduced conceptually with a steady progression to precise definitions and analytical applications. The analytical aspect (problem solving) is tied back to the conceptual before moving on to another topic. Each introductory chapter, for example, opens with an engaging photograph relevant to the subject of the chapter and interesting applications that are easy for most students to visualize.

# Concepts and Calculations

The ability to calculate does not guarantee conceptual understanding. In order to unify conceptual, analytical, and calculation skills within the learning process, we have integrated Strategies and Discussions throughout the text.

## Supplements

Accompanying the main text are a Student Solutions Manual and an Instructor Solutions Manual. The Student Solutions Manual provides worked-out solutions to select end-of-module Problems and Exercises. The Instructor Solutions Manual provides worked-out solutions to all Exercises.

## Learning Objectives

Every module begins with a set of learning objectives. These objectives are designed to guide the instructor in deciding what content to include or assign, and to guide the student with respect to what he or she can expect to learn. After completing the module and end-of-module exercises, students should be able to demonstrate mastery of the learning objectives.

# Call-Outs

Key definitions, concepts, and equations are called out with a special design treatment. Call-outs are designed to catch readers' attention, to make it clear that a specific term, concept, or equation is particularly important, and to provide easy reference for a student reviewing content.

## Key Terms

Key terms are in bold and are followed by a definition in context. Definitions of key terms are also listed in the Glossary, which appears at the end of the module.

## Worked Examples

Worked examples have four distinct parts to promote both analytical and conceptual skills. Worked examples are introduced in words, always using some application that should be of interest. This is followed by a Strategy section that emphasizes the concepts involved and how solving the problem relates to those concepts. This is followed by the mathematical Solution and Discussion.

Many worked examples contain multiple-part problems to help the students learn how to approach normal situations, in which problems tend to have multiple parts. Finally, worked examples employ the techniques of the problem-solving strategies so that students can see how those strategies succeed in practice as well as in theory.

## Problem-Solving Strategies

Problem-solving strategies are first presented in a special section and subsequently appear at crucial points in the text where students can benefit most from them. Problem-solving strategies have a logical structure that is reinforced in the worked examples and supported in certain places by line drawings that illustrate various steps.

## Misconception Alerts

Students come to physics with preconceptions from everyday experiences and from previous courses. Some of these preconceptions are misconceptions, and many are very common among students and the general public. Some are inadvertently picked up through misunderstandings of lectures and texts. The Misconception Alerts feature is designed to point these out and correct them explicitly.

## Take-Home Investigations

Take Home Investigations provide the opportunity for students to apply or explore what they have learned with a hands-on activity.

## Things Great and Small

In these special topic essays, macroscopic phenomena (such as air pressure) are explained with submicroscopic phenomena (such as atoms bouncing off walls). These essays support the modern perspective by describing aspects of modern physics before they are formally treated in later chapters. Connections are also made between apparently disparate phenomena.

## Simulations

Where applicable, students are directed to the interactive PHeT physics simulations developed by the University of Colorado

(http://phet.colorado.edu). There they can further explore the physics concepts they have learned about in the module.

## Summary

Module summaries are thorough and functional and present all important definitions and equations. Students are able to find the definitions of all terms and symbols as well as their physical relationships. The structure of the summary makes plain the fundamental principles of the module or collection and serves as a useful study guide.

## Glossary

At the end of every module or chapter is a glossary containing definitions of all of the key terms in the module or chapter.

## End-of-Module Problems

At the end of every chapter is a set of Conceptual Questions and/or skills-based Problems & Exercises. Conceptual Questions challenge students' ability to explain what they have learned conceptually, independent of the mathematical details. Problems & Exercises challenge students to apply both concepts and skills to solve mathematical physics problems. Online, every other problem includes an answer that students can reveal immediately by clicking on a "Show Solution" button. Fully worked solutions to select problems are available in the Student Solutions Manual and the Teacher Solutions Manual.

In addition to traditional skills-based problems, there are three special types of end-of-module problems: Integrated Concept Problems, Unreasonable Results Problems, and Construct Your Own Problems. All of these problems are indicated with a subtitle preceding the problem.

## Integrated Concept Problems

In Integrated Concept Problems, students are asked to apply what they have learned about two or more concepts to arrive at a solution to a problem. These problems require a higher level of thinking because, before solving a problem, students have to recognize the combination of strategies required to solve it.

## Unreasonable Results

In Unreasonable Results Problems, students are challenged to not only apply concepts and skills to solve a problem, but also to analyze the answer with respect to how likely or realistic it really is. These problems contain a premise that produces an unreasonable answer and are designed to further emphasize that properly applied physics must describe nature accurately and is not simply the process of solving equations.

## Construct Your Own Problem

These problems require students to construct the details of a problem, justify their starting assumptions, show specific steps in the problem's solution, and finally discuss the meaning of the result.

These types of problems relate well to both conceptual and analytical aspects of physics, emphasizing that physics must describe nature. Often they involve an integration of topics from more than one chapter. Unlike other problems, solutions are not provided since there is no single correct answer. Instructors should feel free to direct students regarding the level and scope of their considerations. Whether the problem is solved and described correctly will depend on initial assumptions.

## Acknowledgements

This text is based on the work completed by Dr. Paul Peter Urone in collaboration with Roger Hinrichs, Kim Dirks, and Manjula Sharma. We would like to thank the authors as well as the numerous professors (a partial list follows) who have contributed their time and energy to review and provide feedback on the manuscript. Their input has been critical in maintaining the pedagogical integrity and accuracy of the text.

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# Introduction to Physics 1104

This textbook has been modified from Open Stax College Physics to better match the course Practical Physics Physics 1104 at Douglas College. Here is the official curriculum for Physics 1104 at Douglas College as of August 2018.

This information is also available at http://www.douglascollege.ca/student-services/post-douglas/guidelines/courses

**Course Description:** This course is intended for students who have not taken Physics previously or who have taken some secondary school Physics and want a review. The areas to be covered are mechanics (one and two dimensional motions; vectors; rotational motion; simple machines; work, energy, and power; momentum; equilibrium; Hooke's law; collisions; circular motion; hydrostatics), heat (thermometry; heat transfer; thermal properties of matter), and electricity (electrostatics; direct current concepts and basic circuits).
Method Of Instruction:Lecture, Lab, Partially Online
Contact Hours:4 hours lecture / 3 hours laboratory per week
Semester Length:15 weeks
Prerequisites: BC Foundations of Math 11 (C or higher) or BC Pre-calculus 11 (C or higher)

**COURSE CURRICULUM**

**Learning Outcomes:** Upon completion of the course the student will be able to:

1. Explain/define terms and quantities encountered: displacement, velocity/speed, acceleration, free-fall, scalar, vector resultant, vector component, equilibrium, mass, weight, force, free body diagram, centre of gravity, torque, lever arm, friction, work, kinetic energy, potential energy, power,

momentum, impulse, moment of inertia, angular displacement, angular velocity, angular acceleration, centripetal force, centripetal acceleration, density, pressure, fluid pressure, temperature, thermal energy, specific heat, latent heat, heat conduction, convection, radiation, electric charge, electrical conductor, insulator, electric field, electric potential difference/voltage, resistance, current, electromotive force.

2. Identify the appropriate SI units for the quantities encountered.
3. State the major principles/laws encountered: first and second conditions for equilibrium, Newton's three laws of motion, law of universal gravitation, work-energy theorem, principles of conservation of energy and momentum, Archimedes' principle, Coulomb's law, Ohm's law.
4. Add vector quantities using the geometric and component (trigonometry) methods.
5. Apply the laws/principles to the solution of numerical problems encountered in the textbook and in the laboratory.
6. Perform basic experiments in mechanics, heat and electricity and analyze the data obtained using appropriate graphing techniques, scientific notation, significant figures and experimental uncertainty considerations.

**Course Content**

**Mechanics**

- physical quantities and SI units
- vectors versus scalars
- vector addition
- velocity and acceleration
- uniformly accelerated motion
- Newton's laws of motion
- gravitation
- friction
- first condition for equilibrium

- torque and lever arm
- second condition for equilibrium
- simple machines
- work, energy and power
- conservation of energy
- momentum and impulse
- centripetal force and acceleration
- rotational motion
- density
- pressure
- Archimedes' principle

**Heat**

- temperature and thermometers
- thermal energy and heat capacity
- latent heats and phase changes
- heat transfer mechanisms

**Electricity**

- electric charge
- Coulomb's Law
- electric field
- potential difference
- current
- resistance and Ohm's Law
- electric power
- simple circuit analysis

**Methods Of Instruction:** Classroom time will be divided between the presentation and discussion of basic concepts on the one hand and the application of these concepts in problem solving (working through examples and problems) on the other. Some of the assignments may be on on-line. The laboratory program will involve

weekly, three hour sessions during which students will perform a set number of experiments. This course involves some group work.

PART II

# CHAPTER 1 THE NATURE OF SCIENCE AND PHYSICS

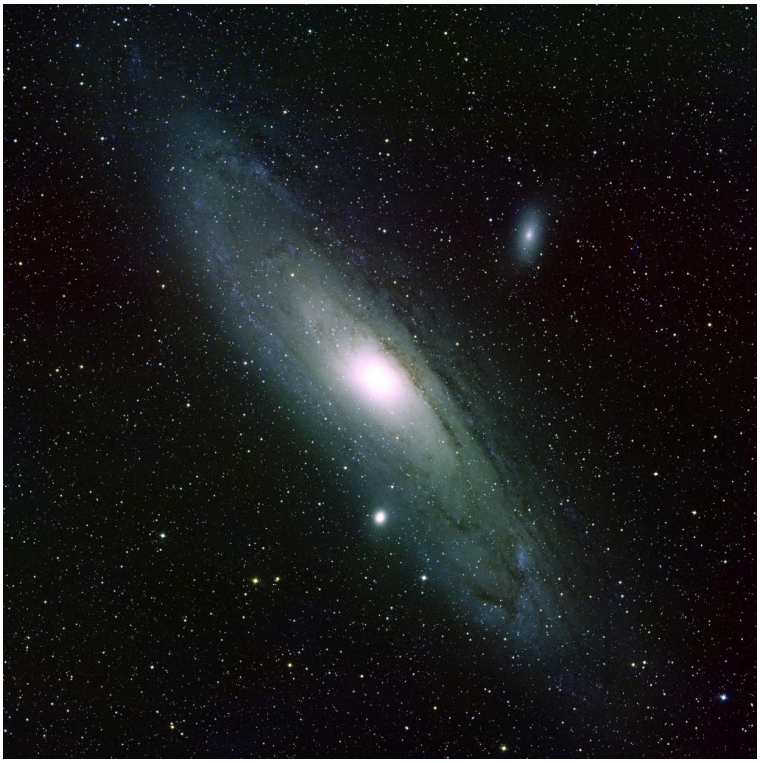


***Figure 1.*** *Andromeda Galaxy in True Colour. Photo credit: Bill Schoening, Vanessa Harvey/REU program/NOAO/AURA/NSF retrieved from NOAO the National Optical Astronomical Observatory at https://www.noao.edu/image_gallery/html/im0424.html*

What is your first reaction when you hear the word "physics"? Did you imagine working through difficult equations or memorizing

formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

For example, take a look at the image above. This image is of the Andromeda Galaxy, which contains billions of individual stars, huge clouds of gas, and dust. Two smaller galaxies are also visible as bright blue spots in the background. At a staggering 2.5 million light years from the Earth, this galaxy is the nearest one to our own galaxy (which is called the Milky Way). The stars and planets that make up Andromeda might seem to be the furthest thing from most people's regular, everyday lives. But Andromeda is a great starting point to think about the forces that hold together the universe. The forces that cause Andromeda Figure 1 to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply raise the walls for a new home. The same gravity that causes the stars of Andromeda to rotate and revolve also causes water to flow over hydroelectric dams here on Earth. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater understanding of the interconnectedness of everything we can see and know in this universe.

Think now about all of the technological devices that you use on a regular basis. Computers, smart phones, GPS systems, MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, "invisibility cloaks" that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical

therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient Greeks, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

**Tour of the Universe and Powers of Ten**

Physics deals with the very large and the very small. Here are links to some short videos and interactive web sites that will help you experience that range. After you watch and play with some of these, please reflect on the questions "Are you big? Are you small?"

Powers of Ten: www.youtube.com/watch?v=0fKBhvDjuy0. This classic short video is narrated by Philip Morrison (9:00).

The Known Universe: www.youtube.com/watch?v=17jymDn0W6U. This video tour from the American Museum of Natural History has realistic animation, music, and captions (6:30).

Wanderers: apod.nasa.gov/apod/ap141208.html. This video provides a tour of the solar system, with narrative by Carl Sagan, imagining other worlds with dramatically realistic paintings (3:50).

The Scale of the Universe https://www.youtube.com/watch?v=uaGEjrADGPA by CaryKH. http://htwins.net/scale2/ is the direct link to an interactive online Flash version, or check out your App store

# 1. 1.1 Physics: An Introduction

*Summary*

- Explain the difference between a principle and a law.
- Explain the difference between a model and theory.



***Figure 1.*** *The flight formations of migratory birds such as Canada geese are governed by the laws of physics. (credit: David Merrett).*

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from

lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

## Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone. Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another.




***Figure 2.*** *The Apple "iPhone" is a common smart phone with a GPS function. Physics describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: @gletham GIS, Social, Mobile Tech Images).*

# Applications of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers.




***Figure 3.*** *The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz).*

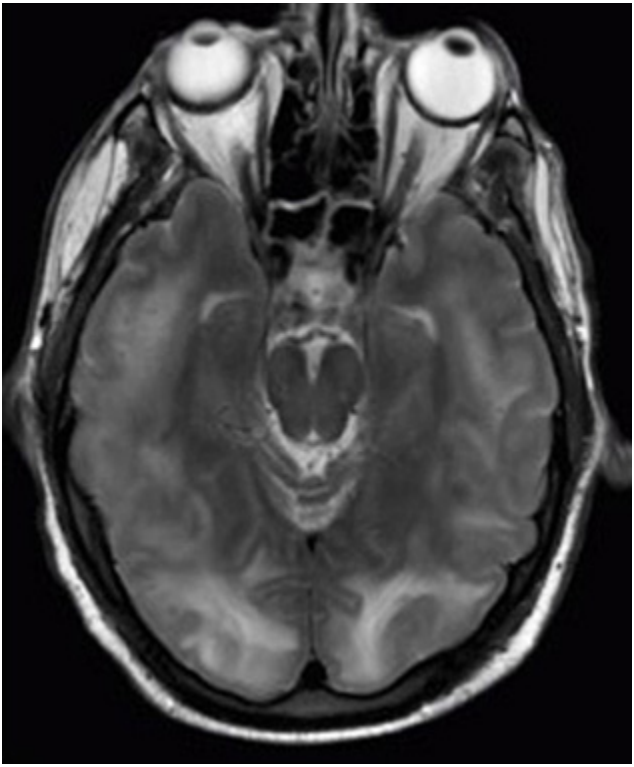


***Figure 4.*** *These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. (credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik).*

Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car's ignition system as well as the transmission of electrical signals through our body's nervous

system are much easier to understand when you think about them in terms of basic physics.

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes.

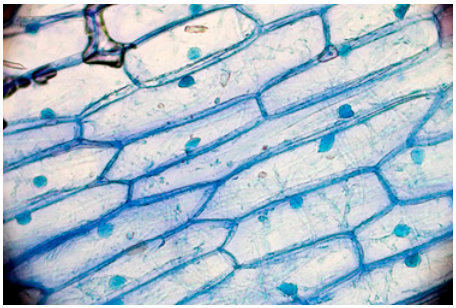


***Figure 5.*** *Physics, chemistry, and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin).*

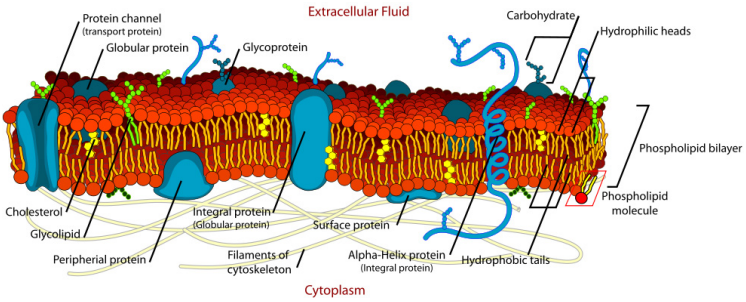



***Figure 6.*** *An artist's rendition of the the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz).*

On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects colour, and how lasers can transmit information.

It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.

## Models, Theories, and Laws; The Role of

## Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavour, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.

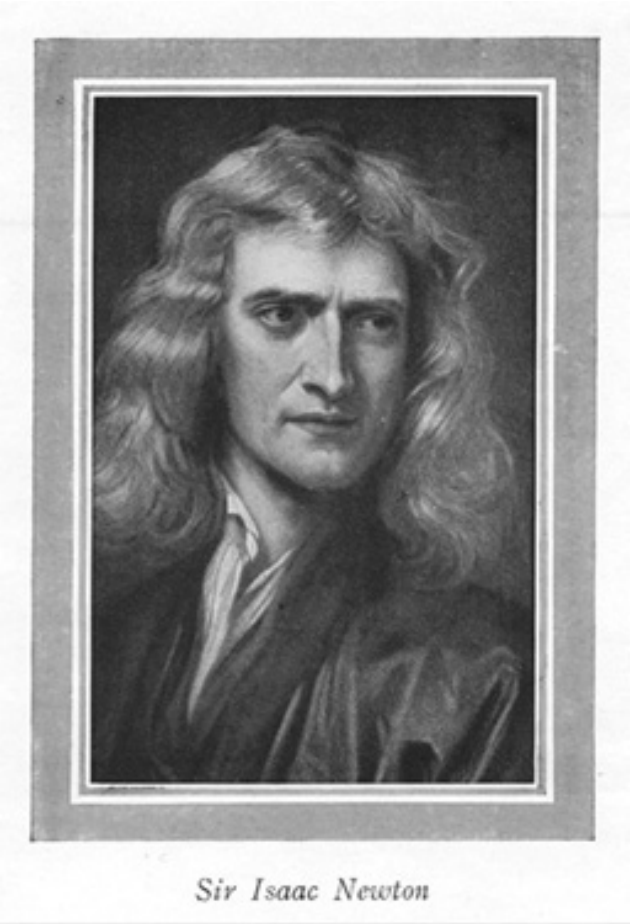



***Figure 7.*** *Isaac Newton (1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. (credit: Arthur Shuster and Arthur E. Shipley: Britain's Heritage of Science. London, 1917.).*



***Figure 8.*** *Marie Curie (1867–1934) sacrificed monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. (credit: Wikimedia Commons).*

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we

look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun.

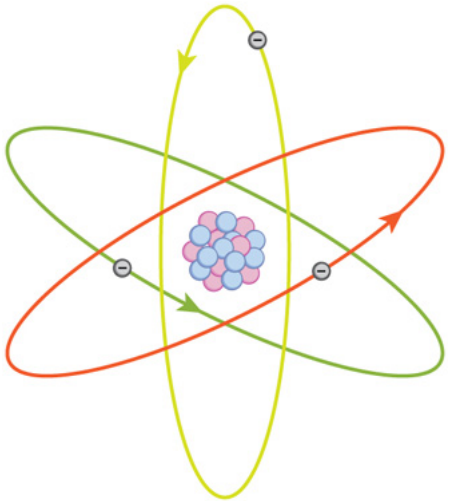


***Figure 9.*** *What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small.*

We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A **law** uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation $\mathbf{F} = m\,\mathbf{a}$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the

foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal's principle, which is applicable only in fluids), but the distinction between laws and principles often is not carefully made.

## MODELS, THEORIES, AND LAWS

Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unobserved.* These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if *experiment* does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

## THE SCIENTIFIC METHOD

As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level.

You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

## The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called "natural philosophy." From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See the figures of Artistotle, Galileo, and Bohr below.) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.

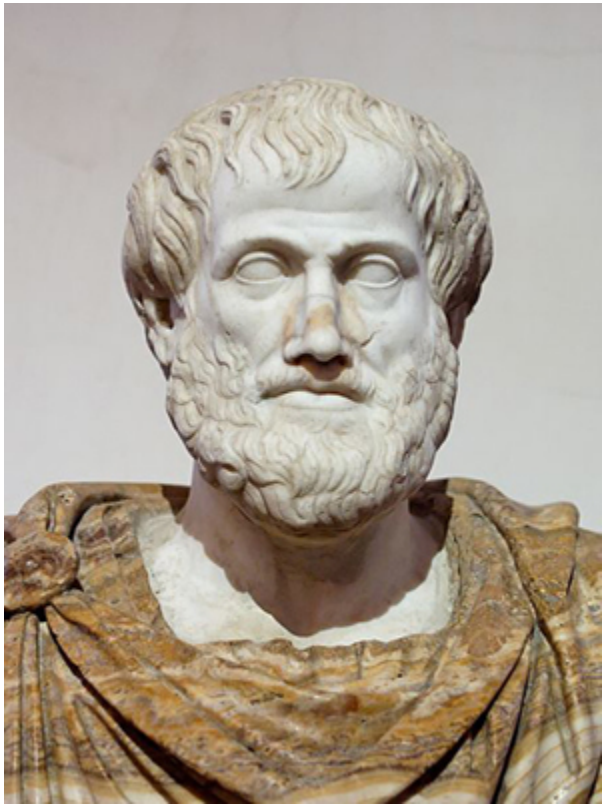


***Figure 10.*** *Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher Aristotle (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. (credit: Jastrow (2006)/Ludovisi Collection).*

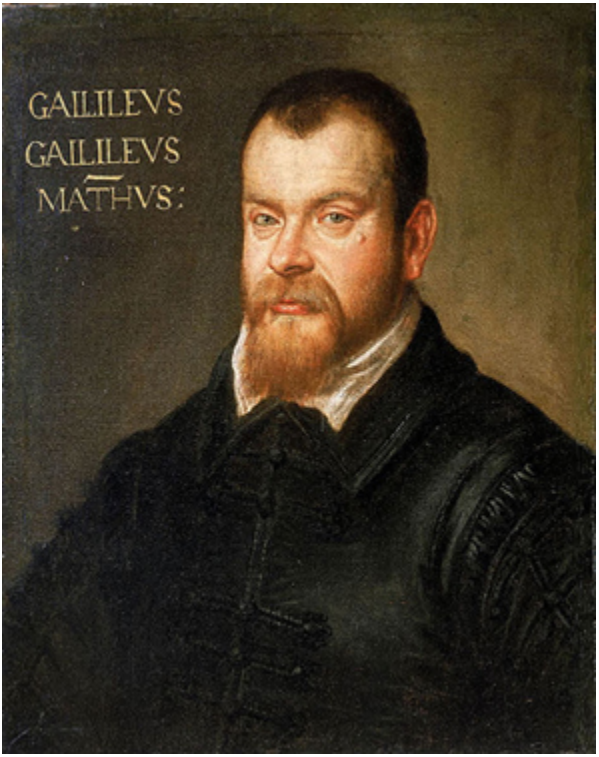



***Figure 11.*** *Galileo Galilei (1564–1642) laid the foundation of modern experimentation and made contributions in mathematics, physics, and astronomy. (credit: Domenico Tintoretto).*



***Figure 12.*** *Niels Bohr (1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics. (credit: United States Library of Congress Prints and Photographs Division).*

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable,

while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom's properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually "picture" the atom.

## LIMITS ON THE LAWS OF CLASSICAL PHYSICS

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.

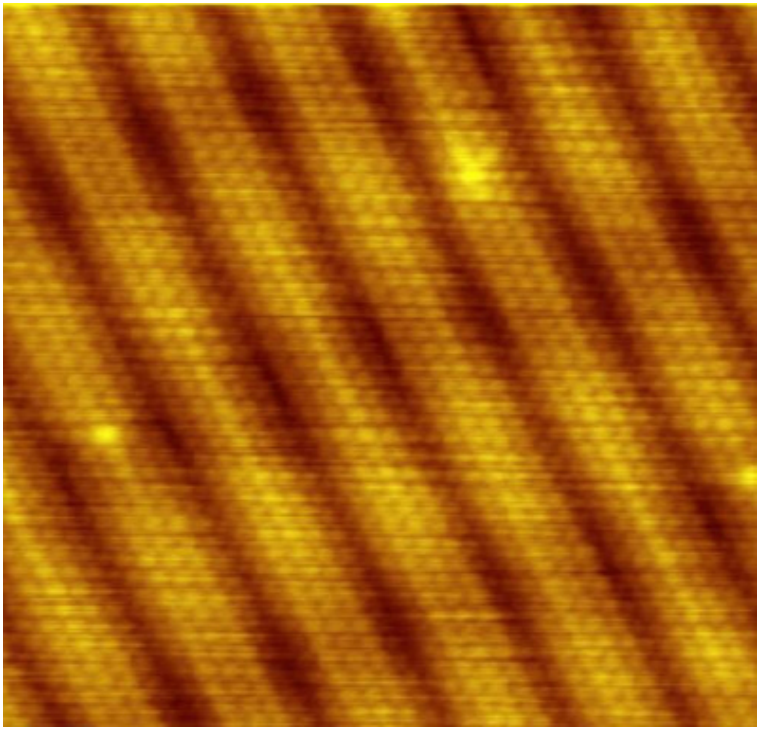


***Figure 13.*** *Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (credit: Erwinrossen).*

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

**Modern physics** itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light ( 300,000,000 m/s or 3.0 x $10^8$ m/s) or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics*, and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results. We will find, however, that we can do a great deal of modern physics with the algebra and trigonometry used in this text.

### *Check Your Understanding*

**1:** A friend tells you he has learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you he had learned about a scientific theory rather than a law?

## Summary

- Science seeks to discover and describe the underlying order and simplicity in nature.

- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

## *Conceptual Questions*

**1:** Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?

**2:** How does a model differ from a theory?

**3:** If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?

**4:** What determines the validity of a theory?

**5:** Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?

**6:** Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

**7:** Classical physics is a good approximation to modern physics under certain circumstances. What are they?

**8:** When is it *necessary* to use relativistic quantum mechanics?

**9:** Can classical physics be used to accurately describe a satellite moving around the Earth at a speed of 7500 m/s? Explain why or why not.

## Glossary

**classical physics**
: physics that was developed from the Renaissance to the end of the 19th century

**physics**
: the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

**model**
: representation of something that is often to difficult (or impossible) to display directly

**theory**
: an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

**law**
: a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence and repeated examples

**scientific method**
: a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the

results of the experiment and draws a conclusion

**modern physics**

the study of relativity, quantum mechanics, or both

**relativity**

the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

**quantum mechanics**

the study of objects smaller than can be seen with a microscope

## *Solutions*

**Check Your Understanding**

**1:** Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

**Conceptual Question**

9. Yes classical physics can explain the motion of most satellites, as the gravity field is weak and the speed of 7500 m/s is less than 1% of the speed of light. Note: 1% of 300,000,000 m/s is 3,000,000 m/s. Do note that the GPS satellites are so accurate that they have use relativistic physics.

# 2. 1.2 Physical Quantities and Units

*Summary*

- Perform unit conversions both in the SI and English units
- Explain the most common prefixes in the SI units and be able to write them in scientific notation

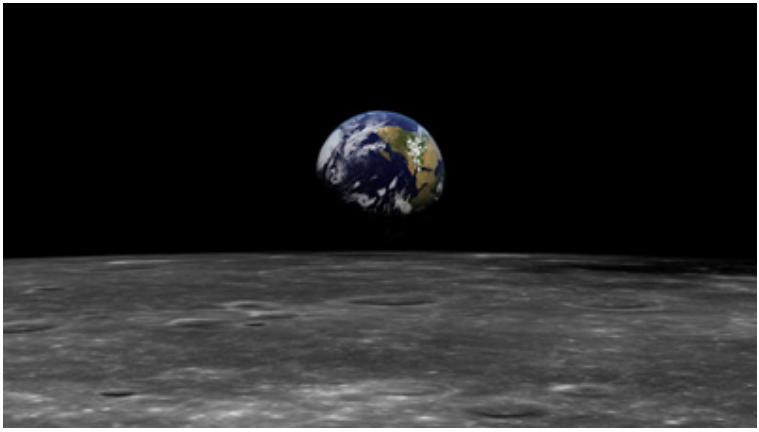


***Figure 1.*** *The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit:* NASA*).*

The range of objects and phenomena studied in physics is immense.

From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—all physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See Figure 2 below.)

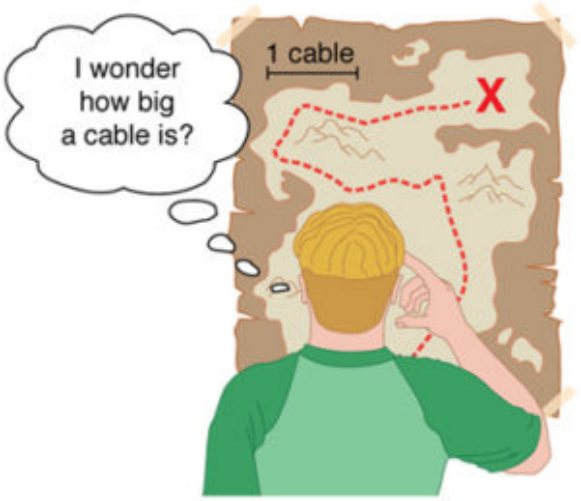



*Figure 2. Distances given in unknown units are maddeningly useless.*

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym "SI" is derived from the French *Système International*.

## SI Units: Fundamental and Derived Units

The metric or SI system is administered in France by the Bureau International des Poids and Mesures or BIPM. You can read more about them at https://www.bipm.org/en/about-us/

Table 1 below shows the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

| Length | Mass | Time | Electric Current |
|---|---|---|---|
| meter (m) | kilogram (kg) | second (s) | ampere (A) |

**Table 1.** Fundamental SI Units.

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

# Units of Time, Length, and Mass: The Second, Meter, and Kilogram

## The Second

The SI unit for time, the second (abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth's rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.

## The Meter

The SI unit for length is the meter (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by

krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

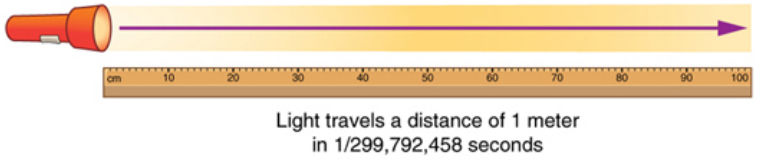



***Figure 4.*** *The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.*

## The Kilogram

The SI unit for mass is the kilogram (abbreviated kg); it is defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses can be ultimately traced to a comparison with the standard mass. As of 2018 the BIPM is working on creating a standard for the kilogram that depends on standard constants via Einstein's famous $E=mc^2$ equation. The initial modules in this textbook are concerned with mechanics, fluids, heat, and waves.

In these subjects all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

## The Ampere

Electric current and its accompanying unit, the ampere, will be introduced in Chapter 14 Electric Current, Resistance, and Ohm's Law.

# Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. The table below gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term **order of magnitude** refers to the scale of a value expressed in the metric system. Each power of 10 in the metric system represents a different order of magnitude. For example,$10^1$, $10^2$, $10^3$, and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the *same* order of magnitude. For example, the number 800 can be written as $8 \times 10^2$, and the number 450 can be written as $4.5 \times 10^2$. Thus, the numbers 800 and 450 are of the same

order of magnitude: $10^2$. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of $10^{-9}$ m while the diameter of the Sun is on the order of $10^9$ m.

## THE QUEST FOR MICROSCOPIC STANDARDS FOR BASIC UNITS

The fundamental units described in this chapter are those that produce the greatest accuracy and precision in measurement. There is a sense among physicists that, because there is an underlying microscopic substructure to matter, it would be most satisfying to base our standards of measurement on microscopic objects and fundamental physical phenomena such as the speed of light. A microscopic standard has been accomplished for the standard of time, which is based on the oscillations of the cesium atom.

The standard for length was once based on the wavelength of light (a small-scale length) emitted by a certain type of atom, but it has been supplanted by the more precise measurement of the speed of light. If it becomes possible to measure the mass of atoms or a particular arrangement of atoms such as a silicon sphere to greater precision than the kilogram standard, it may become possible to base mass measurements on the small scale. There are also possibilities that electrical phenomena on the small scale may someday

allow us to base a unit of charge on the charge of electrons and protons, but at present current and charge are related to large-scale currents and forces between wires.

| Prefix | Symbol | Value | Example (some are app |
|---|---|---|---|
| exa | E | $10^{18}$ | exameter |
| peta | P | $10^{15}$ | petasecond |
| tera | T | $10^{12}$ | terawatt |
| giga | G | $10^{9}$ | gigahertz |
| mega | M | $10^{6}$ | megacurie |
| kilo | k | $10^{3}$ | kilometer |
| hecto | h | $10^{2}$ | hectoliter |
| deka | da | $10^{1}$ | dekagram |
| – | – | $10^{0}$ (=1) | |
| deci | d | $10^{-1}$ | deciliter |
| centi | c | $10^{-2}$ | centimeter |
| milli | m | $10^{-3}$ | millimeter |
| micro | μ | $10^{-6}$ | micrometer |
| nano | n | $10^{-9}$ | nanogram |
| pico | p | $10^{-12}$ | picofarad |

| Prefix | Symbol | Value | Example (some are app |
|---|---|---|---|
| femto | f | $10^{-15}$ | femtometer |
| atto | a | $10^{-18}$ | attosecond |

**Table 2.** Metric Prefixes for Powers of 10 and their Symbols.

## Known Ranges of Length, Mass, and Time

The vastness of the universe and the breadth over which physics applies are illustrated by the wide range of examples of known lengths, masses, and times show in the table above. Examination of this table will give you some feeling for the range of possible topics and numerical values.

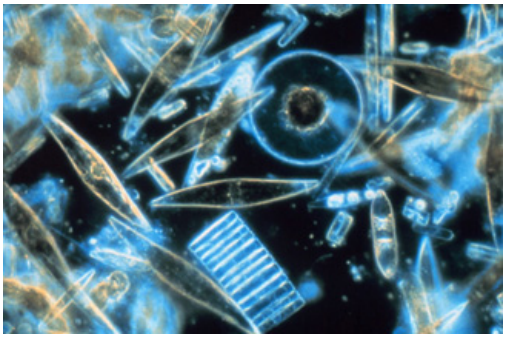


***Figure 5.*** *Tiny phytoplankton swims among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. (credit: Prof. Gordon T. Taylor, Stony Brook University; NOAA Corps Collections).*

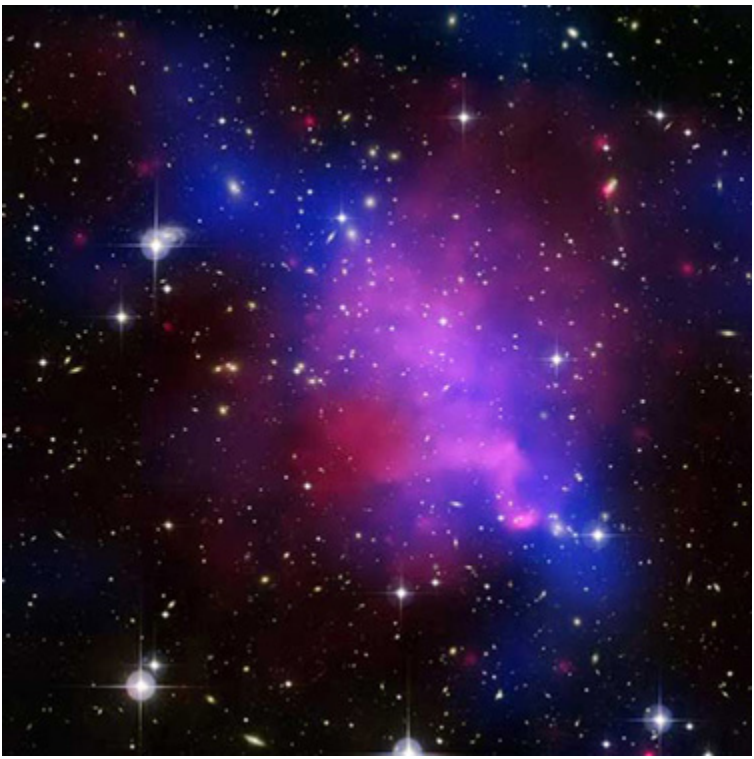


***Figure 6.*** *Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. (credit: NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al.).*

## Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you

will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meter*s and we want to convert to *kilometer*s.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:

$$\mathbf{80\ \cancel{m} \times \frac{1\ km}{1000\ \cancel{m}} = 0.080\ km}$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

| Lengths in meters | | Masses in kilograms (more precise values in parentheses) | | Times in seconds (more precise values in parentheses) | |
|---|---|---|---|---|---|
| $10^{-18}$ | Present experimental limit to smallest observable detail | $10^{-30}$ | Mass of an electron ($9.11\times10^{-31}$ kg) | $10^{-23}$ | Time for light to cross a proton |
| $10^{-15}$ | Diameter of a proton | $10^{-27}$ | Mass of a hydrogen atom ($1.67\times10^{-27}$ kg) | $10^{-22}$ | Mean life of an extremely unstable nucleus |
| $10^{-14}$ | Diameter of a uranium nucleus | $10^{-15}$ | Mass of a bacterium | $10^{-15}$ | Time for one oscillation of visible light |
| $10^{-10}$ | Diameter of a hydrogen atom | $10^{-5}$ | Mass of a mosquito | $10^{-13}$ | Time for one vibration of an atom in a solid |
| $10^{-8}$ | Thickness of membranes in cells of living organisms | $10^{-2}$ | Mass of a hummingbird | $10^{-8}$ | Time for one oscillation of an FM radio wave |
| $10^{-6}$ | Wavelength of visible light | 1 | Mass of a liter of water (about a quart) | $10^{-3}$ | Duration of a nerve impulse |
| $10^{-3}$ | Size of a grain of sand | $10^{2}$ | Mass of a person | 1 | Time for one heartbeat |
| 1 | Height of a 4-year-old child | $10^{3}$ | Mass of a car | $10^{5}$ | One day ($8.64\times10^{4}$ s) |
| $10^{2}$ | Length of a football field | $10^{8}$ | Mass of a large ship | $10^{7}$ | One year (y) ($3.16\times10^{7}$ s) |
| $10^{4}$ | Greatest ocean depth | $10^{12}$ | Mass of a large iceberg | $10^{9}$ | About half the life expectancy of a human |
| $10^{7}$ | Diameter of the Earth | $10^{15}$ | Mass of the nucleus of a comet | $10^{11}$ | Recorded history |
| $10^{11}$ | Distance from the Earth to the Sun | $10^{23}$ | Mass of the Moon ($7.35\times10^{22}$ kg) | $10^{17}$ | Age of the Earth |

| Lengths in meters | | Masses in kilograms (more precise values in parentheses) | | Times in seconds (more precise values in parentheses) | |
|---|---|---|---|---|---|
| $10^{16}$ | Distance traveled by light in 1 year (a light year) | $10^{25}$ | Mass of the Earth ($5.97\times10^{24}$ kg) | $10^{18}$ | Age of the universe |
| $10^{21}$ | Diameter of the Milky Way galaxy | $10^{30}$ | Mass of the Sun ($1.99\times10^{30}$ kg) | | |
| $10^{22}$ | Distance from the Earth to the nearest large galaxy (Andromeda) | $10^{42}$ | Mass of the Milky Way galaxy (current upper limit) | | |
| $10^{26}$ | Distance from the Earth to the edges of the known universe | $10^{53}$ | Mass of the known universe (current upper limit) | | |

**Table 3.** Approximate Values of Length, Mass, and Time.

## Example 1: Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

**Strategy**

First we calculate the average speed using the given

units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

**Solution for (a)**

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\textbf{average speed} = \frac{\textbf{distance}}{\textbf{time}}.$$

(2) Substitute the given values for distance and time.

$$\textbf{average speed} = \frac{\textbf{10.0 km}}{\textbf{20.0 min}}$$

$$= \textbf{0.500}\,\frac{\textbf{km}}{\textbf{min}}$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/hr. Thus,

$$\textbf{average speed} = \textbf{0.500}\,\frac{\textbf{km}}{\textbf{min}} \times \frac{\textbf{60 min}}{\textbf{1 h}} = \textbf{30.0}\,\frac{\textbf{km}}{\textbf{h}}.$$

**Discussion for (a)**

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units

in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

$$\frac{\text{km}}{\text{min}} \times \frac{\text{1 hr}}{\text{60 min}} = \frac{\text{1 km} \cdot \text{hr}}{\text{60 min}^2},$$

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/h does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

**Solution for (b)**

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to

m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

$$\textbf{Average speed} = 30.0\frac{\textbf{km}}{\textbf{h}} \times \frac{\textbf{1 h}}{\textbf{3,600 s}} \times \frac{\textbf{1,000 m}}{\textbf{1 km}},$$

$$\textbf{Average speed} = 8.33\frac{\textbf{m}}{\textbf{s}}.$$

**Discussion for (b)**

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces? The module Chapter 1.4 Accuracy, Precision, and Significant Figures will help you answer these questions.

## NONSTANDARD UNITS

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different "weights and measures." Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

**PhET Interactive Simulations and a Math Review**

There are many interactive simulations available from the University of Colorado Boulder called PheT or Physics Education https://phet.colorado.edu/en/simulations

Starting with some basic math, you might want to review some skills as math is the language of physics. Some of the older simulations use Flash which might not run on all computers.

https://phet.colorado.edu/en/simulation/arithmetic

Ratios and proportionality https://phet.colorado.edu/en/simulation/proportion-playground

Fractions https://phet.colorado.edu/en/simulation/fraction-matcher

### *Check Your Understanding 1*

**1:** Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

### *Check Your Understanding 2*

**1:** One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

## Summary

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the

meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.

- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

## *Conceptual Questions*

**1:** Identify some advantages of metric units.

## *Problems & Exercises*

**1:** The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?

**2:** A car is traveling at a speed of 33 m/s. (a) What is its speed in kilometers per hour? (b) Is it exceeding the 90 km/h speed limit?

**3:** Show that 1.0 m/s=3.6 km/h. Hint: Show the explicit steps involved in converting 1.0 m/s=3.6 km/h.

**4:** American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)

**5:** Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)

**6:** What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)

**7:** Mount Everest, at 29,028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3,281 feet.)

**8:** The speed of sound is measured to be 342 m/s on a certain day. What is this in km/h?

**9:** Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?

**10:** (a) Table 3 gives you the average distance between the Earth and the Sun is about $1 \times 10^{11}$ meters Then calculate the average speed of the Earth in its orbit in meters per second assuming a circular orbit that takes 1 year or $3.15 \times 10^{7}$ seconds.

# Glossary

**physical quantity**
: a characteristic or property of an object that can be measure or calculated from other measurements

**units**
: a standard used for expressing and comparing measurements

**SI units**
: the international system of units that scientist in most countries have agreed to use; includes units such as meters, liters, and grams

**English units**
: system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

**fundamental units**
: units that can only be expressed relative to the procedure used to measure them

**derived units**
: units that can be calculated using algebraic combinations of the fundamental units

**second**
: the SI unit for time, abbreviated (s)

**meter**
: the SI unit for length, abbreviated (m)

**kilogram**
: the SI unit for mass, abbreviated (kg)

**metric system**
: a system in which values can be calculated in factors of 10

**order of magnitude**
: refers to the size of a quantity as it related to a power of 10

**conversion factor**
: a ratio expression how many of one unit are equal to another unit

## *Solutions*

**Check Your Understanding 1**

**1:** The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or $10^{-3}$ seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

**Check Your Understanding 2**

**1:** The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

**Problems & Exercises**

**1:** (a) 27.8 m/s  b) 62.1 miles per hour or mph

**3:**

$$\frac{\mathbf{1.0\ m}}{\mathbf{s}} = \frac{\mathbf{1.0\ m}}{\mathbf{s}} \times \frac{\mathbf{3600\ s}}{\mathbf{1\ hr}} \times \frac{\mathbf{1\ km}}{\mathbf{1000\ m}} =$$

3.6 km/h

**5:** length: 377 ft; $4.53 \times 10^3$ in  width: 280 ft ; $3.3 \times 10^3$ inches

**7:** 8.847 km

**9:** (a) $1.3 \times 10^{-9}$ m  (b) 40 km/My  or 40 kilometers per mega year.

10:  a) speed = distance / time = $2 \pi r$ / time = 20000 m/s or 20 km/s  b)

# 3. 1.3 Other Units: Temperature and Density

*Learning Objectives*

1. Learn about the various temperature scales that are commonly used in physics and chemistry.
2. Define density and use it as a conversion factor.

*This content is originally from OpenStax College Chemistry 1st Canadian Edition.*

There are other units in chemistry that are important, and we will cover others in the course of the entire book. One of the fundamental quantities in science is temperature. Temperature is a measure of the average amount of energy of motion, or *kinetic energy*, a system contains. Temperatures are expressed using scales that use units called degrees, and there are several temperature scales in use. In the United States, the commonly used temperature scale is the *Fahrenheit scale* (symbolized by °F and spoken as "degrees Fahrenheit"). On this scale, the freezing point of liquid water (the temperature at which liquid water turns to solid ice) is 32 °F, and the boiling point of water (the temperature at which liquid water turns to steam) is 212 °F.

Science also uses other scales to express temperature. The Celsius scale (symbolized by °C and spoken as "degrees Celsius") is a temperature scale where 0 °C is the freezing point of water and 100 °C is the boiling point of water; the scale is divided into 100 divisions between these two landmarks and extended higher and lower. By

comparing the Fahrenheit and Celsius scales, a conversion between the two scales can be determined:

$$°C = (°F - 32) \times \frac{5}{9}$$

$$°F = \left(°C \times \frac{9}{5}\right) + 32$$

Using these formulas, we can convert from one temperature scale to another. The number 32 in the formulas is exact and does not count in significant figure determination.

### Example 12

1. What is 98.6 °F in degrees Celsius?
2. What is 25.0 °C in degrees Fahrenheit?

Solution

1. Using the first formula from above, we have

   °C = (98.6 − 32) × 5/9 = 66.6 × 5/9 = 37.0 °C

2. Using the second formula from above, we have

   °F = (25.0 × 9/5) + 3/2 = 45.0 + 32 = 77.0 °F

## *Check Your Understanding 1*

1. Convert 0 °F to degrees Celsius.
2. Convert 212 °C to degrees Fahrenheit.

The fundamental unit of temperature (another fundamental unit of science, bringing us to four) in SI is the kelvin (K). The Kelvin temperature scale (note that the name of the scale capitalizes the word *Kelvin*, but the unit itself is lowercase) uses degrees that are the same size as the Celsius degree, but the numerical scale is shifted up by 273.15 units. That is, the conversion between the Kelvin and Celsius scales is as follows:

$K = °C + 273.15$

$°C = K - 273.15$

For most purposes, it is acceptable to use 273 instead of 273.15. Note that the Kelvin scale does not use the word *degrees*; a temperature of 295 K is spoken of as "two hundred ninety-five kelvins" and not "two hundred ninety-five degrees Kelvin."

The reason that the Kelvin scale is defined this way is because there exists a minimum possible temperature called absolute zero. The Kelvin temperature scale is set so that 0 K is absolute zero, and temperature is counted upward from there. Normal room temperature is about 295 K, as seen in the following example.

## Example 13

If normal room temperature is 72.0 °F, what is room temperature in degrees Celsius and kelvins?

Solution

First, we use the formula to determine the temperature in degrees Celsius:

$°C = (72.0 - 32) \times 5/9 = 40.0 \times 5/9 = 22.2\ °C$

Then we use the appropriate formula above to determine the temperature in the Kelvin scale:

$K = 22.2\ °C + 273.15 = 295.4\ K$

So, room temperature is about 295 K.

## *Check Your Understanding 2*

What is 98.6 °F on the Kelvin scale?

Figure 2.9 compares the three temperature scales. Note that science uses the Celsius and Kelvin scales almost exclusively; virtually no practicing chemist expresses laboratory-measured temperatures with the Fahrenheit scale. (In fact, the United States is one of the few countries in the world that still uses the Fahrenheit

scale on a daily basis. The other two countries are Liberia and Myanmar [formerly Burma].

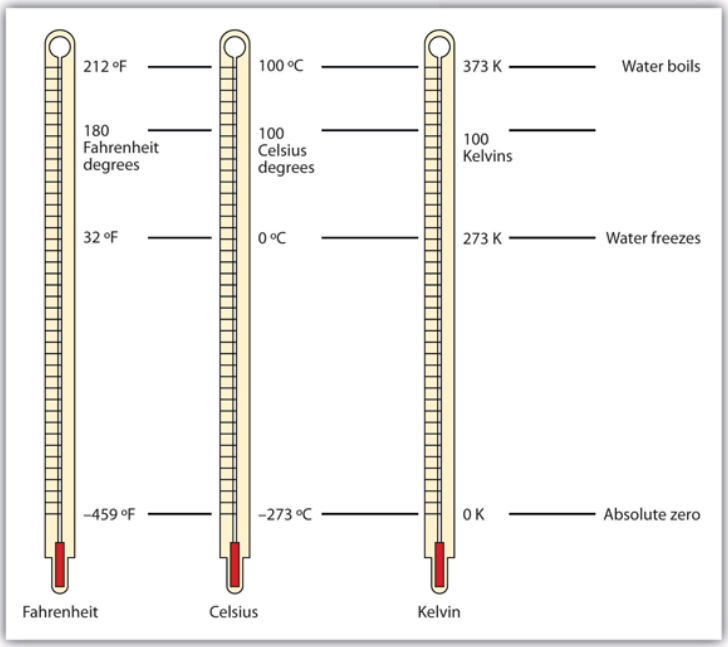



***Figure 1.*** *Fahrenheit, Celsius, and Kelvin Temperatures*

A comparison of the three temperature scales.

Density is a physical property that is defined as a substance's mass divided by its volume:

density = mass/volume or d = m/V

Density is usually a measured property of a substance, so its numerical value affects the significant figures in a calculation. Notice that density is defined in terms of two dissimilar units, mass and volume. That means that density overall has derived units, just like velocity. Common units for density include g/mL, $g/cm^3$, g/L,

kg/L, and even kg/$m^3$. Densities for some common substances are listed in Table 2.2.

Table 2.2 Densities of Some Common Substances

| **Substance** | **Density (g/mL or g/$cm^3$)** |
|---|---|
| water | 1.0 |
| gold | 19.3 |
| mercury | 13.6 |
| air | 0.0012 |
| cork | 0.22–0.26 |
| aluminum | 2.7 |
| iron | 7.87 |

Because of how it is defined, density can act as a conversion factor for switching between units of mass and volume. For example, suppose you have a sample of aluminum that has a volume of 7.88 $cm^3$. How can you determine what mass of aluminum you have without measuring it? You can use the volume to calculate it. If you multiply the given volume by the known density (from Table 2.2 "Densities of Some Common Substances"), the volume units will cancel and leave you with mass units, telling you the mass of the sample:

$7.88\ cm^3 \times 2.7\ g/cm^3 = 21\ g$ of aluminum

where we have limited our answer to two significant figures.

## Example 14

What is the mass of 44.6 mL of mercury?

Solution

Use the density from Table 2.2 as a conversion factor to go from volume to mass:

$44.6 \text{ mL} \times 13.6 \text{ g/mL} = 607 \text{ g}$

The mass of the mercury is 607 g.

## *Check Your Understanding 3*

What is the mass of 25.0 $cm^3$ of iron?

Density can also be used as a conversion factor to convert mass to volume—but care must be taken. We have already demonstrated that the number that goes with density normally goes in the numerator when density is written as a fraction. Take the density of gold, for example:

$$d = 19.3 \frac{g}{mL} = \frac{19.3\ g}{1\ mL}$$

Although this was not previously pointed out, it can be assumed that there is a 1 in the denominator:

That is, the density value tells us that we have 19.3 grams for every 1 milliliter of volume, and the 1 is an exact number. When we want to use density to convert from mass to volume, the numerator and denominator of density need to be switched—that is, we must take the *reciprocal* of the density. In so doing, we move not only the units but also the numbers:

$$\frac{1}{d} = \frac{1\ mL}{19.3\ g}$$

This reciprocal density is still a useful conversion factor, but now the mass unit will cancel and the volume unit will be introduced. Thus, if we want to know the volume of 45.9 g of gold, we would set up the conversion as follows:

$$45.9\ \cancel{g} \times \frac{1\ mL}{19.3\ \cancel{g}} = 2.38\ mL$$

Note how the mass units cancel, leaving the volume unit, which is what we're looking for.

## Example 15

A cork stopper from a bottle of wine has a mass of 3.78 g. If the density of cork is 0.22 g/cm$^3$, what is the volume of the cork?

Solution

To use density as a conversion factor, we need to take the reciprocal so that the mass unit of density is in the denominator. Taking the reciprocal, we find

$$\frac{1}{d} = \frac{1\ cm^3}{0.22\ g}$$

We can use this expression as the conversion factor. So

$$3.78\ \cancel{g} \times \frac{1\ cm^3}{0.22\ \cancel{g}} = 17.2\ cm^3$$

### *Check Your Understanding 4*

What is the volume of 3.78 g of gold?

Care must be used with density as a conversion factor. Make sure the mass units are the same, or the volume units are the same, before using density to convert to a different unit. Often, the unit of the given quantity must be first converted to the appropriate unit before applying density as a conversion factor.

## Food and Drink App: Cooking Temperatures

Because degrees Fahrenheit is the common temperature scale in the United States, kitchen appliances, such as ovens, are calibrated in that scale. A cool oven may be only 150°F, while a cake may be baked at 350°F and a chicken roasted at 400°F. The broil setting on many ovens is 500°F, which is typically the highest temperature setting on a household oven.

People who live at high altitudes, typically 2,000 ft above sea level or higher, are sometimes urged to use slightly different cooking instructions on some products, such as cakes and bread, because water boils at a lower temperature the higher in altitude you go, meaning that foods cook slower. For example, in Cleveland water typically boils at 212°F (100°C), but in Denver, the Mile-High City, water boils at about 200°F (93.3°C), which can significantly lengthen cooking times. Good cooks need to be aware of this.

At the other end is pressure cooking. A pressure cooker is a closed vessel that allows steam to build up additional pressure, which increases the temperature at which water boils. A good

pressure cooker can get to temperatures as high as 252°F (122°C); at these temperatures, food cooks much faster than it normally would. Great care must be used with pressure cookers because of the high pressure and high temperature. (When a pressure cooker is used to sterilize medical instruments, it is called an *autoclave*.)

Other countries use the Celsius scale for everyday purposes. Therefore, oven dials in their kitchens are marked in degrees Celsius. It can be confusing for US cooks to use ovens abroad—a 425°F oven in the United States is equivalent to a 220°C oven in other countries. These days, many oven thermometers are marked with both temperature scales.

## *Key Takeaways*

- Chemistry uses the Celsius and Kelvin scales to express temperatures.
- A temperature on the Kelvin scale is the Celsius temperature plus 273.15.
- The minimum possible temperature is absolute zero and is assigned 0 K on the Kelvin scale.
- Density relates a substance's mass and volume.
- Density can be used to calculate volume from a given mass or mass from a given volume.

## *Problems & Exercises*

1. Perform the following conversions.

(a) 255°F to degrees Celsius (b) −255°F to degrees Celsius (c) 50.0°C to degrees Fahrenheit (d) −50.0°C to degrees Fahrenheit

2. Perform the following conversions.

(a) 1,065°C to degrees Fahrenheit (b) −222°C to degrees Fahrenheit (c) 400.0°F to degrees Celsius (d) 200.0°F to degrees Celsius

3. Perform the following conversions.

(a) 100.0°C to kelvins (b) −100.0°C to kelvins (c) 100 K to degrees Celsius (d) 300 K to degrees Celsius

4. Perform the following conversions.

(a) 1,000.0 K to degrees Celsius (b) 50.0 K to degrees Celsius (c) 37.0°C to kelvins (d) −37.0°C to kelvins

5. Convert 0 K to degrees Celsius. What is the significance of the temperature in degrees Celsius?

6. Convert 0 K to degrees Fahrenheit. What is the significance of the temperature in degrees Fahrenheit?

7. The hottest temperature ever recorded on the surface of the earth was 136°F in Libya in 1922. What is the temperature in degrees Celsius and in kelvins?

8. The coldest temperature ever recorded on the surface of the earth was −128.6°F in Vostok, Antarctica, in 1983. What is the temperature in degrees Celsius and in kelvins?

9. Give at least three possible units for density.

10. What are the units when density is inverted? Give three examples.

11. A sample of iron has a volume of 48.2 $cm^3$. What is its mass?

12. A sample of air has a volume of 1,015 mL. What is its mass?

13. The volume of hydrogen used by the *Hindenburg*, the German airship that exploded in New Jersey in 1937, was $2.000 \times 10^8$ L. If hydrogen gas has a density of 0.0899 g/L, what mass of hydrogen was used by the airship?

14. The volume of an Olympic-sized swimming pool is $2.50 \times 10^9$ $cm^3$. If the pool is filled with alcohol ($d$ = 0.789 g/$cm^3$), what mass of alcohol is in the pool?

15. A typical engagement ring has 0.77 $cm^3$ of gold. What mass of gold is present?

16. A typical mercury thermometer has 0.039 mL of mercury in it. What mass of mercury is in the thermometer?

17. What is the volume of 100.0 g of lead if lead has a density of 11.34 g/$cm^3$?

18. What is the volume of 255.0 g of uranium if uranium has a density of 19.05 g/$cm^3$?

19. What is the volume in liters of 222 g of neon if neon has a density of 0.900 g/L?

20. What is the volume in liters of 20.5 g of sulfur

hexafluoride if sulfur hexafluoride has a density of 6.164 g/L?

21. Which has the greater volume, 100.0 g of iron ($d$ = 7.87 g/cm$^3$) or 75.0 g of gold ($d$ = 19.3 g/cm$^3$)?

22. Which has the greater volume, 100.0 g of hydrogen gas ($d$ = 0.0000899 g/cm$^3$) or 25.0 g of argon gas ($d$ = 0.00178 g/cm$^3$)?

## *Solutions*

**Check Your Understanding 1**

1. −17.8 °C
2. 414 °F

**Check Your Understanding 2**

310.2 K

**Check Your Understanding 3**

197 g

**Check Your Understanding 4**

0.196 cm$^3$**Problems & Exercises**

**1.** (a) 124°C (b) −159°C (c) 122°F (d) −58°F

**3.** (a) 373 K (b) 173 K (c) −173°C (d) 27°C

**5.** −273°C. This is the lowest possible temperature in degrees Celsius.

**7.** 57.8°C; 331 K

**9.** g/mL, g/L, and kg/L (answers will vary)

**11.** 379 g

**13.** $1.80 \times 10^7$ g

**15.** 15 g

**17.** 8.818 $cm^3$

**19.** 247 L

**21.** The 100.0 g of iron has the greater volume.

# 4. 1.4 Accuracy, Precision, and Significant Figures

## *Summary*

- Determine the appropriate number of significant figures in both addition and subtraction, as well as multiplication and division calculations.
- Calculate the percent uncertainty of a measurement.



***Figure 1.*** *A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The "known masses" are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (credit: Serge Melki).*

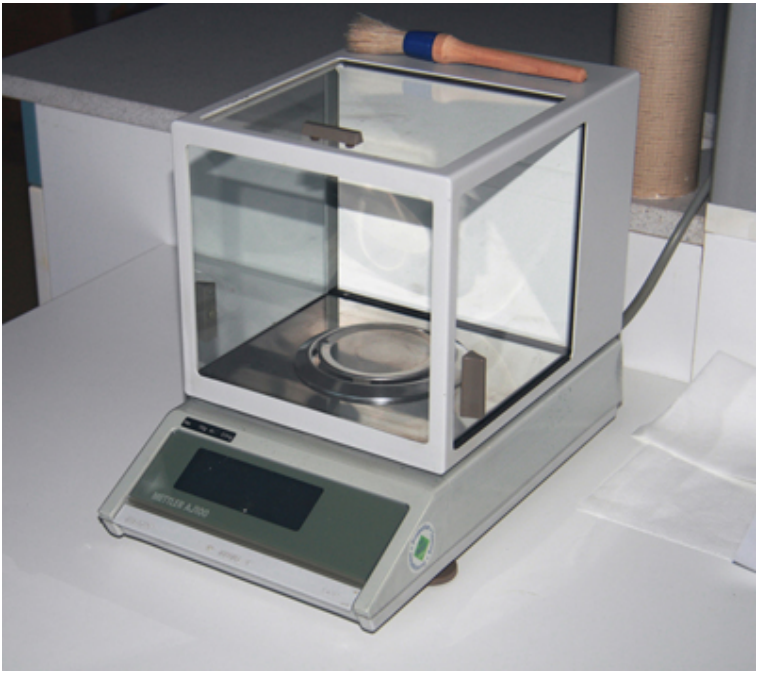


***Figure 2.*** *Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit: Karel Jakubec).*

## Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These

measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The **precision** of a measurement system is refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the centre of a bull's-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In Figure 3 you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the centre of the target. This indicates a low precision, high accuracy measuring system. However, in Figure 4 the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.

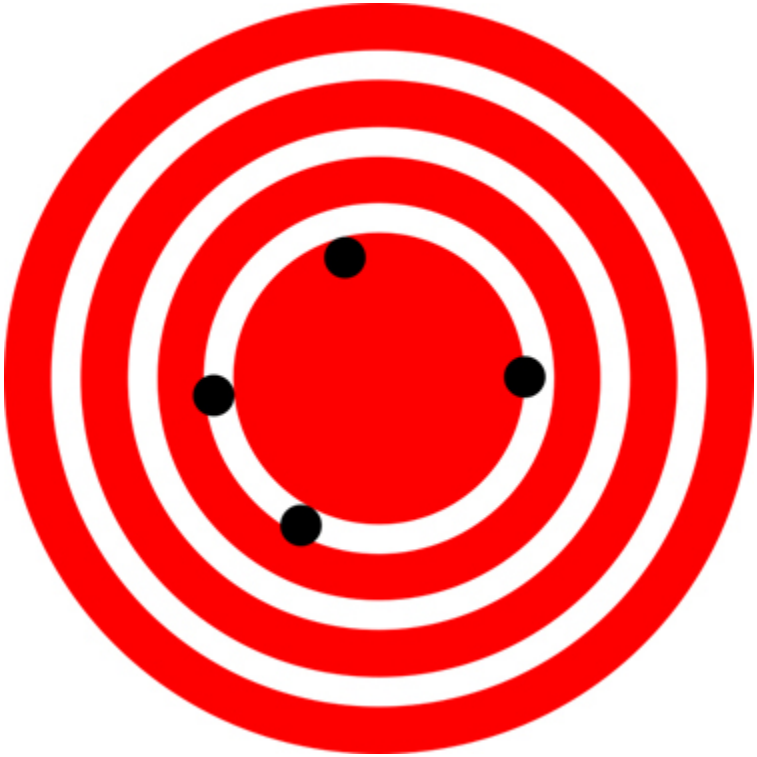


***Figure 3.*** *A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (credit: Dark Evil).*



***Figure 4.*** *In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit: Dark Evil).*

## Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of

as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement, **A**, is often denoted as $\delta\mathbf{A}$ ("delta **A**"), so the measurement result would be recorded as $\mathbf{A} \pm \delta\mathbf{A}$. In our paper example, the length of the paper could be expressed as 11 in. ± 0.2.

The factors contributing to uncertainty in a measurement include:

1. Limitations of the measuring device,
2. The skill of the person making the measurement,
3. Irregularities in the object being measured,
4. Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

## MAKING CONNECTIONS: REAL-WORLD CONNECTIONS – FEVER OR CHILLS?

Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check his or her temperature with a thermometer. What if the uncertainty of the thermometer were 3.0 °C? If the child's temperature reading was 37.0 °C (which is normal body temperature), the "true" temperature could be anywhere from a hypothermic 34.0 °C to a dangerously high 40.0 °C. A thermometer with an uncertainty of 3.0 °C would be useless.

## Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement **A** is expressed with uncertainty, δ**A**, the **percent uncertainty** (%unc) is defined to be:

$$\textbf{\% unc} = \frac{\delta \mathbf{A}}{\mathbf{A}} \times 100\%$$

## Example 1: Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: **4.8 lb**
- Week 2 weight: **5.3 lb**
- Week 3 weight: **4.9 lb**
- Week 4 weight: **5.4 lb**

You determine that the weight of the 5-lb bag has an uncertainty of ±0.4 lb. What is the percent uncertainty of the bag's weight?

**Strategy**

First, observe that the expected value of the bag's weight, **A**, is 5 lb. The uncertainty in this value, δ**A**, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight:

$$\textbf{\% unc} = \frac{\boldsymbol{\delta A}}{\mathbf{A}} \times \mathbf{100\%}$$

**Solution**

Plug the known values into the equation:

$$\textbf{\% unc} = \frac{\mathbf{0.4\ lb}}{\mathbf{5\ lb}} \times \mathbf{100\%} = \mathbf{8\%}$$

**Discussion**

We can conclude that the weight of the apple bag is 5 lb ±8%. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

## Uncertainties in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements going into the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used for multiplication or division. This method says that *the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation*. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2% and 1%, respectively, then the area of the floor is 12.0 m and has an uncertainty of 3%. (Expressed as an area this is 0.36 $m^2$, which we round to 0.4 $m^2$ since the area of the floor is given to a tenth of a square meter.)

*Check Your Understanding 1*

**1:** A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of ±0.05 s. Runners on the track coach's team regularly clock 100-m sprints of 11.49 s to 15.01 s. At the school's last track meet, the first-place sprinter came in at 12.04 s and the second-place sprinter came in at 12.07 s. Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?

## Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest 0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be **36.7 cm**. You could not express this value as **36.71 cm** because your measuring tool was not precise enough to

measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between **36.6 cm** and **36.7 cm**, and he or she must estimate the value of the last digit. Using the method of **significant figures**, the rule is that *the last digit written down in a measurement is the first digit with some uncertainty*. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value **36.7 cm** has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

## Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) *Zeros are significant except when they serve only as placekeepers.*

*Check Your Understanding 2*

**2:** Determine the number of significant figures in the following measurements:

a: 0.0009

b: 15,450.0

c: $6\times10^3$

d: 87.990

e: 30.42

## Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, *the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value*. There are two different rules, one for multiplication and division and the other for addition and subtraction, as discussed below.

**1. For multiplication and division:** *The result should have the same number of significant figures as the quantity having the least significant figures entering into the calculation.* For example, the area of a circle can be calculated from its radius using $\mathbf{A=\pi r^2}$. Let us see how many significant figures the area has if the radius has only two—say, $\mathbf{r=1.2\ m}$. Then,

$$\mathbf{A=\pi r^2 = 3.1415927 \times (1.2\ m)^2 = 4.5238934\ m^2}$$

is what you would get using a calculator that has an eight-digit

output. But because the radius has only two significant figures, it limits the calculated quantity to two significant figures or

$$A=\pi r^2 = 3.1415927 \times (1.2\ \text{m})^2 = 4.5\ \text{m}^2$$

even though $\pi$ is good to at least eight digits.

**2. For addition and subtraction:** *The answer can contain no more decimal places than the least precise measurement.* Suppose that you buy 7.56-kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg. Then you drop off 6.052-kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$\begin{array}{r} 7.56\ \text{kg} \\ -6.052\ \text{kg} \\ \underline{+\ \ 13.7\ \text{kg}} \\ 15.208\ \text{kg} \end{array} \quad = 15.2\ \text{kg}$$

Next, we identify the least precise measurement: 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

## Significant Figures in this Text

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will

also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and more than three significant figures will be used. Finally, if a number is *exact*, such as the two in the formula for the circumference of a circle, $\mathbf{c=2\pi r}$, it does not affect the number of significant figures in a calculation.

## *Check Your Understanding 3*

**1:** Perform the following calculations and express your answer using the correct number of significant digits.

(a) A woman has two bags weighing 13.5 pounds and one bag with a weight of 10.2 pounds. What is the total weight of the bags?

(b) The force $\vec{F}$ on an object is equal to its mass $m$ multiplied by its acceleration $\vec{a}$. If a wagon with mass 55 kg accelerates at a rate of 0.0255 $m/s^2$, what is the force on the wagon? (The unit of force is called the newton, and it is expressed with the symbol N.)

PHET EXPLORATION: ESTIMATION

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PhET Interactive Simulation

***Figure 5.*** Estimation.

## Summary

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.

- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

## *Conceptual Questions*

**1:** What is the relationship between the accuracy and uncertainty of a measurement?

**2:** Prescriptions for vision correction are given in units called *diopters* (D). Determine the meaning of that unit. Obtain information (perhaps by calling an optometrist or performing an internet search) on the minimum uncertainty with which corrections in diopters are determined and the accuracy with which corrective lenses can be produced. Discuss the sources of uncertainties in both the prescription and accuracy in the manufacture of lenses.

## *Problems & Exercises*

**Express your answer to problems in this section to the correct number of significant figures and proper units.**

**1:** Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?

**2:** A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?

**3:** (a) A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads 90 km/h? (b) Convert this range to miles per hour. (1 km = 0.6214 mi)

**4:** An infant's pulse rate is measured to be 130 ± 5 beats/min. What is the percent uncertainty in this measurement?

**5:** (a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats does he or she have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?

**6:** A can contains 375 mL of soda. How much is left after 308 mL is removed?

**7:** State how many significant figures are proper in the results of the following calculations: (a) (106.7)(98.2)\(46.210)(1.01) (b) $(18.7)^2$ (c) $(1.60 \times 10^{-19})(3712)$.

**8:** (a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?

**9:** (a) If your speedometer has an uncertainty of 2.0 km/h at a speed of 90 km/h, what is the percent uncertainty? (b)

If it has the same percent uncertainty when it reads 60 km/h, what is the range of speeds you could be going?

**10:** (a) A person's blood pressure is measured to be 120 ± 2 mm Hg. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of 80 mm Hg?

**11:** A person measures his or her heart rate by counting the number of beats in 30 s. If 40 ± 1 beats are counted in 30.0 ± 0.5 s, what is the heart rate and its uncertainty in beats per minute?

**12:** What is the area of a circle 3.102 cm in diameter?

**13:** If a marathon runner averages 9.5 mi/h, how long does it take him or her to run a 26.22-mi marathon?

**14:** A marathon runner completes a 42.188-km course in 2 h, 30 min, and 12 s. There is an uncertainty of 25 m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?

**15:** The sides of a small rectangular box are measured to be 1.80 ± 0.01 cm, 2.05 ± 0.02 cm, and 3.1 ± 0.1 cm long. Calculate its volume and uncertainty in cubic centimetres.

**16:** When non-metric units were used in the United Kingdom, a unit of mass called the *pound-mass* (lbm) was employed, where 1 lbm = 0.4539 kg. (a) If there is an uncertainty of 0.0001 kg in the pound-mass unit, what is its

percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?

**17:** The length and width of a rectangular room are measured to be 3.955 ± 0.005 m and 3.050 ± 0.005 m. Calculate the area of the room and its uncertainty in square meters.

**18:** A car engine moves a piston with a circular cross section of 7.500 ± 0.002 cm diameter a distance of 3.250 ± 0.001 cm to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimetres? (b) Find the uncertainty in this volume.

## Glossary

**accuracy**
: the degree to which a measure value agrees with the correct value for that measurement

**method of adding percents**
: the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

**percent uncertainty**
: the ratio of the uncertainty of a measurement to the measure value, express as a percentage

**precision**
: the degree to which repeated measurements agree with each other

**significant figures**
: express the precision of a measuring tool used to measure a value

**uncertainty**
: a quantitative measure of how much your measured values deviate from a standard or expected value

## *Solutions*

**Check Your Understanding 1**

**1:** No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

**Check Your Understanding 2**

**1:** (a) 1; the zeros in this number are place keepers that indicate the decimal point

(b) 6; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant

(c) 1; the value $10^3$ signifies the decimal place, not the number of measured values

(d) 5; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant

(e) 4; any zeros located in between significant figures in a number are also significant

**Check Your Understanding 3**

**1:** (a) 37.2 pounds; Because the number of bags is an exact value, it is not considered in the significant figures.

(b) 1.4 N; Because the value 55 kg has only two significant

figures, the final value must also contain two significant figures.

**Problems & Exercises**

**1:** 2 kg

**3:** (a) 85.5 to 94.5 km/h (b) 53.1 to 58.7 mi/h

**5:** (a) 7.6 x $10^6$ beats (b) 7.57 x 10^7 beats (c) 7.57 x 10^7\ beats

**7.** (a) 3 b) 3 (c) 3

**9:** (a) 2.2 % (b) 59 to 61 km/h

**11:** 80 $\pm$ 3 beats/min

**13:** 2.8 h

**15:** 11.4 $\pm$ 0.5 $cm^3$

**17:** 12.06 $\pm$ 0.04 $m^2$

# 5. 1.5 Approximation

*Summary*

- Make reasonable approximations based on given data.

On many occasions, physicists, other scientists, and engineers need to make **approximations** or "guesstimates" for a particular quantity. What is the distance to a certain destination? What is the approximate density of a given item? About how large a current will there be in a circuit? Many approximate numbers are based on formulae in which the input quantities are known only to a limited accuracy. As you develop problem-solving skills (that can be applied to a variety of fields through a study of physics), you will also develop skills at approximating. You will develop these skills through thinking more quantitatively, and by being willing to take risks. As with any endeavor, experience helps, as well as familiarity with units. These approximations allow us to rule out certain scenarios or unrealistic numbers. Approximations also allow us to challenge others and guide us in our approaches to our scientific world. Let us do two examples to illustrate this concept.

## Example 1: Approximate the Height of a Building

Can you approximate the height of one of the buildings on your campus, or in your neighborhood? Let us make an approximation based upon the height of a person. In this example, we will calculate the height of a 39-story building.

**Strategy**

Think about the average height of an adult male. We can approximate the height of the building by scaling up from the height of a person.

**Solution**

Based on information in the example, we know there are 39 stories in the building. If we use the fact that the height of one story is approximately equal to about the length of two adult humans (each human is about 2-m tall), then we can estimate the total height of the building to be

$$\frac{\mathbf{2\ m}}{\mathbf{1\ person}} \times \frac{\mathbf{2\ person}}{\mathbf{1\ story}} \times \mathbf{39\ stories} = \mathbf{156\ m}$$

**Discussion**

You can use known quantities to determine an approximate measurement of unknown quantities. If your hand measures 10 cm across, how many hand

lengths equal the width of your desk? What other measurements can you approximate besides length?

## Example 2: Approximating Vast Numbers: a Trillion Dollars



***Figure 1.*** *A bank stack contains one-hundred $100 bills, and is worth $10,000. How many bank stacks make up a trillion dollars? (credit: Andrew Magill).*

The U.S. federal deficit in the 2008 fiscal year was a little greater than $10 trillion. Most of us do not have any

concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in $100 bills. If you made 100-bill stacks and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

**Strategy**

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped $100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

**Solution**

(1) Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is:

$$\begin{aligned} \textbf{volume of stack} &= \textbf{length} \times \textbf{width} \times \textbf{height}, \\ \textbf{volume of stack} &= \textbf{6 in.} \times \textbf{3 in.} \times \textbf{0.5 in.}, \\ \textbf{volume of stack} &= \textbf{9 in.}^{\textbf{3}}\textbf{.} \end{aligned}$$

(2) Calculate the number of stacks. Note that a trillion dollars is equal to $\$1 \times 10^{12}$, and a stack of one-hundred

$100 bills is equal to $10,000, or $\$1 \times 10^4$. The number of stacks you will have is:

$$\$1 \times 10^{12} \text{(a trillion dollars)} / \$1 \times 10^4 \text{ per stack} = 1 \times 10^8 \text{ stacks.}$$

(3) Calculate the area of a football field in square inches. The area of a football field is 100 yd × 50 yd, which gives 5000 $\text{yd}^2$ Because we are working in inches, we need to convert square yards to square inches:

$$\text{Area} = 5000 \text{ yd}^2 \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{12 \text{ in.}}{1 \text{ ft.}} \times \frac{12 \text{ in.}}{1 \text{ ft.}} = 6,480,000 \text{ in.}^2$$

$$\text{Area} \approx 6 \times 10^6 \text{ in.}^2$$

This conversion gives us $6 \times 10^6$ $\text{in.}^2$ for the area of the field. (Note that we are using only one significant figure in these calculations.)

(4) Calculate the total volume of the bills. The volume of all the $100-bill stacks is
9 $\text{in.}^3$ /stack $\times 10^8$ stacks=$9 \times 10^8$ $\text{in.}^3$.

(5) Calculate the height. To determine the height of the bills, use the equation:

$$\text{volume of bills} = \text{area of field} \times \text{height of money:}$$

$$\text{Height of money} = \frac{\text{volume of bills}}{\text{area of field}},$$

$$\text{Height of money} = \frac{9 \times 10^8 \text{ in.}^3}{6 \times 10^6 \text{ in.}^2} = 1.33 \times 10^2 \text{ in.},$$

$$\text{Height of money} \approx 1 \times 10^2 \text{ in.} = 100 \text{ in.}$$

The height of the money will be about 100 inches high. Converting this value to feet gives

$$100 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = 8.33 \text{ ft} \approx 8 \text{ ft.}$$

**Discussion**

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough "guesstimates" versus carefully calculated approximations?

### *Check Your Understanding*

**1:** Using mental math and your understanding of fundamental units, approximate the area of a regulation basketball court. Describe the process you used to arrive at your final approximation.

## Summary

Scientists often approximate the values of quantities to perform calculations and analyze system.

## *Problems & Exercises*

**1:** How many heartbeats are there in a lifetime?

**2:** A generation is about one-third of a lifetime. Approximately how many generations have passed since the year 0 AD?

**3:** How many times longer than the mean life of an extremely unstable atomic nucleus is the lifetime of a human? (Hint: The lifetime of an unstable atomic nucleus is on the order of $10^{-22}$ seconds.)

**4:** Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint: The mass of a hydrogen atom is on the order of $10^{-27}$ kg and the mass of a bacterium is on the order of $10^{-15}$ kg.)



***Figure 2.*** *This color-enhanced photo shows Salmonella typhimurium (red) attacking human cells. These bacteria are commonly known for causing foodborne illness. Can you estimate the number of atoms in each bacterium? (credit: Rocky Mountain Laboratories, NIAID, NIH).*

**5:** Approximately how many atoms thick is a cell membrane, assuming all atoms there average about twice the size of a hydrogen atom?

**6:** (a) What fraction of Earth's diameter is the greatest ocean depth? (b) The greatest mountain height?

**7:** (a) Calculate the number of cells in a hummingbird assuming the mass of an average cell is ten times the mass of a bacterium. (b) Making the same assumption, how many cells are there in a human?

**8:** Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?

## Glossary

**approximation**
an estimated value based on prior experience and reasoning

## *Solutions*

**Check Your Understanding**

**1:** An average male is about two meters tall. It would take approximately 15 men laid out end to

end to cover the length, and about 7 to cover the width. That gives an approximate area of 420 $m^2$.

**Problems & Exercises**

**1:** Sample answer: $2 \times 10^9$ heartbeats if an average human lifetime is taken to be about 70 years.

**3:** Sample answer: $2 \times 10^{31}$ if an average human lifetime is taken to be about 70 years.

**5:** Sample answer: 50 atoms

**7:** Sample answers: (a) $10^{12}$ cells/hummingbird (b) $10^{16}$ cells/human

# 6. 1.6 Graphing Introduction

Graphs are a simple and elegant way to express a lot of information.

The PHET group in Colorado have a large, and growing, number of simulations to help you with your math basics.

You can check them all out at

https://phet.colorado.edu/en/simulations/category/math/mathconcepts

There are two main ones you will need in this class.

Play around with this PHET simulation. It will help you see what equations look like.

https://phet.colorado.edu/en/simulation/graphing-lines

For help with analyzing linear graphs, check out

https://phet.colorado.edu/en/simulation/graphing-slope-intercept

PART III

# CHAPTER 2 ONE-DIMENSIONAL KINEMATICS



***Figure 1.*** *The motion of an American kestrel through the air can be described by the bird's displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons).*

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars*? *Where will a football land if it is thrown at a certain angle*? But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with **kinematics** which is

defined as the *study of motion without considering its causes*. The word "kinematics" comes from a Greek term meaning motion and is related to other English words such as "cinema" (movies) and "kinesiology" (the study of human motion). In one-dimensional kinematics we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In Chapter 3 Two-Dimensional Kinematics, we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion); for example, that of a car rounding a curve.

# 7. 2.1 Displacement

*Summary*

- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position and the path between the two.



***Figure 1.*** *These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia).*

## Position

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person

in an airplane, for example, we use the airplane, not the Earth, as the reference frame. Please see the figures below.

## Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as **displacement**. The word "displacement" implies that an object has moved, or has been displaced.

### DISPLACEMENT

Displacement is the *change in position* of an object:

$$\Delta x = x_f - x_0$$

where $\Delta x$ is displacement, $x_f$ is the final position, and $x_0$ is the initial position.

In this text the upper case Greek letter $\Delta$ (delta) always means "change in" whatever quantity follows it; thus, $\Delta x$ means *change in position*. Always solve for displacement by subtracting initial position $x_0$ from final $x_f$.

Note that the SI unit for displacement is the meter (m) (see Chapter 1 Physical Quantities and Units), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that

when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.

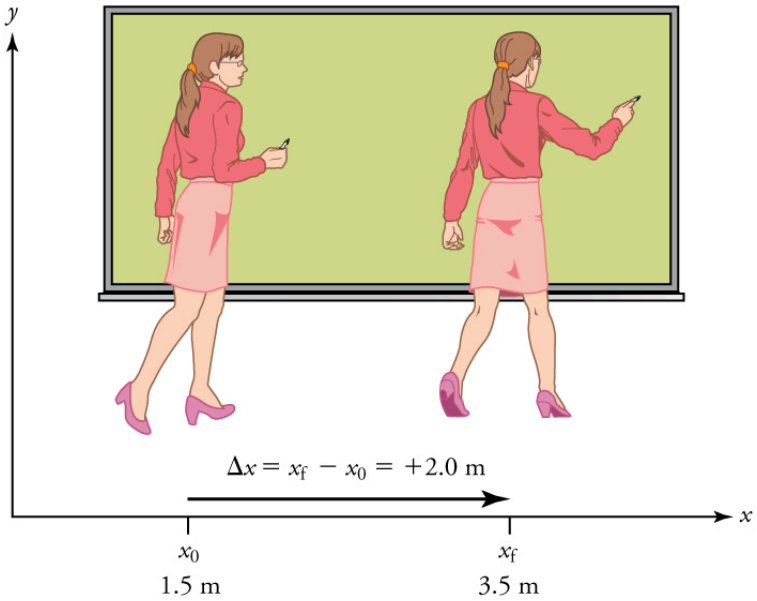



*Figure 2. A professor paces left and right while lecturing. Her position relative to Earth is given by x. The +2.0 m displacement of the professor relative to Earth is represented by an arrow pointing to the right.*

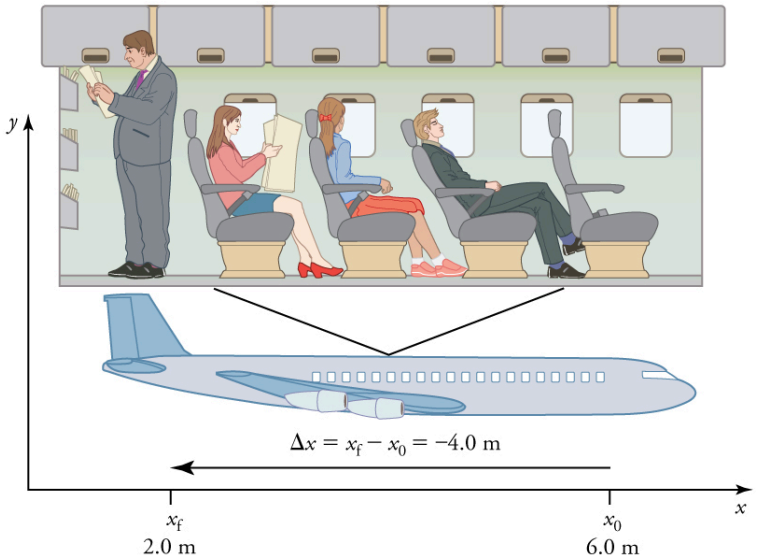



***Figure 3.*** *A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by* **x**. *The* **–4.0–m** *displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor shown above (he moves twice as far).*

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is $\mathbf{x_0 = 1.5\ m}$ and her final position is $\mathbf{x_f = 3.5\ m}$. Thus her displacement is

$$\mathbf{\Delta x = x_{final} - x_{initial} = x_f - x_o} = 3.5\text{ m} - 1.5\text{ m} = +2.0\text{ m}$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's

initial position is $x_0 = 6.0$ **m** and his final position is $x_f = 2.0$ **m**, so his displacement is

$$\Delta x = x_{final} - x_{initial} = x_f - x_0 = 2.0 \text{ m} - 6.0 \text{ m} = -4.0 \text{ m}$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

## Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

### MISCONCEPTION ALERT: DISTANCE TRAVELED VS. MAGNITUDE OF DISPLACEMENT

It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of

the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

## *Check Your Understanding 1*

**1:** A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?

## Section Summary

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.
- In symbols, displacement $\Delta x$ is defined to be

$$\Delta x = x_{\text{final}} - x_{\text{initial}} = x_f - x_0$$

where $x_0$ is the initial position and $x_f$ is the final position. In this text, the Greek letter $\Delta$ (delta) always means "change in" what ever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.

- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

### *Conceptual Questions*

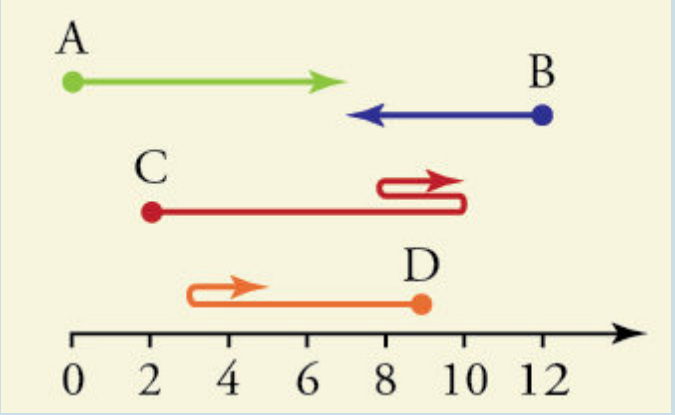
**1:** Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

**2:** Under what circumstances does distance traveled

equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

**3:** Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to 50 μm/s ($50 \times 10^{-6}$ m/s) have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

## *Problems & Exercises*




*Four objects travelling along a one dimensional path, with the distance axis labelled.*

**1:** Find the following for path A: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

**2:** Find the following for path B: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

**3:** Find the following for path C: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

**4:** Find the following for path D: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

## Glossary

**kinematics**
the study of motion without considering its causes

**position**
the location of an object at a particular time

**displacement**
the change in position of an object

**distance**
the magnitude of displacement between two positions

**distance traveled**
the total length of the path traveled between two positions

## *Solutions*

### Check Your Understanding 1

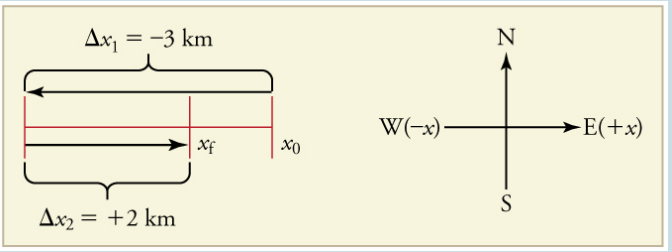



***Figure 5.***

**1:** (a) The rider's displacement is $\Delta x = x_{final} - x_{initial} = x_f - x_0$. The displacement is negative because we take east to be positive and west to be negative. Or you could just say "1 km to the West". Note that the drawing clearly showed that

West was chosen to be negative. (b) The distance traveled is 3 km + 2 km = 5 km. (c) The magnitude of the displacement is 1 km.

**Problems & Exercises**

**1:** (a) 7 m (b) 7 m (c) + 7 m

**2:** (a) 5 m (b) 5 m (c) − 5 m

**3:** This is badly drawn so the answers are debatable. Assuming it went from a position of 2 m to 10 then back to 8 and then back again to 10 m that gives a) distance of 12 m b) magnitude of the displacement as 8 m and c) a displacement of +8 m or 8 metres ot the right.

**4:** (a) 8 m (b) 4 m (c) − 4 m

# 8. 2.2 Vectors, Scalars, and Coordinate Systems

*Summary*

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.




***Figure 1.*** *The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the* ***x****-coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr).*

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

The direction of a vector in one-dimensional motion is given simply by a plus (+) or minus (−) sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the

magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a 20 °C temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a -20 °C temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

## Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet shown above, it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

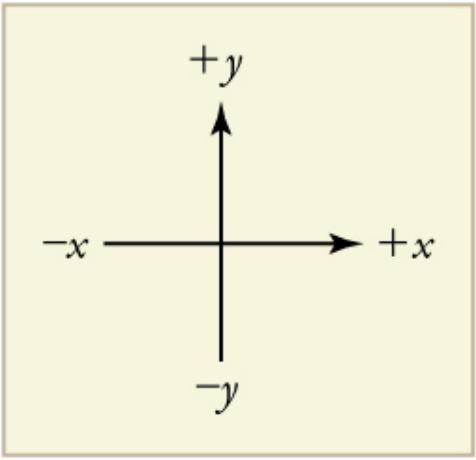



***Figure 2.*** *It is usually convenient to consider motion upward or to the right as positive* **(+)** *and motion downward or to the left as negative* **(−)**.

This are called the Cartesian Coordinates in honour of Rene Descartes who first proposed them in the 17th Century. He revolutionized mathematics at the time by combining geometry with algebra. A circle could be described in a equation. You can read more on them at https://en.wikipedia.org/wiki/Cartesian_coordinate_system.

Yes that is "I think, therefore I am" Rene Descartes.



*Portrait of Rene Descartes painted sometime between 1649-1700 by Frans Hals (Retrieved from Wikimedia Commons)*

Retrieved from https://en.wikipedia.org/wiki/Ren%C3%A9_Descartes#/media/File:Frans_Hals_-_Portret_van_Ren%C3%A9_Descartes.jpg

## *Check Your Understanding*

**1:** A person's speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

## Section Summary

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

### *Conceptual Questions*

**1:** A student writes, "A *bird that is diving for prey has a speed of* -10 m/s." What is wrong with the student's statement? What has the student actually described? Explain.

**2:** What is the speed of the bird in the previous question?

**3:** Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

**4:** A weather forecast states that the temperature is predicted to be -5 °C the following day. Is this temperature a vector or a scalar quantity? Explain.

## Glossary

**scalar**
a quantity that is described by magnitude, but not direction

**vector**
a quantity that is described by both magnitude and direction

## *Solutions*

**Check Your Understanding**

**1:** Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

**Conceptual Questions**

2. The speed is 10 m/s.

# 9. 2.3 Time, Velocity, and Speed

## *Summary*

- Explain the relationship between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial position, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.



***Figure 1.*** *The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr).*

There is more to motion than distance and displacement. Questions such as, “How long does a foot race take?” and “What was the runner’s speed?” cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

## Time

As discussed earlier, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple—**time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time $\Delta t$** is the difference between the ending time and beginning time,

$$\Delta t = t_f - t_0$$

where $\Delta t$ is the change in time or elapsed time, $t_f$ is the time at the end of the motion, and $t_0$ is the time at the beginning of the motion. (As usual, the delta symbol, $\Delta$, means the change in the quantity that follows it.)

Life is simpler if the beginning time $t_0$ is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0=0$, then $\Delta t = t_f \equiv t$.

In this text, for simplicity's sake,

- motion starts at time equal to zero $t_0=0$
- the symbol $t$ is used for elapsed time unless otherwise specified ( $\Delta t = t_f \equiv t$ )

## Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

### AVERAGE VELOCITY

Average velocity is *displacement (change in position) divided by the time of travel*,

$$\bar{v} = = v_{\text{average}} = \Delta x/\Delta t = (x_f - x_0)/(t_f - t_0)$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, $\Delta x$ is the change in position (or displacement), and $x_f$ and $x_0$ are the final and beginning positions at times $t_f$ and $t_0$, respectively. If the starting time $t_0$ is taken to be zero, then the average velocity is simply

$$\bar{v} = = v_{\text{average}} = \Delta x/\Delta t = (x_f - x_0)/t$$

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move –4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

$$v_{\text{average}} = \text{displacement} / \text{time} = (-4\text{ m}) / (5\text{ s}) = -0.8\text{ m/s}$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

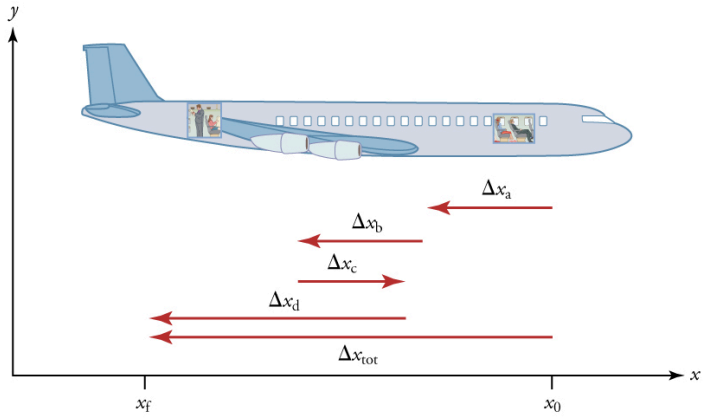



***Figure 2.*** *A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.*

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity or the velocity at a specific instant.* A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity v** is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, $\boldsymbol{v}$, at a precise instant $\boldsymbol{t}$ can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

# Speed

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

**Instantaneous speed** is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of –3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

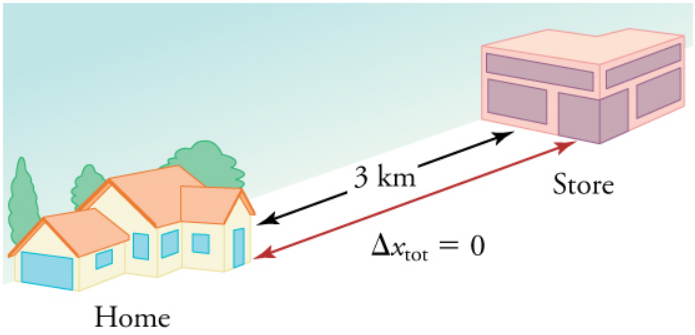



***Figure 3.*** *During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.*

**Graphing**

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. You might want to review the basics of graphing. Refer to a good math textbook or play around with the PHET simulation

https://phet.colorado.edu/en/simulation/graphing-slope-intercept

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*https://pressbooks.bccampus.ca/douglasphys1104summer2021/?p=87*

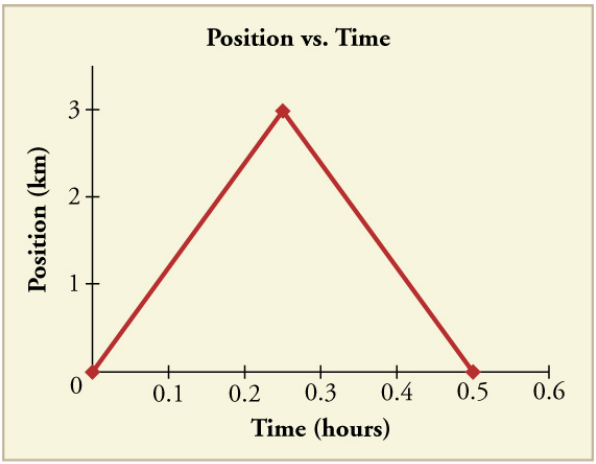
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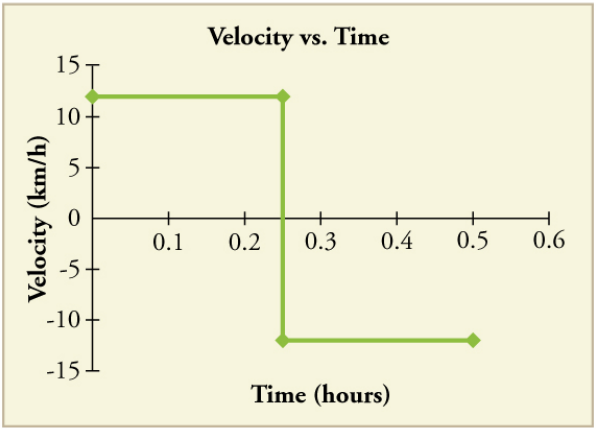
https://phet.colorado.edu/en/simulation/graphing-lines



*An interactive or media element has been excluded from this version of the text. You can view it online here:*
*https://pressbooks.bccampus.ca/douglasphys1104summer2021/?p=87*

For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in Figure 4 below. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)


Position vs. Time
Position (km)
3
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
Time (hours)


Velocity vs. Time
Velocity (km/h)
15
10
5
0
-5
-10
-15
0.1
0.2
0.3
0.4
0.5
0.6
Time (hours)

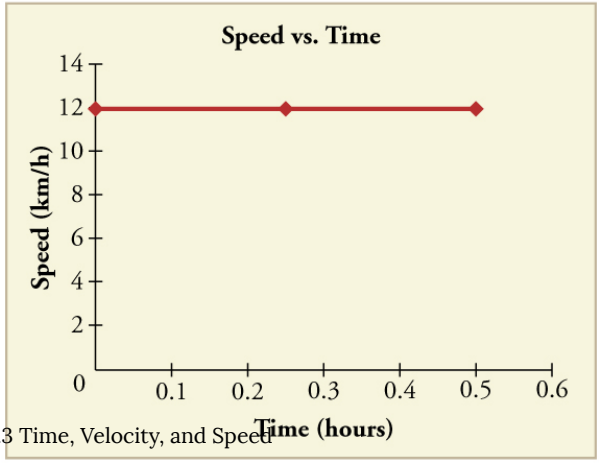

Speed vs. Time
Speed (km/h)
14
12
10
8
6
4
2
0
0.1
0.2
0.3
0.4
0.5
0.6
Time (hours)

***Figure 4.*** *Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.*

## MAKING CONNECTIONS: TAKE-HOME INVESTIGATION — GETTING A SENSE OF SPEED

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

### *Check Your Understanding*

**1:** A commuter train travels from Baltimore to Washington, DC, and back to Balitmore in 2 hours. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in miles per hour and then m/s?

## Section Summary

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is $\Delta t = t_f - t_0$ where $t_f$ is the final time and $t_0$ is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just $t$.
- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is

$$\bar{v} = = v_{\text{average}} = \Delta x/\Delta t = (x_f - x_0)/(t_f - t_0)$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity $v$ is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction

specified.

- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

## *Conceptual Questions*

**1:** Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

**2:** There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

**3:** Does a car's odometer measure position or displacement? Does its speedometer measure speed or velocity?

**4:** If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

**5:** How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

## *Problems & Exercises*

**1:** (a) Calculate Earth's average speed relative to the Sun. (b) What is its average velocity over a period of one year? Assume that it moves in a circular orbit with a radius of 1.50 x $10^{11}$ meters and it takes one year = 3.16 x $10^{7}$ seconds to complete the one orbit.

**2:** A helicopter blade spins at exactly 100 revolutions per minute in a circle. Its tip is 5.00 m from the centre of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference in metres per second. Remember that there are 60 seconds in one minute and that in one revolution the tip travels in a circle – a distance of 2 π r. (b) What is its average velocity over one revolution?

**3:** The North American and European continents are moving apart at a rate of about 3 cm/year. At this rate how many years will it take them to drift 500 km farther apart than they are at present?

**4:** Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

**5:** On May 26, 1934, a streamlined, stainless steel diesel

train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

**6:** Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by $3.84 \times 10^6$ m (1%)?

**7:** A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed in km/h? Remember that there are 60 minutes in one hour. (b) If the straight-line distance from her home to the university is 10.3 km in a direction 25.0° south of east, what was her average velocity? (c) If she returned home by the same path 8.00 hours after she left her home, what were her average speed and velocity for the entire trip?

**8:** The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance? This is one of the factors in your reaction time. It takes time for humans to respond to any stimulus.

**9:** Conversations with astronauts on the lunar surface

were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light ($3.00 \times 10^8$ m/s).

**10:** A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

**11:** The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit $5.30 \times 10^{-11}$ m in diameter. (a) If the average speed of the electron in this orbit is known to be $2.20 \times 10^6$ m/s, calculate the number of revolutions per second it makes about the nucleus. Hint: Find the distance travelled in one circular orbit first. (b) What is the electron's average velocity?

## Glossary

**average speed**
distance traveled divided by time during which motion occurs

**average velocity**
displacement divided by time over which displacement occurs

**instantaneous velocity**
velocity at a specific instant, or the average velocity over an infinitesimal time interval

**instantaneous speed**
magnitude of the instantaneous velocity

**time**
change, or the interval over which change occurs

**model**
simplified description that contains only those elements necessary to describe the physics of a physical situation

**elapsed time**
the difference between the ending time and beginning time

## *Solutions*

**Check Your Understanding**

**1:** (a) The average velocity of the train is zero because $\Delta x = 0$. The train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

speed = distance / time = 2 x 40 miles / (2 hours) = 40 miles/hour

1 mile = 1609 meters

1 hour = 3600 seconds

speed = (2 x 40 miles x 1609 m/mile) / (2 hours x 3600 s/ hour) = 18 m/s to two significant figures. You could argue this is a 1 significant figure question and give the answer of 20 m/s.

**Problems & Exercises**

**1:** (a) $2 \pi r / 3.16 \times 10^7$ s = 3.0 x $10^4$ m/s (b) 0 m/s because in one year the displacement is 0 m.

**2**. 52.3 m/s b) 0 as the displacement is zero.

**3:** 2 x $10^7$ years

**4**. 1 x $10^7$ years

**5:** 34.689 m/s = 124.88 km/h

6. 1 x $10^8$ years. One significant figure as 4 cm/year has only one significant figure.

**7:** (a) 40.0 km/h (b) 34.3 km/h , 25 $^o$ S of E. (c) average speed=3.00 km/h, average v = 0 m/s as the displacement is 0.

8: 6.1 x $10^{-2}$ s

9: 384,000 km Remember that you were asked for the distance between Earth and Moon, a one way trip, though the time given in the question was for the two way trip there and back again.

10: a) 15.0 m/ 2.50 s = 6.00 m/s forward then − 3.00 m / 1.75 s = − 1.71 m/s then 21.0 m / 5.20 s = 4.04 m/s Overall

displacement = ( + 15.0 − 3.00 + 21.0 m) / (2.50 + 1.75 + 5.20 s) = 3.49 m/s

**11:** (a) 6.61 x $10^{15}$ rev/s (b) 0 m/s

# 10. 2.4 Acceleration

*Summary*

- Define and distinguish instantaneous acceleration, average acceleration, and deceleration.
- Calculate acceleration given initial time, initial velocity, and final velocity.




***Figure 1.*** *A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr).*

In everyday conversation, to accelerate means to speed up. The

accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

## AVERAGE ACCELERATION

**Average Acceleration** is *the rate at which velocity changes*,

$\bar{a} = a_{\text{average}} = \Delta v/\Delta t = (v_f - v_0)/(t_f - t_0)$

where $\bar{a}$ is average acceleration, $a$ is velocity, and $t$ is time. (The bar over the $a$ means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are $\mathbf{m/s^2}$, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

## ACCELERATION AS A VECTOR

Acceleration is a vector in the same direction as the *change* in velocity, $\Delta \boldsymbol{v}$. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration**.




***Figure 2.*** *A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)Misconception Alert: Deceleration vs. Negative Acceleration.*

## MISCONCEPTION ALERT: DECELERATION VS. NEGATIVE ACCELERATION

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration *in the negative direction in the chosen coordinate system*. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider the car shown below here:

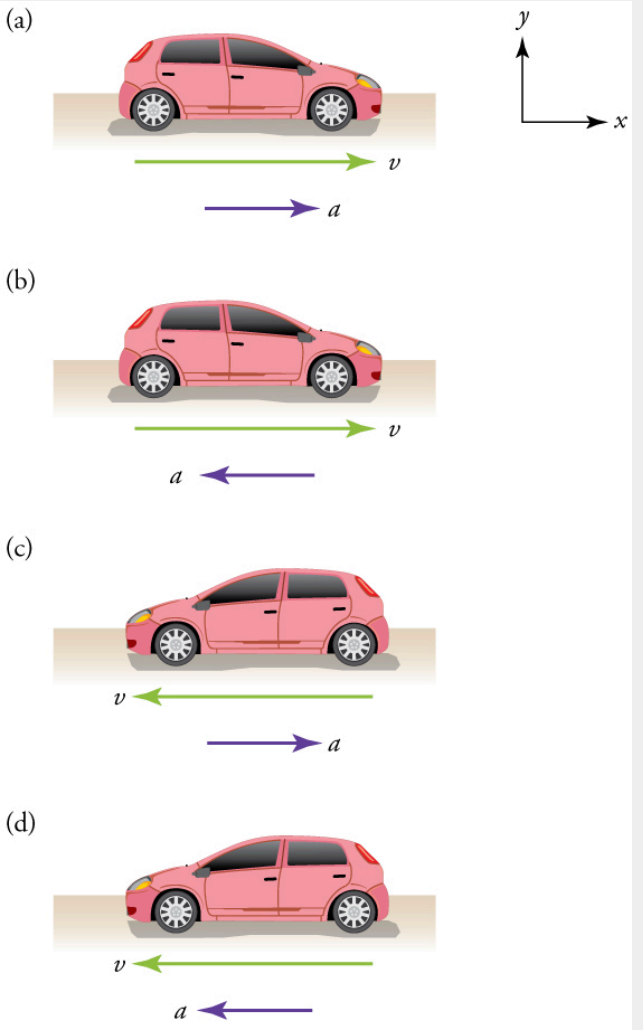



***Figure 3.*** *(a) This car is speeding up as it moves toward the right.*

*It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (not decelerating).*

## Example 1: Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?



***Figure 4.*** *(credit: Jon Sullivan, PD Photo.org).*

**Strategy**

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.

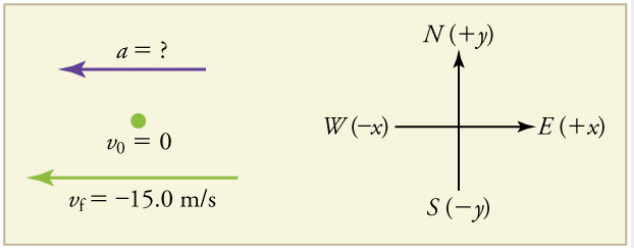



***Figure 5.***

We can solve this problem by identifying **Δv** and **Δt** from the given information and then calculating the average acceleration directly from the equation $\bar{a} =$ **=a average = Δv/Δt = (vf − vo)/ (tf − to)**

**Solution**

1. Identify the knowns. **$v_0$ = 0**, **$v_f$ = -15.0 m/s** (the negative sign indicates direction toward the west), **Δt = 1.80 s**.

2. Find the change in velocity. Since the horse is going from zero to -15.0 m/s, its change in velocity equals its final velocity: **Δv = $v_f$ = -15.0 m/s**.

3. Plug in the known values (**Δv** and **Δt**) and solve for the unknown $\bar{a}$.

$$\bar{a} = \text{a average} = \Delta v/\Delta t = (v_f - v_o)/(t_f - t_o)$$
$$= (-15.0 \text{ m/s} - 0 \text{ m/s}) / (1.80 \text{ s}) = -8.33 \text{ m/s}^2$$

**Discussion**

The negative sign for acceleration indicates that

acceleration is toward the west. An acceleration of 8.33 $m/s^2$ due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as 8.33 $m/s^2$. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

## Instantaneous Acceleration

**Instantaneous acceleration** a, or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity in the earlier section on speed and velocity —that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. Figure 6 below shows graphs of instantaneous acceleration versus time for two very different motions. In Figure 6 a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about 1.8 $m/s^2$). In Figure 6 b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of +3.0 $m/s^2$ and -2.0 $m/s^2$, respectively.

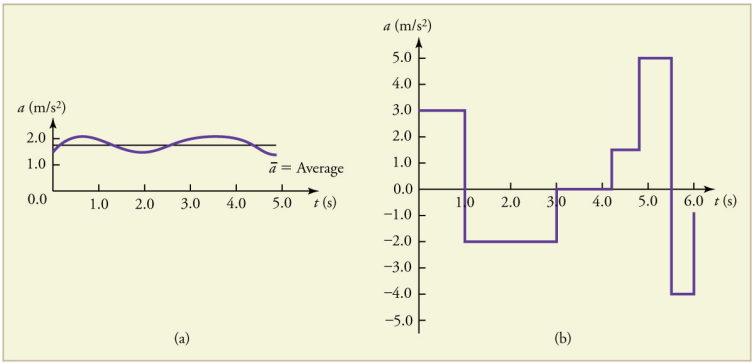



***Figure 6.*** *Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.*

The next several examples consider the motion of the subway train shown below in Figure 7. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.

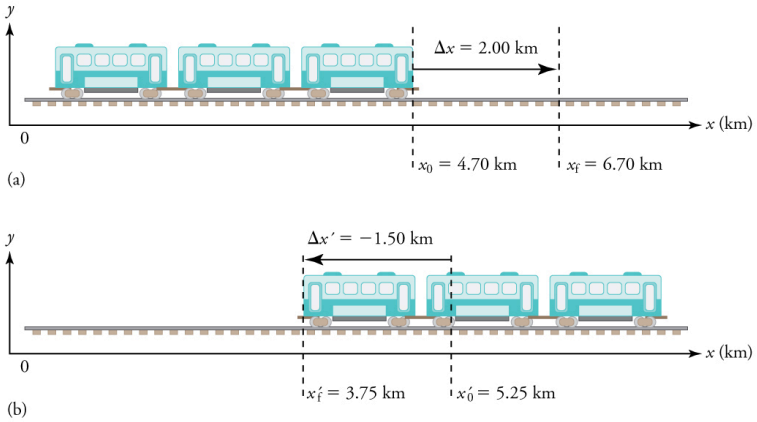



***Figure 7.*** *One-dimensional motion of a subway train. We have chosen the x-axis so that + means to the right and – means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from* $\mathbf{x_0}$ *to* $\mathbf{x_f}$*. Its displacement* $\mathbf{\Delta x}$ *is +2.0 km. (b) The train moves to the left from* $\mathbf{x'_0}$ *to* $\mathbf{x'_f}$*. Its displacement* $\mathbf{\Delta x'}$ *is* **–1.5 km***. (Note that the prime symbol (′) is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.).*

## Example 2: Calculating Displacement: A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of Figure 7 above?

**Strategy**

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_f - x_0$. This is straightforward since the initial and final positions are given.

**Solution**

1. Identify the knowns. In the figure we see that $x_f$ = **6.70 km** and $x_0$ = **4.70 km** for part (a), and $x'_f$ = **3.75 km** and $x'_0$ = **5.25 km** for part (b).

2. Solve for displacement in part (a).

$$\Delta x = x_f - x_o = 6.70 \text{ km} - 4.70 \text{ km} = +2.00 \text{ km}$$

3. Solve for displacement in part (b).

$$\Delta x = x'_f - x'_o = 3.75 \text{ km} - 5.25 \text{ km} = -1.50 \text{ km}$$

**Discussion**

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

## Example 3: Calculating Distance Traveled with Displacement: A Subway

## Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train?

**Strategy**

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in Example 2. Distance traveled is the total length of the path traveled between the two positions. In the case of the subway train, the distance traveled is the same as the distance between the initial and final positions of the train.

**Solution**

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.

2. The displacement for part (b) was −1.5 km. Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

**Discussion**

Distance is a scalar. It has magnitude but no sign to indicate direction.

## Example 4: Calculating Acceleration: A Subway Train Speeding Up

Suppose the subway train in Figure 7(a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

**Strategy**

It is worth it at this point to make a simple sketch:

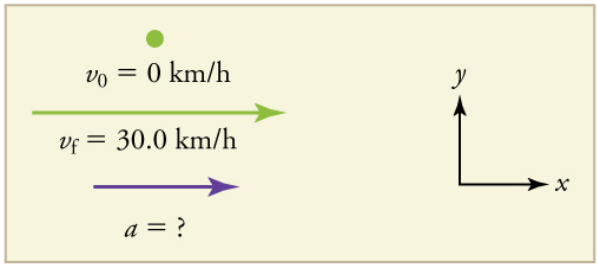



***Figure 8.***

This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

**Solution**

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_f = 30.0\ \text{km/h}$, and $\Delta t = 20.0\ \text{s}$.

2. Calculate $\Delta v$. Since the train starts from rest, its change in velocity is $\Delta v = +30.0\ \text{km/h}$, where the plus sign means velocity to the right.

3. Plug in known values and solve for the unknown,

$$\bar{a} = = a_{\text{average}} = \Delta v/\Delta t = (v_f - v_0)/(t_f - t_0)$$
$$= \frac{+30.0\ \text{km/h}}{20.0\ \text{s}}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds.

$$\text{average acceleration} = (30.0\ \text{km} \times (1000\ /\text{k}) - 0\ ) / (20.0\ \text{s}) = 0.417\ \text{m/s}^2$$

**Discussion**

The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

## Example 5: Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the subway train in Figure 7(a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

**Strategy**

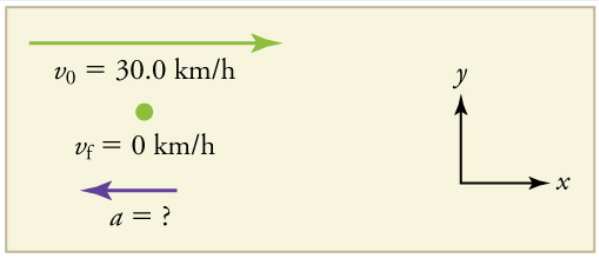



***Figure 9.***

In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

**Solution**

1. Identify the knowns. $\boldsymbol{v_0 = 30.0\ km/h}$,$\boldsymbol{v_f = 0\ km/h}$ (the train is stopped, so its velocity is 0), and $\boldsymbol{\Delta t = 8.00\ s}$.

2. Solve for the change in velocity, $\Delta v$.

$$\Delta v = v_f - v_0 = 0 - 30.0 \text{ km/h} = -30.0 \text{ km/h}$$

3. Plug in the knowns, $\Delta v$ and $\Delta t$, and solve for $\bar{a}$.

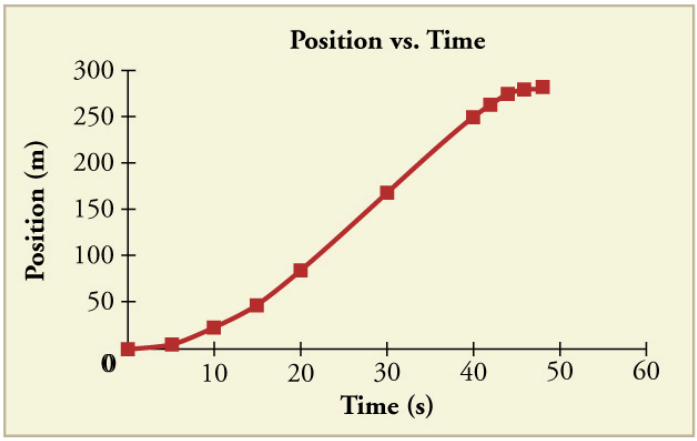
average acceleration = (- 30.0 km/h ) / ( 8.00 s)

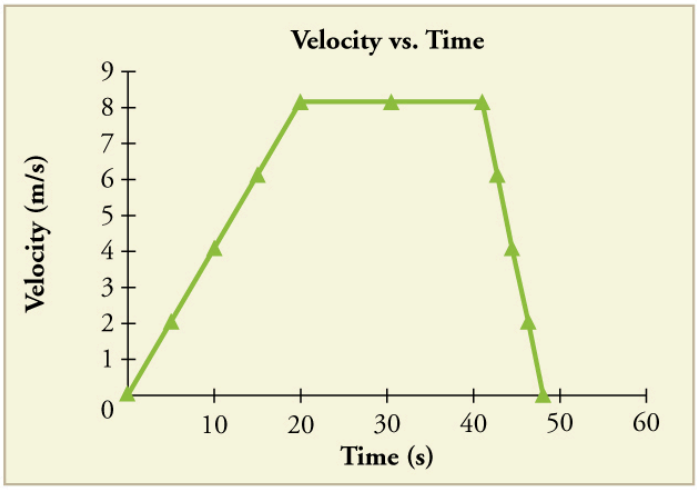
4. Convert the units to meters and seconds.

average acceleration = (- 30.0 x 1000 m/( 3600 s ) / ( 8.00 s) = – 1.04 m/s

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the *change* in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains are shown in Figure 10 below. (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)


Position vs. Time
Position (m)
300
250
200
150
100
50
0
10
20
30
40
50
60
Time (s)


Velocity vs. Time
Velocity (m/s)
9
8
7
6
5
4
3
2
1
0
10
20
30
40
50
60
Time (s)

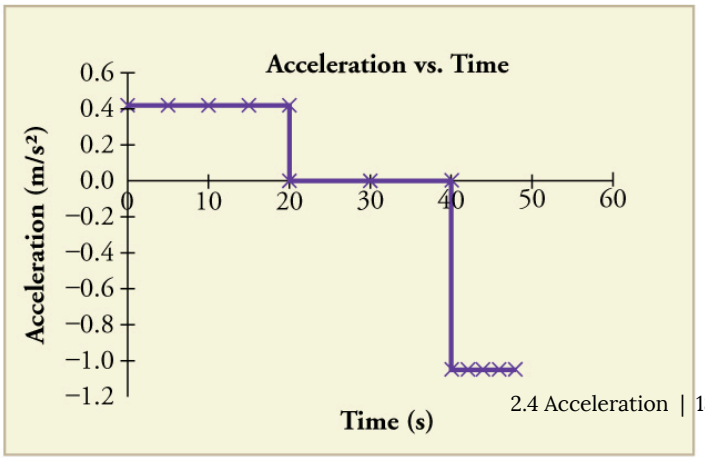

Acceleration vs. Time
Acceleration (m/s²)
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
0
10
20
30
40
50
60
Time (s)

***Figure 10.*** *(a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.*

## Example 6: Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of Example 2 and shown again below, if it takes 5.00 min to make its trip?

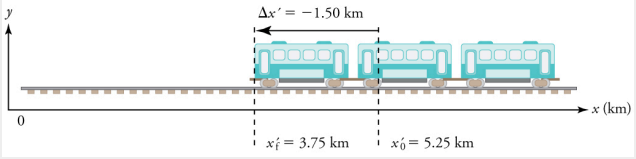



***Figure 11.***

**Strategy**

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

**Solution**

1. Identify the knowns. $\boldsymbol{x'_f = 3.75\ km}$, $\boldsymbol{x'_0 = 5.25\ km}$, $\boldsymbol{\Delta t = 5.00\ min}$.

2. Determine displacement, $\boldsymbol{\Delta x'}$. We found $\boldsymbol{\Delta x'}$ to be **-1.5 km**

3. Solve for average velocity.

$$\text{average } v = \boldsymbol{\Delta x' / \Delta t'} = (-1.50\ \text{km}) / (5.00\ \text{min})$$

4. Convert units.

average v = $\boldsymbol{\Delta x' / \Delta t'}$ = ( − 1.50 km ) / (5.00 min)
average v = ( − 1.50 x 1000 m ) / (5.00 min x ( 60 second / 1 minute) = − 5.00 m/s
or
average v = $\boldsymbol{\Delta x' / \Delta t'}$ = ( − 1.50 km ) / (5.00 min x (1 hour/60 minutes) = − 18.0 km/h

**Discussion**

The negative velocity indicates motion to the left.

## Example 7: Calculating Deceleration: The Subway Train

Finally, suppose the train in Figure 11 slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

**Strategy**

Once again, let's draw a sketch:

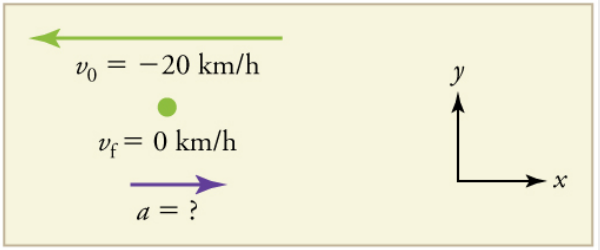



***Figure 12.***

As before, we must find the change in velocity and the change in time to calculate average acceleration.

**Solution**

1. Identify the knowns. $v_0$ **= -20 km/h**, $v_f$ **= 0 km/h**, $\Delta t$ **= 10.0 s**.

2. Calculate $\Delta v$. The change in velocity here is actually positive, since

$$\Delta v = v_f - v_o = 0 - (-20.0 \text{ km/h}) = +20.0 \text{ km/h}$$

3. Solve for $\bar{a}$.

$$\text{average } a = \Delta v / \Delta t = (+20.0 \text{ km/h}) / (10.0 \text{ s})$$

4. Convert units

$$\text{average } a = \Delta v / \Delta t = (+20.0 \text{ km/h}) / (10.0 \text{ s})$$

$$\text{average } a = \Delta v / \Delta t = (+20.0 \times 1000 \text{ m}/(3600 \text{ s}) / (10.0 \text{ s}) = +0.556 \text{ m/s}^2$$

**Discussion**

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the *change* in velocity, which is positive here. As in Example 5 this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

## Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative

acceleration as the slowing of an object. This was not the case in Example 7 where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in Figure 11 sped up by an acceleration to the left. In that case, both **v** and **a** are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

### *Check Your Understanding*

**1:** An airplane lands on a runway traveling east. Describe its acceleration.

### PHET EXPLORATIONS: MOVING MAN SIMULATION

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

https://phet.colorado.edu/en/simulation/moving-man



PhET Interactive Simulation

***Figure 13.*** *Moving Man*.

## Section Summary

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is
$$\bar{a} = = a_{\text{average}} = \Delta v/\Delta t = (v_f - v_0)/(t_f - t_0)$$
- The SI unit for acceleration is $m/s^2$.
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration $a$ is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

## *Conceptual Questions*

**1:** Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

**2:** Is it possible for velocity to be constant while acceleration is not zero? Explain.

**3:** Give an example in which velocity is zero yet acceleration is not.

**4:** If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

**5:** Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

## *Problems & Exercises*

**1:** A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

**2: Professional Application**

Dr. John Paul Stapp was U.S. Air Force officer who

studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of $g$ (9.80 m/s$^2$) by taking its ratio to the acceleration of gravity.

**3:** A commuter backs her car out of her garage with an acceleration of 1.40 m/s$^2$. (a) How long does it take her to reach a speed of 2.00 m/s assuming she started from rest? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?

**4:** Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s and in multiples of $g$ (9.80 m/s$^2$)?

## Glossary

**acceleration**
: the rate of change in velocity; the change in velocity over time

**average acceleration**
: the change in velocity divided by the time over which it changes

**instantaneous acceleration**
: acceleration at a specific point in time

**deceleration**
: acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

## *Solutions*

**Check Your Understanding**

**1:** If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

**Conceptual Questions**

4. Right Postive

5. Positive Negative

**Problems & Exercises**

**1:** 4.29 $m/s^2$

**2:** 56.4 $m/s^2$ or 5.76 g or 5.76 times the acceleration of gravity, decleration was 201 m/s2 or 20.6 g or 20.6 times the acceleration of gravity

**3:** (a) 1.43 s (b) -2.50 $m/s^2$

4: 108 $m/s^2$ or 11.1 g

# 11. 2.5 Graphical Analysis of One-Dimensional Motion

*Summary*

- Describe a straight-line graph in terms of its slope and y-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of displacement, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

PHET EXPLORATIONS: Simulations to help you understand the concept

Graphing Slope-Intercept




*Graphing Slope Intercept*.

Direct link: https://phet.colorado.edu/en/simulation/graphing-slope-intercept

## Graphing Straight Lines



**PhET Interactive Simulation**

*Graphing Straight Lines*.

Direct link: https://phet.colorado.edu/en/simulation/graphing-lines

## EQUATION GRAPHER

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y=bx$) to see how they add to generate the polynomial curve. Please note that this uses Flash so it might not run on all computers.




***Figure 14.*** *Equation Grapher*.

## Slopes and General Relationships

The two PHET simulations mentioned in the previous chapter are embedded here, so you can play with them now.

*An interactive or media element has been excluded from this version of the text. You can view it online here:*
*https://pressbooks.bccampus.ca/douglasphys1104summer2021/?p=128*

*An interactive or media element has been excluded from this version of the text. You can view it online here:*

https://pressbooks.bccampus.ca/douglasphys1104summer2021/?p=128

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an **independent variable** and the vertical axis a **dependent variable**. If we call the horizontal axis the ***x***-axis and the vertical axis the ***y***-axis, as in Figure 1 a straight-line graph has the general form

$$y = mx + b$$

Here ***m*** is the **slope,** defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter ***b*** is used for the ***y*-intercept**, which is the point at which the line crosses the vertical axis.

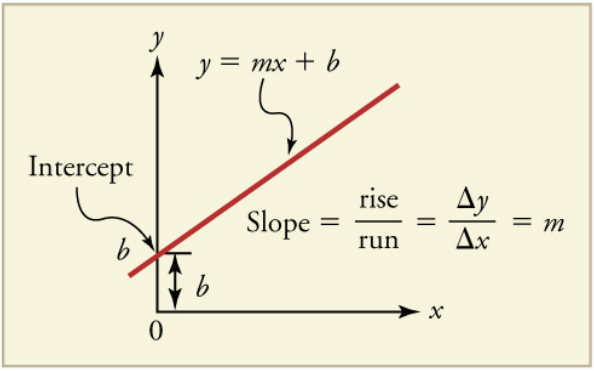



***Figure 1.*** *A straight-line graph. The equation for a straight line is* ***y*** *=* ***mx*** *+* ***b.***

# Graph of Displacement vs. Time ($a$ = 0, so $v$ is constant)

Time is usually an independent variable that other quantities, such as displacement, depend upon. A graph of displacement versus time would, thus, have **x** on the vertical axis and **t** on the horizontal axis. Figure 2 shown below is just such a straight-line graph. It shows a graph of displacement versus time for a jet-powered car on a very flat dry lake bed in Nevada.

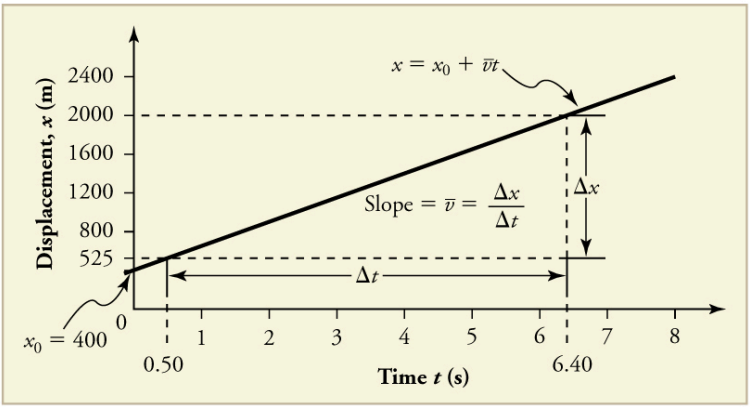



***Figure 2.*** *Graph of displacement versus time for a jet-powered car on the Bonneville Salt Flats.*

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity or $\bar{v}$ and the intercept is displacement at time zero—that is, $x_0$. Substituting these symbols into **y = mx + b** gives

$$x = \bar{v}t + x_0$$

or

$$x = x_0 + \bar{v}t.$$

Thus a graph of displacement versus time gives a general relationship among displacement, velocity, and time, as well as giving detailed numerical information about a specific situation.

## THE SLOPE OF X VS. T

The slope of the graph of displacement **x** vs. time **t** is velocity **v**.

slope = rise / run = $\Delta x / \Delta t = v_{average}$ or v bar (by definition)

Notice that this equation is the same as that derived algebraically from other motion equations the earlier section.

From the figure we can see that the car has a displacement of 25 m at 0.50 s and 2000 m at 6.40 s. Its displacement at other times can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

## Example 1: Determining Average Velocity from a Graph of Displacement

## versus Time: Jet Car

Find the average velocity of the car whose position is graphed above in Figure 2.

**Strategy**

The slope of a graph of ***x*** vs. ***t*** is average velocity, since slope equals rise over run. In this case, rise = change in displacement and run = change in time, so that

$$\text{slope} = \text{rise} / \text{run} = \Delta x / \Delta t = v_{\text{average}}$$

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

**Solution**

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)

2. Substitute the ***x*** and ***t*** values of the chosen points into the equation. Remember in calculating change (**Δ**) we always use final value minus initial value.

$$\text{slope} = \text{rise} / \text{run} = \Delta x / \Delta t = v_{\text{average}} = (2000 \text{ m} - 525 \text{ m}) / (6.4 \text{ s} - 0.50 \text{ s})$$

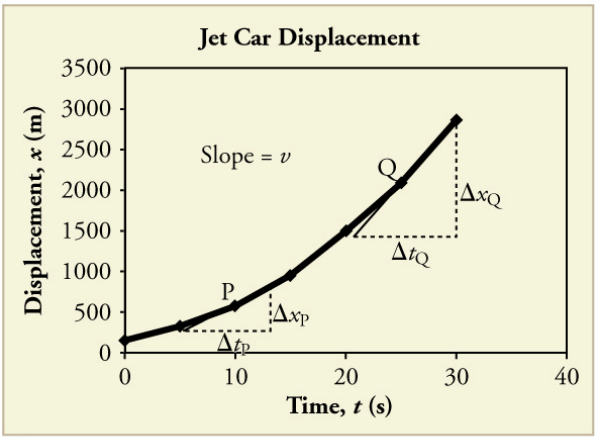
$$\text{yielding the average velocity} = 250 \text{ m/s}$$

**Discussion**

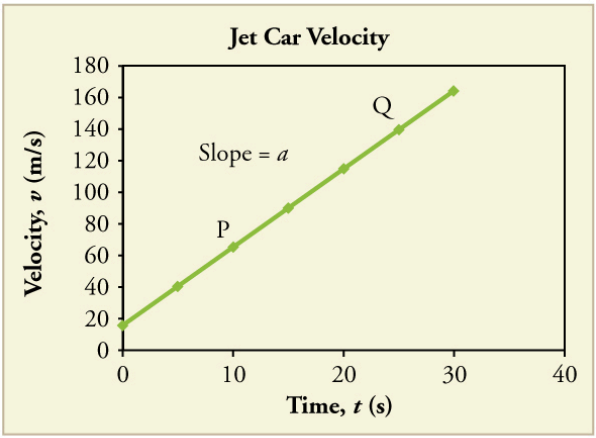
This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

## Graphs of Motion when $\alpha$ is constant but $a \neq o$

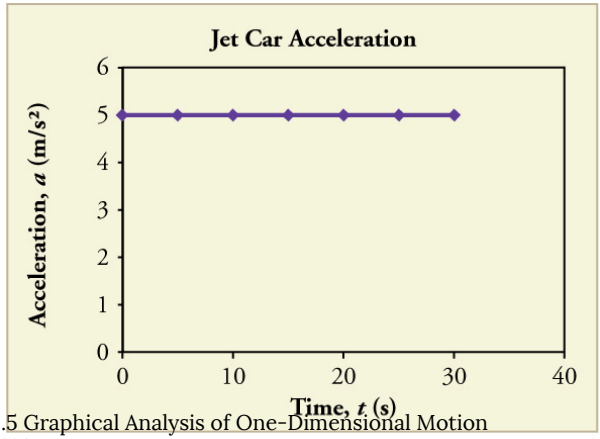
The graphs in Figure 3 below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the displacement and velocity are initially 200 m and 15 m/s, respectively.


Jet Car Displacement
Displacement, x (m)
3500
3000
2500
2000
1500
1000
500
0
Slope = v
Q
P
$\Delta x_Q$
$\Delta t_Q$
$\Delta x_P$
$\Delta t_P$
0
10
20
30
40
Time, t (s)

(a)


Jet Car Velocity
Velocity, v (m/s)
180
160
140
120
100
80
60
40
20
0
Slope = a
Q
P
0
10
20
30
40
Time, t (s)

(b)


Jet Car Acceleration
Acceleration, a (m/s²)
6
5
4
3
2
1
0
0
10
20
30
40
Time, t (s)

(c)

***Figure 3.*** *Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an* $x$ *vs.* $t$ *graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the* $v$ *vs.* $t$ *graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of* $5.0\ m/s^2$ *over the time interval plotted.*



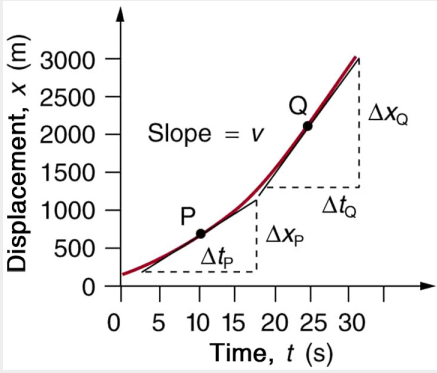
***Figure 4.*** *A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr).*

The graph of displacement versus time in Figure 3 (a) shown above is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a displacement-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown above for two points in Figure 3 (a). If this is done at every point on the curve and the values are plotted against time, then the graph of

velocity versus time shown in Figure 3 (b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in Figure 3(c).

## Example 2: Determining Instantaneous Velocity from the Slope at a Point: Jet Car

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the **x** vs. **t** graph in the graph below.




| t (s) | x (m) |
|---|---|
| 0 | 200 |
| 5 | 338 |
| 10 | 600 |
| 15 | 988 |
| 20 | 1500 |
| 25 | 2138 |
| 30 | 2900 |

***Figure 5.*** *The slope of an* **x** *vs.***t** *graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.*

### Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This

principle is illustrated in the figure shown above where Q is the point at **t=25 s**.

**Solution**

1. Find the tangent line to the curve at **t = 25 s**.

2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, $v$.

$$v_Q = \Delta x_Q / \Delta t_Q = (3120 \text{ m} - 1300 \text{ m}) / (32 \text{ s} - 19 \text{ s})$$

$$\text{Thus } v_Q = (1820 \text{ m}) / (13 \text{ s}) = 140 \text{ m/s}$$

**Discussion**

This is the value given in this figure's table for $v$ at **t = 25 s**. The value of 140 m/s for $v_Q$ is plotted in Figure 5(b) . The entire graph of $v$ vs. $t$ can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a $v$ vs. $t$ graph, rise = change in velocity $\Delta v$ and run = change in time $\Delta t$.

## THE SLOPE OF V VS. T

The slope of a graph of velocity $v$ vs. time $t$ is acceleration $a$.

**slope = average acceleration = $\Delta v / \Delta t$**

Since the velocity versus time graph in Figure 3 (b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in Figure 3(c)

Additional general information can be obtained from Figure 6 and the expression for a straight line, $y = mx + b$.

In this case, the vertical axis $y$ is **V**, the intercept $b$ is $v_0$, the slope $m$ is $a$, and the horizontal axis $x$ is $t$. Substituting these symbols yields

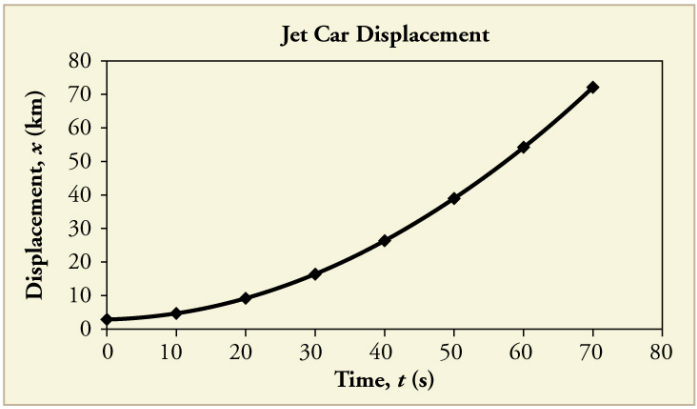
$$v = a t + v_0 \quad \text{or often written} \quad v = v_0 + a t$$

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in earlier sections.

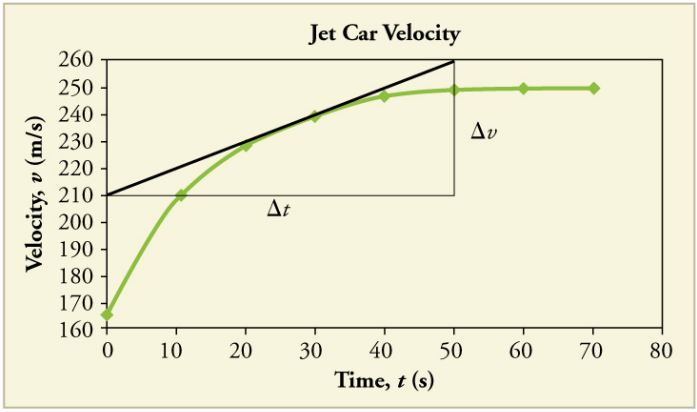
It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

## Graphs of Motion Where Acceleration is Not Constant

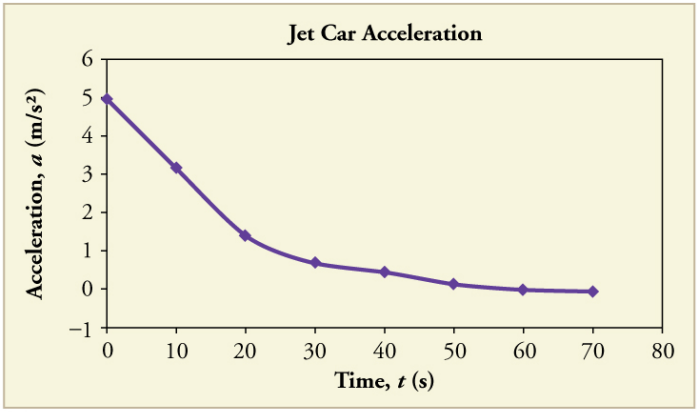
Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in Figure 6 below. Time again starts at zero, and the initial displacement and velocity are 2900 m and 165 m/s, respectively. (These were the final displacement and velocity of the car in the motion graphed in Figure 3. Acceleration gradually decreases from 5.0 $m/s^2$ to zero when the car hits 250 m/s. The slope of the $\boldsymbol{x}$ vs. $\boldsymbol{t}$ graph increases until $\boldsymbol{t = 55\ s}$, after which time the slope is constant. Similarly, velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.


Jet Car Displacement
Displacement, x (km)
0
10
20
30
40
50
60
70
80
Time, t (s)

(a)


Jet Car Velocity
Velocity, v (m/s)
160
170
180
190
200
210
220
230
240
250
260
Δv
Δt
0
10
20
30
40
50
60
70
80
Time, t (s)

(b)


Jet Car Acceleration
Acceleration, a (m/s²)
−1
0
1
2
3
4
5
6
0
10
20
30
40
50
60
70
80
Time, t (s)

(c)

***Figure 6.*** *Graphs of motion of a jet-powered car as it reaches its top velocity. This motion begins where the motion in Figure 3 ends. (a) The slope of this graph is velocity; it is plotted in the next graph. (b) The velocity gradually approaches its top value. The slope of this graph is acceleration; it is plotted in the final graph. (c) Acceleration gradually declines to zero when velocity becomes constant.*

## Example 3: Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the ***v*** vs. ***t*** graph in Figure 6(b).

**Strategy**

The slope of the curve at **t = 25 s** is equal to the slope of the line tangent at that point, as illustrated in Figure 6(b)

**Solution**

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, ***a***.

$$\text{slope} = \Delta v / \Delta t = (260 \text{ m/s} - 210 \text{ m/s}) / (51 - 1.0 \text{ s}) = 1.0 \text{ m/s}^2$$

**Discussion:** Note that this value for **a** is consistent with the value plotted in Figure 6 (c) at **t = 25 s**.

A graph of displacement versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time

can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

### *Check Your Understanding*

**1:** A graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b)What would a graph of the ship's acceleration look like?




***Figure 7.***

## Section Summary

- Graphs of motion can be used to analyze motion.

- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement $x$ vs. time $t$ is velocity $v$.
- The slope of a graph of velocity $v$ vs. time $t$ graph is acceleration $a$.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

## *Conceptual Questions*

**1:** (a) Explain how you can use the graph of position versus time in Figure 8 below o describe the change in velocity over time. Identify (b) the time ($t_a$, $t_b$, $t_c$, $t_d$, or $t_e$) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.

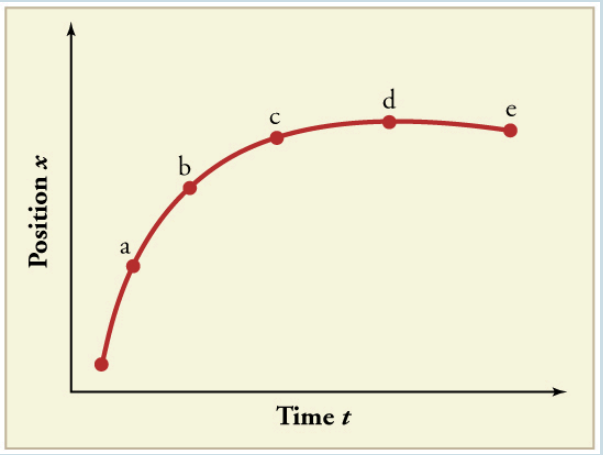



***Figure 8.***

**2:** (a) Sketch a graph of velocity versus time corresponding to the graph of displacement versus time given in Figure 9 below (b) Identify the time or times ($t_a$, $t_b$, $t_c$, etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?

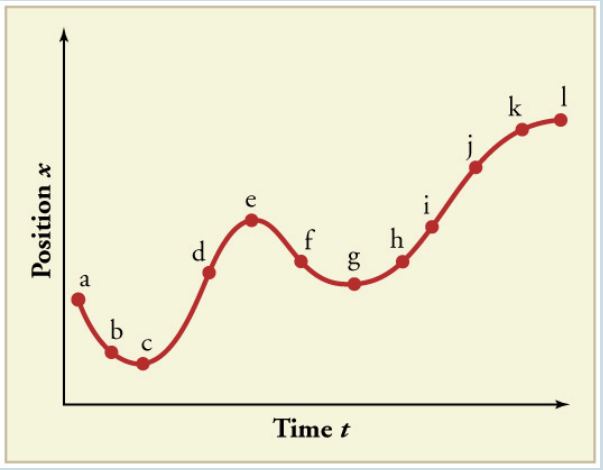



***Figure 9.***

**3:** (a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in Figure 10 below. (b) Based on the graph, how does acceleration change over time?

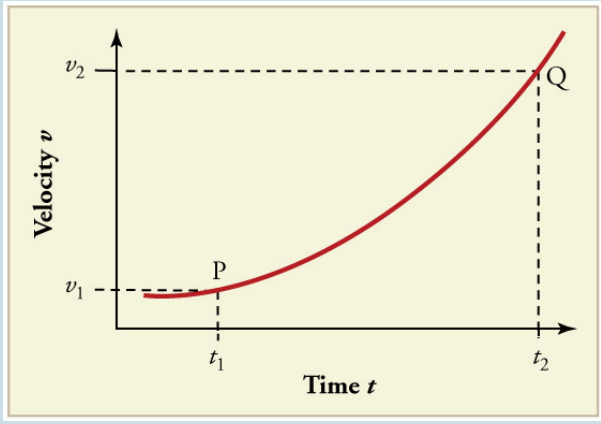



***Figure 10.***

**4:** (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in Figure 11 below. (b) Identify the time or times ( $t_a$, $t_b$, $t_c$, etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?

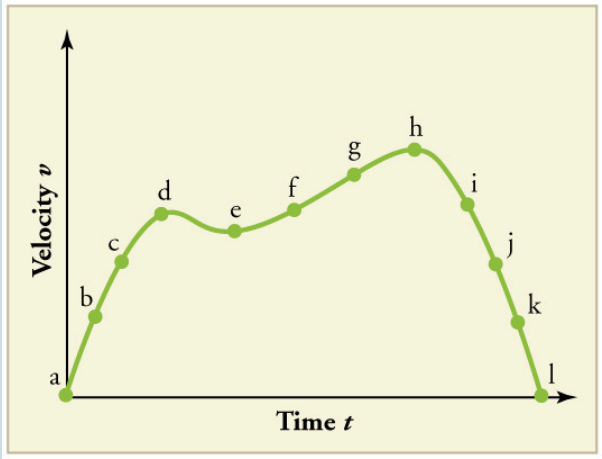



***Figure 11.***

**5:** Consider the velocity vs. time graph of a person in an elevator shown in Figure 12. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion you have used earlier in this chapter for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.

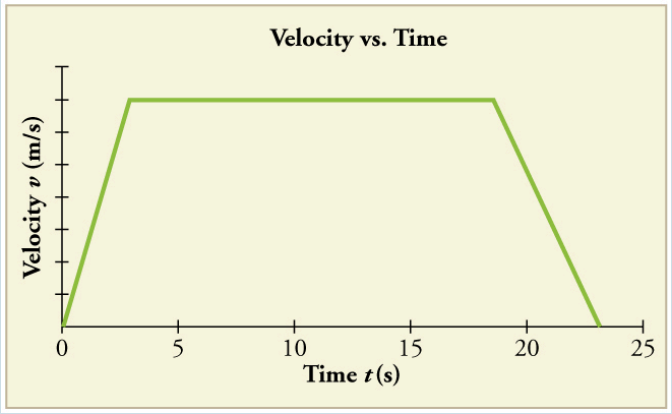



***Figure 12.***

**6:** A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

## *Problems & Exercises*

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

**1:** (a) By taking the slope of the curve in Figure 13 below,

verify that the velocity of the jet car is 115 m/s at t = 20 s.
(b) By taking the slope of the curve at any point in Figure 14. verify that the jet car's acceleration is 5.0 $m/s^2$.

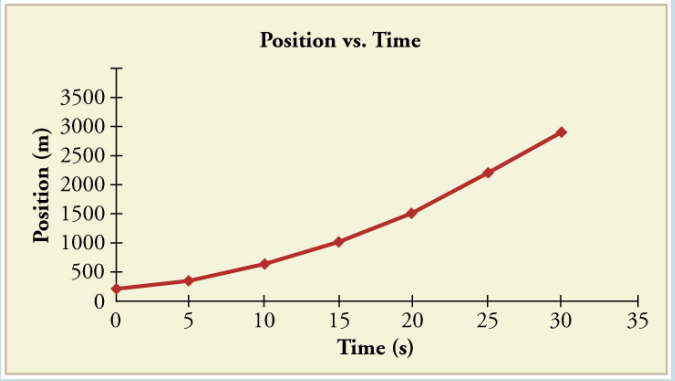



***Figure 13.***

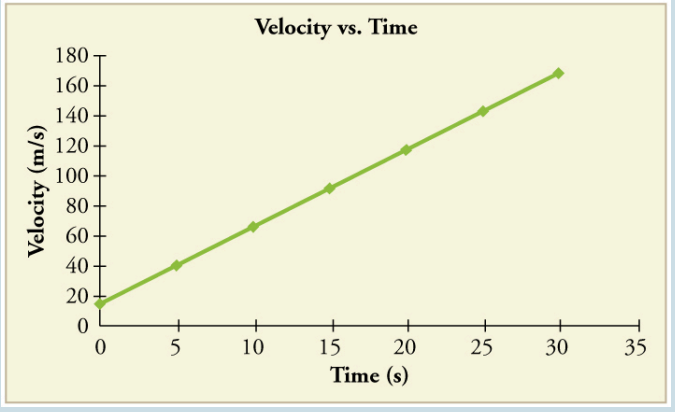



***Figure 14.***

**2:** Using approximate values, calculate the slope of the curve in Figure 15 below to verify that the velocity at t = 10.0 s is 0.208 m/s. Assume all values are known to 3 significant figures.

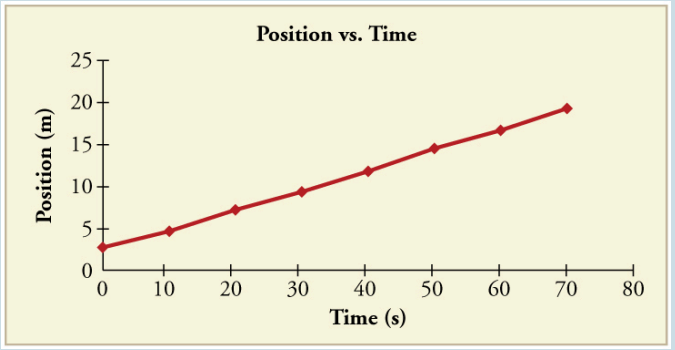



***Figure 15.***

**3:** Using approximate values, calculate the slope of the curve in Figure 15 above to verify that the velocity at t = 30.0 s is 0.238 m/s. Assume all values are known to 3 significant figures.

**4:** By taking the slope of the curve in Figure 16 below, verify that the acceleration is 3.2 $m/s^2$ at t = 10 s.

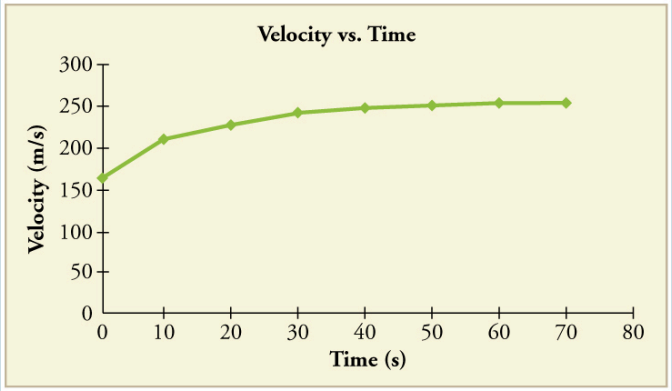



***Figure 16.***

**5:** Construct the displacement graph for the subway shuttle train as shown in the previous section, Chapter 2.7 Figure 7(a). Your graph should show the position of the train, in kilometres, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.

**6:** (a) Take the slope of the curve in Figure 17 below to find the jogger's velocity at t = 2.5 s. (b) Repeat at 7.5 s. These values must be consistent with the graph in Figure 18, also below.

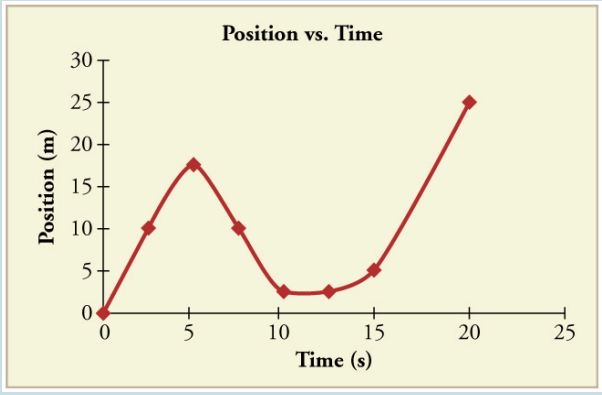



***Figure 17.***

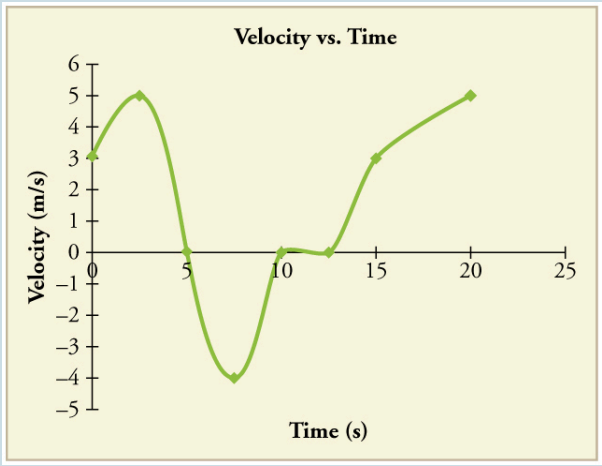



***Figure 18.***

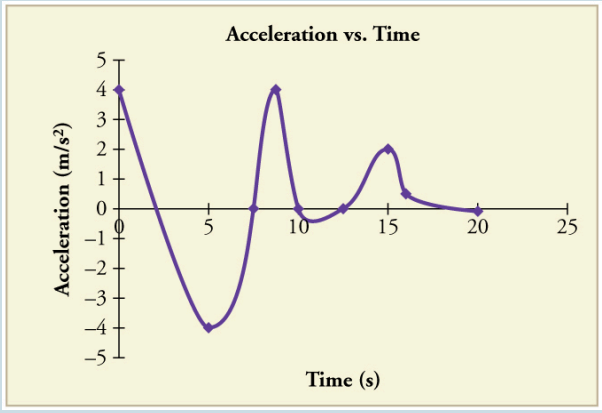



***Figure 19.***

**7:** A graph of $v(t)$ is shown for a world-class track sprinter in a 100-m race is shown below in Figure 20. What is the runner's (a) average velocity for the first 4 s? (b) instantaneous velocity at t = 5 s? (c) average acceleration between 0 and 4 s? d) acceleration at t = 5 s? (e) time for the race?

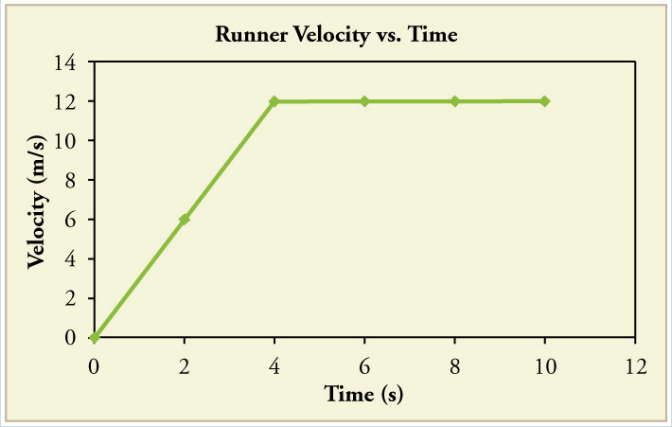



***Figure 20.***

**8:** Figure 21 below shows the displacement graph for a particle for 5 s. Draw the corresponding velocity and acceleration graphs.

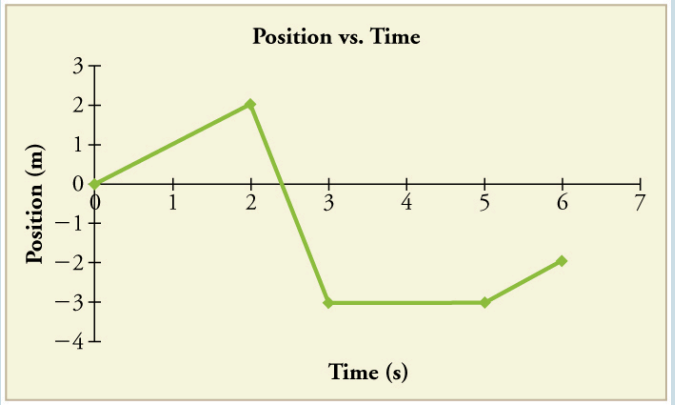



***Figure 21.***

## Glossary

**independent variable**
: the variable that the dependent variable is measured with respect to; usually plotted along the $x$-axis

**dependent variable**
: the variable that is being measured; usually plotted along the $y$-axis

**slope**
: the difference in $y$-value (the rise) divided by the difference in $x$-value (the run) of two points on a straight line

**y-intercept**
: the $y$-value when $x = 0$, or when the graph crosses the $y$-axis

## *Solutions*

**Check Your Understanding**

**1:** (a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

(b) A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.

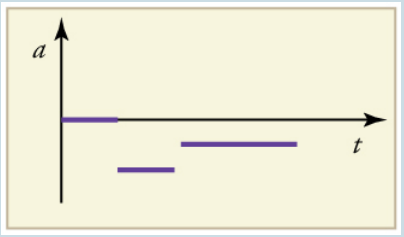



***Figure 22.***

**Concept Questions**

1: (b) $t_a$ 1(c) $t_d$ 1(d) $t_e$

2: (b) $t_a$ and or $t_d$ 1(c) $t_c$ $t_e$ $t_g$ 1d) $t_a$ $t_b$ $t_f$

4: (c) $t_d$ $t_e$ $t_h$

**Problems & Exercises**

**1:** (a) 115 m/s (b) 5.0 $m/s^2$

**3:** v = (11.7 – 6.95) x $10^3$ m / ( 40.0 x – 20.0 s) = 238 m/s

**5:**

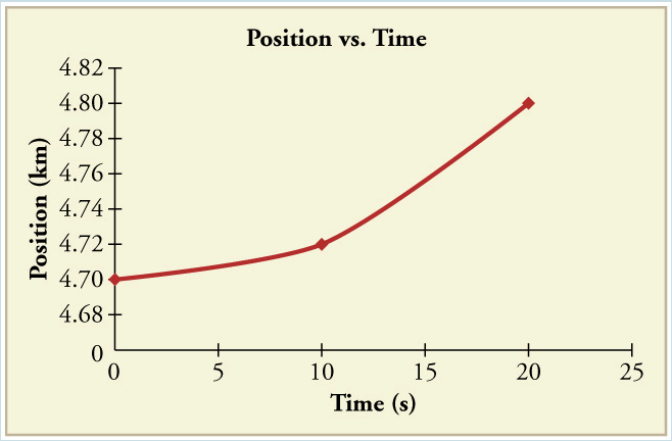



***Figure 23.***

**6:** The graphs are hard to read. About a) $v_{2.5}$ = (18 m / 5 s) = + 3.6 m/s b) $v_{7.5}$ = (2 – 18 m) / (10 – 5 s) = – 2.8 m/s Those values are consistent with the graphs.

**7:** (a) 6 m/s (b) 12 m/s c) 3 $m/s^2$ d) 0 $m/s^2$ e) 10 s

**8:** From t = 0 to t = 2 seconds, average velocity = 1 m/s and the acceleration is zero.

From t = 2 to t = 3 seconds the average velocity is -5 m/s and the acceleration is zero.

From t = 3 to t = 5.0 seconds the average velocity = 0 m/s and the acceleration = 0 m/s.

From t = 5 to t = 6 seconds, the average velocity is + 1 m/s. It is constant in that time and the acceleration is zero.

The acceleration is zero for all the straight line segments.

There was an infinitely large acceleration in the very short time it took the velocities to change.

# 12. 2.6 Motion Equations for Constant Acceleration in One Dimension

## *Summary*

- Calculate displacement of an object moving at a constant velocity, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.



***Figure 1.*** *Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr).*

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

## Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_f - t_0$, taking $t_0 = 0$ means

that $\Delta t = t_f$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, $x_0$ *is the initial position* and $v_0$ *is the initial velocity*. We put no subscripts on the final values. That is, $t$ *is the final time*, $x$ *is the final position*, and $v$ *is the final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

$$\left.\begin{aligned} \Delta t &= t \\ \Delta x &= x - x_0 \\ \Delta v &= v - v_0 \end{aligned}\right\}$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

$$\bar{a} = a = \textbf{constant},$$

so we use the symbol $a$ for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration *is* constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

## SOLVING FOR DISPLACEMENT ($\Delta x$) AND FINAL POSITION ($x$) FROM AVERAGE VELOCITY WHEN ACCELERATION ($a$) IS CONSTANT

To get our first two new equations, we start with the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t}.$$

Substituting the simplified notation for $\Delta x$ and $\Delta t$ yields

$$\bar{v} = \frac{x - x_0}{t}.$$

Solving for $x$ yields

$$x = x_0 + \bar{v}t,$$

where the average velocity is

$$\bar{v} = \frac{v_0 + v}{2} \text{ (constant } a\text{)}.$$

The equation $\bar{v} = \frac{v_0 + v}{2}$ reflects the fact that, when acceleration is constant, $v$ is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your

average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0 + v}{2}$ to check this, we see that

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30\ \text{km/h} + 60\ \text{km/h}}{2} = 45\ \text{km/h},$$

which seems logical.

## Example 1: Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

**Strategy**

Draw a sketch.

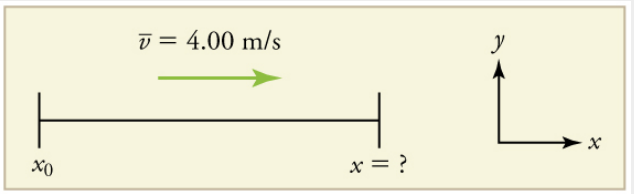



***Figure 2.***

The final position **x** is given by the equation

$$x = x_0 + \bar{v}t.$$

To find **x**, we identify the values of $x_0$, $\bar{v}$, and **t** *from the statement of the problem and substitute them into the equation.*

**Solution**

1. Identify the knowns. $\bar{v} = 4.00\ \text{m/s}$, $\Delta t = 2.00$ **min**, and $x_0 = 0$ **m**.

2. Enter the known values into the equation.

$$x = x_0 + \bar{v}t = 0 + (4.00\ \text{m/s})(120\ \text{s}) = 480\ \text{m}$$

**Discussion**

Velocity and final position are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.

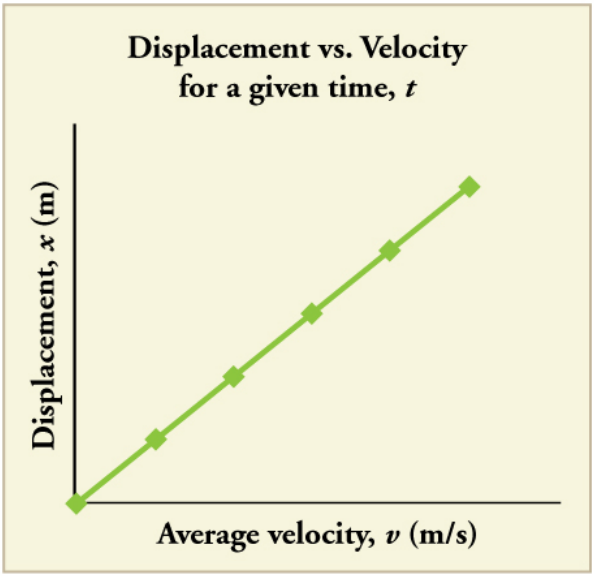



***Figure 3.*** *There is a linear relationship between displacement and average velocity. For a given time* **t**, *an object moving twice as fast as another object will move twice as far as the other object.*

## SOLVING FOR FINAL VELOCITY

We can derive another useful equation by manipulating the definition of acceleration.

$$a = \Delta v / \Delta t$$

Substituting the simplified notation for $\Delta v$ and $\Delta t$ gives us

$$a = \Delta v / \Delta t = (v - v_0) / t \quad \textbf{for a constant acceleration}$$

Solving for $v$ yields $v = v_0 + a t$

## Example 2: Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at 1.50 $m/s^2$ for 40.0 s. What is its final velocity?

**Strategy**

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.

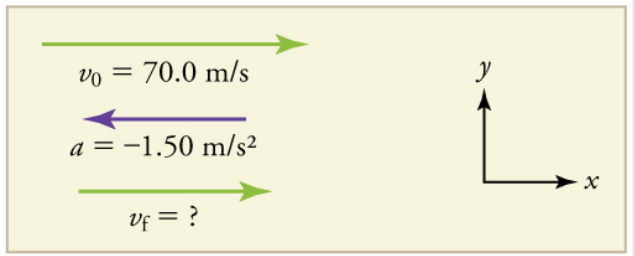



***Figure 4.***

**Solution**

1. Identify the knowns. $v_0 = 70.0$ **m/s**, $a = -1.50$ **m/s**$^2$, **t = 40.0 s**.

2. Identify the unknown. In this case, it is final velocity, $v_f$.

3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + at$.

4. Plug in the known values and solve.

$v = v_0 + a\,t = (70.0\text{ m/s}) + (-1.50\text{ m/s}^2)(40.0\text{ s}) = 10.0\text{ m/s}$

**Discussion**

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.

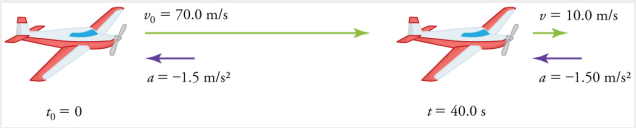



***Figure 5.*** *The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.*

In addition to being useful in problem solving, the equation $\boldsymbol{v = v_0 + at}$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity ($\boldsymbol{v = v_0}$), as expected (i.e., velocity is constant)
- if **a** is negative when the initial velocity is positive, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

## MAKING CONNECTIONS: REAL WORLD CONNECTION



***Figure 6.*** *The Space Shuttle Endeavor blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr).*

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified–short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

## SOLVING FOR FINAL POSITION WHEN VELOCITY IS NOT CONSTANT ( a ≠ o )

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

$$v = v_0 + at$$

Adding $v_0$ to each side of this equation and dividing by 2 gives

$$v + v_0 = v_0 + at + v_0$$

$$(1/2)(v + v_0) / t = v_0 + (1/2)\, a\, t$$

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}at$$

Since $\frac{v_0 + v}{2} = \bar{v}$ for constant acceleration, then

$$\bar{v} = v_0 + \frac{1}{2}at.$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $\Delta x = x - x_0$

$$(1/2)(v + v_0) / t = \Delta x / t = (x - x_0) / t = v_0 + (1/2)\, a\, t$$

yielding $$x = x_0 + v_0 t + 1/2\, a\, t^2$$

## Example 3: Calculating Displacement of an Accelerating Object: Dragsters

Dragsters can achieve average accelerations of 26.0 $m/s^2$. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?




***Figure 7.*** *U.S. Army Top Fuel pilot Tony "The Sarge" Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.).*

**Strategy**

Draw a sketch.

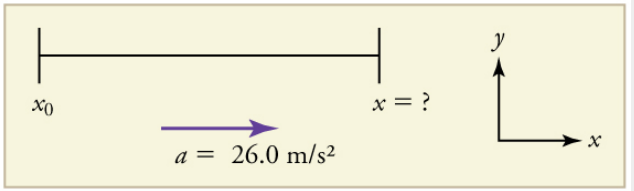



***Figure 8.***

We are asked to find displacement, which is $x$ if we take $x_0$ to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0 t + \frac{1}{2} a t^2$ once we identify $v_0$, $a$, and $t$ from the statement of the problem.

**Solution**

1. Identify the knowns. Starting from rest means that $v_0 = 0$, $a$ is given as $26.0\ \text{m/s}^2$ and $t$ is given as $5.56\ \text{s}$.

2. Plug the known values into the equation to solve for the unknown $x$:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

Since the initial position and velocity are both zero, this simplifies to

$$x = \frac{1}{2} a t^2$$

Substituting the identified values of $a$ and $t$ gives

$$x = \frac{1}{2}(26.0\ \text{m/s}^2)(5.56\ \text{s})^2 = 402\ \text{m}$$

**Discussion**

If we convert 402 m to miles, we find that the distance

covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0 t + \frac{1}{2} a t^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In Example 3 above, the dragster covers only one fourth of the total distance in the first half of the elapsed time
- if acceleration is zero, then the initial velocity equals average velocity $(v_0 = \bar{v})$ and $x = x_0 + v_0 t + \frac{1}{2} a t^2$ becomes $x = x_0 + v_0 t$

## SOLVING FOR FINAL VELOCITY WHEN VELOCITY IS NOT CONSTANT ( $a \neq 0$ )

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 + at$ for $t$, we get $t = (v - v_0) / a$

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v}t$, we get

$$v^2 = v_0^2 + 2a(x - x_0) \quad \text{or} \quad v^2 = v_0^2 + 2a\Delta x$$

## Example 4: Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in Example 3 without using information about time.

**Strategy**

Draw a sketch.

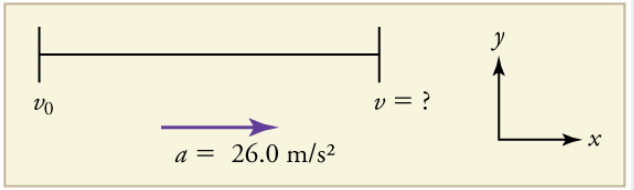



***Figure 9.***

The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

**Solution**

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402\ \text{m}$ (this was the answer in Example 3). Finally, the average acceleration was given to be $a = 26.0\ \text{m/s}^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for $v$.

$$v^2 = 0^2 + 2(26.0\ \text{m/s}^2)(402\ \text{m})$$

Thus $v^2 = 2.09 \times 10^4\ \text{m}^2/\text{s}^2$

To get $v$, we take the square root:

$$v = \sqrt{2.09 \times 10^4\ \text{m}^2/\text{s}^2} = 145\ \text{m/s}$$

.

**Discussion**

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts

- For a fixed deceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to stop. (This is why we have reduced speed zones near schools.)

## Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

SUMMARY OF KINEMATIC EQUATIONS(CONSTANT *a*)

$$x = x_0 + \bar{v}t$$
$$\bar{v} = \frac{v_0 + v}{2}$$
$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

## Example 5: Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of 7.00 $m/s^2$, whereas on wet concrete it can decelerate at only 5.00 $m/s^2$. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

**Strategy**

Draw a sketch.

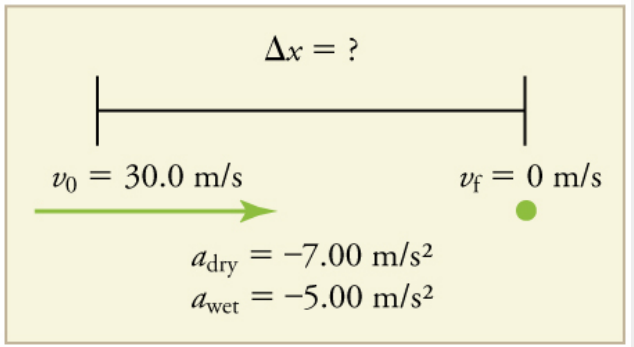



***Figure 10.***

In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

**Solution for (a)**

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0\ \text{m/s}$; $v=0$; $a = -7.00\ \text{m/s}^2$ ( $a$ is negative because it is in a direction opposite to velocity). We take $x_0$ to be $0$. We are looking for displacement $\Delta x$, or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is $v^2 = v_0^2 + 2a(x - x_0)$

This equation is best because it includes only one unknown, $x$. We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for $x$, but they require us to know the stopping time, $t$, which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for $(x - x_0)$

$$(x - x_0) = ( v^2 - v_0^2 ) / 2a$$

4. Enter known values $(x - 0) = ( 0^2 - (30.0\ \text{m/s})^2 ) / ( 2\ x - 7.00\ \text{m/s}^2)$

Thus, x=64.3 m on dry concrete.

**Solution for (b)**

This part can be solved in exactly the same manner as

Part A. The only difference is that the deceleration is **-5.00 m/s$^2$**. The result is $x_{wet}$ = 90.0 m on wet concrete.

**Solution for (c)**

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that Formula does not parse = 30.0 m/s ;$t_{reaction}$ =0.500 s; $a_{reaction}$ = 0 m/s$^2$. We take $x_o$ to be 0. We are looking for $x_{reaction}$.

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$ works well because the only unknown value is **x** , which is what we want to solve for.

3. Plug in the knowns to solve the equation.

$$x = 0 + (30.0\ m/s)(0.500\ s) = 15.0\ m$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

4. Add the displacement during the reaction time to the displacement when braking.

$$x_{braking} + x_{reaction} = x_{total}$$

(a) 64.3 m + 15.0 m = 79.3 m when dry

(b) 90.0 m + 15.0 m = 105 m when wet

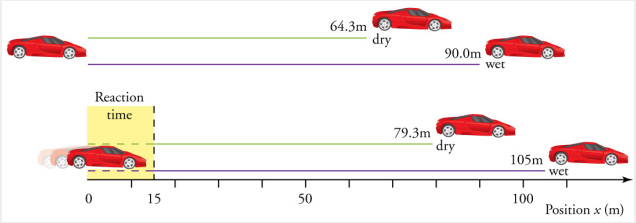



***Figure 11.*** *The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.*

**Discussion**

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

## Example 6: Calculating Time: A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at 2.00 $m/s^2$, how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

**Strategy**

Draw a sketch.

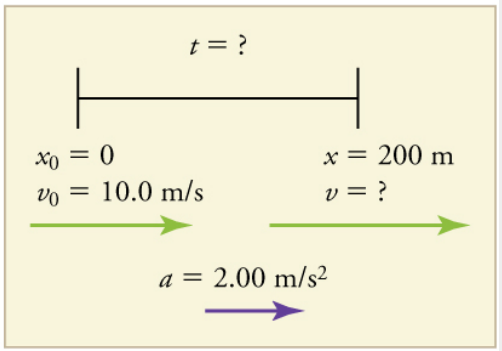



***Figure 12.***

We are asked to solve for the time **t**. As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown, **t**).

**Solution**

1. Identify the knowns and what we want to solve for. We know that $v_0 = 10\ \text{m/s}$; $a = 2.00\ \text{m/s}^2$; and $x = 200\ \text{m}$.

2. We need to solve for $t$. Choose the best equation. $x = x_0 + v_0 t + \frac{1}{2}at^2$ works best because the only unknown in the equation is the variable $t$ for which we need to solve.

3. We will need to rearrange the equation to solve for $t$. In this case, it will be easier to plug in the knowns first.

$$200\ \text{m} = 0\ \text{m} + (10.0\ \text{m/s})t + \frac{1}{2}(2.00\ \text{m/s}^2)t^2$$

4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking $t = t\ \text{s}$, where $t$ is the magnitude of time and s is the unit. Doing so leaves

$$200 = 10t + t^2.$$

5. Use the quadratic formula to solve for $t$.

(a) Rearrange the equation to get 0 on one side of the equation.

$$t^2 + 10t - 200 = 0$$

This is a quadratic equation of the form

$$at^2 + bt + c = 0,$$

where the constants are $a = 1.00$, $b = 10.0$, and $c = -200$.

(b) Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This yields two solutions for **t**, which are

**t** = 10.0 and -20.0

In this case, then, the time is **t = t** in seconds, or

**t** = 10.0 s and -20.0 s

A negative value for time is unreasonable, since it would mean that the event happened 20 s before the motion began. We can discard that solution. Thus,

**t=10.0 s**

**Discussion**

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. The next section called Problem-Solving Basics discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

## MAKING CONNECTIONS: TAKE-HOME EXPERIMENT–BREAKING NEWS

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v/\Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

## *Check Your Understanding*

**1:** A manned rocket accelerates at a rate of 20 $m/s^2$ during launch. How long does it take the rocket to reach a velocity of 400 m/s?

## Section Summary

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,

$$\left.\begin{aligned} \Delta t &= t \\ \Delta x &= x - x_0 \\ \Delta v &= v - v_0 \end{aligned}\right\}$$

- The following kinematic equations for motion with constant $a$ are useful:

$$x = x_0 + \bar{v}t$$
$$\bar{v} = \frac{v_0 + v}{2}$$
$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

- In vertical motion, $y$ is substituted for $x$.

### *Problems & Exercises*

**1:** An Olympic-class sprinter starts a race with an acceleration of 4.50 m/s$^2$. (a) What is her speed 2.40 s

later? (b) Sketch a graph of her position vs. time for this period.

**2:** A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is $2.10 \times 10^4$ m/s$^2$, and 1.85 ms (1 ms =$10^{-3}$ s) elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

**3:** A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of $6.20 \times 10^5$ m/s$^2$ for $8.10 \times 10^{-4}$ s. What is its muzzle velocity (that is, its final velocity)?

**4:** (a) A light-rail commuter train accelerates at a rate of 1.35 m/s$^2$. How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of 1.65 m/s$^2$. How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in m/s$^2$?

**5:** While entering a freeway, a car accelerates from rest at a rate of 2.40 m/s$^2$ for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

**6:** At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of 2.00 $m/s^2$. (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?

**7: Professional Application:**

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

**8:** In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes $3.33 \times 10^{-2}$ s, calculate the distance over which the puck accelerates.

**9:** A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

**10:** Freight trains can produce only relatively small accelerations and decelerations. (a) What is the final velocity of a freight train that accelerates at a rate of 0.0500 $m/s^2$ for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of 0.550 $m/s^2$, how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

**11:** A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.

**12:** A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of 0.350 $m/s^2$, how far will it travel before becoming airborne? (b) How long does this take?

**13: Professional Application:**

A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in $m/s^2$ and in multiples of $g$ ($g$=9.80 $m/s^2$). (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of $g$?

**14:** An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?

**15:** In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-

threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

**16:** Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m. This means the acceleration was g = 9.80 m/$s^2$ down. (b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

**17:** An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of 0.150 m/$s^2$ as it goes through. The station is 210 m long. (a) How long is the nose of the train in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is 130 m long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?

**18:** Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in Example 3 and Example 4. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? *Hint*: Consider whether the

assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

**19:** A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of 0.500 $m/s^2$ for 7.00 s. (a) What is his final velocity? (b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save? (c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

**20:** In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of 183.58 mi/h. The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

**21:** (a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s.

Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

## *Solutions*

**Check Your Understanding**

**1:** To answer this, choose an equation that allows you to solve for time $\boldsymbol{t}$, given only $\boldsymbol{a}$, $\boldsymbol{v_0}$, and $\boldsymbol{v}$.

$$\boldsymbol{v = v_0 + at}$$

Rearrange to solve for $\boldsymbol{t}$.

$$\boldsymbol{t = \frac{v - v_0}{a} = \frac{400\ \text{m/s} - 0\ \text{m/s}}{20\ \text{m/s}^2} = 20\ \text{s}}$$

**Problems & Exercises**

**1:** (a) $\boldsymbol{10.8\ \text{m/s}}$

(b)

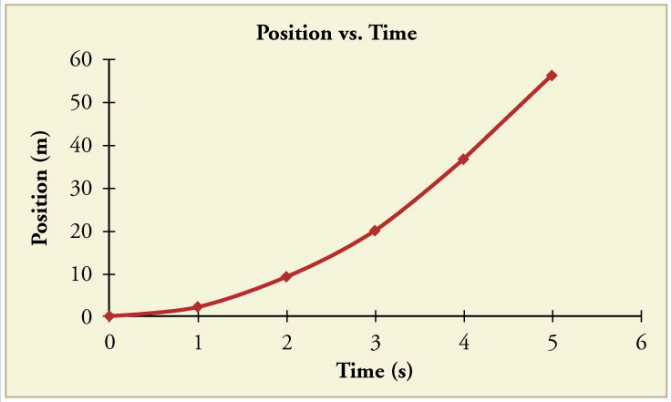



***Figure* 13.**

**2:** 38.9 m/s (about 87 miles per hour)

**3**: 502 m/s

**4:** (a) 16.5 s (b) 13.5 s(c) -2.68 $m/s^2$

**5:** (c) 173 m (d) 28.8 m/s or 106 km/h reasonable

**6:** (a) 20.0 m (b ) -1.00 m/s (c) This result does not really make sense. If the runner starts at 9.00 m/s and decelerates at 2.00 $m/s^2$, then she will have stopped after 4.50 s. If she continues to decelerate, she will be running backwards.

**8:** 0.799 m

**9:** 6.87 $m/s^2$ 52.3 m

**10:** (a) 28.0 m/s (b) 50.9 s (c) 7.68 km to accelerate and 713 m to decelerate

**11**: 0.00769 s and 130 $m/s^2$

**12:** (a) 51.4 m (b) 17.1 s

**13:** (a) $v_o$ = 0.600 m/s $v_f$ = 0 m/s x = 2.00 mm so a = 90.0 $m/s^2$ = 9.18 g (b) t = 0.0667 s (c) $v_o$ = 0.600 m/s $v_f$ = 0 m/s

x = 4.50 mm so a = 40.0 $m/s^2$ = 4.08 g

**14:** (a) -80.4 $m/s^2$ (b) 9.33 x $10^{-2}$ s

**15:** $v_o$ = 54 m/s $v_{final}$ = 0 m/s $\Delta y$ = 3.0 m so a = − 486 $m/s^2$

**16:** (a) 7.7 m/s (b) 1500 $m/s^2$ This is about 3 times the deceleration of the pilots, who were falling from thousands of meters high!

**17:** (a) and (b)$v_o$ = 22.0 m/s a = − 0.150 $m/s^2$ $\Delta x$ = 210 m so vf = 20.5 m/s and t = 9.88 s

**18:** (a) 32.6 m/s**2** (b) 162 m/s (c) v>$v_{max}$ , because the assumption of constant acceleration is not valid for a dragster. A dragster changes gears, and would have a greater acceleration in first gear than second gear than third gear, etc. The acceleration would be greatest at the beginning, so it would not be accelerating at 32.6 $m/s^2$ during the last few meters, but substantially less, and the final velocity would be less than 162 m/s.

**19**: (a) $v_o$ = 11.5 m/s a = 0.500 $m/s^2$ t = 7.00 s so $v_{final}$ = 15.0 m/s and $\Delta x$ = 92.75 = 92.8 m

**20:** 104 s

**21:** (a) v=12.2 m/s ; a=4.07 $m/s^2$ (b) v=11.2 m/s

# 13. 2.7 Problem-Solving Basics for One-Dimensional Kinematics


## *Summary*

- Apply problem-solving steps and strategies to solve problems of one-dimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.

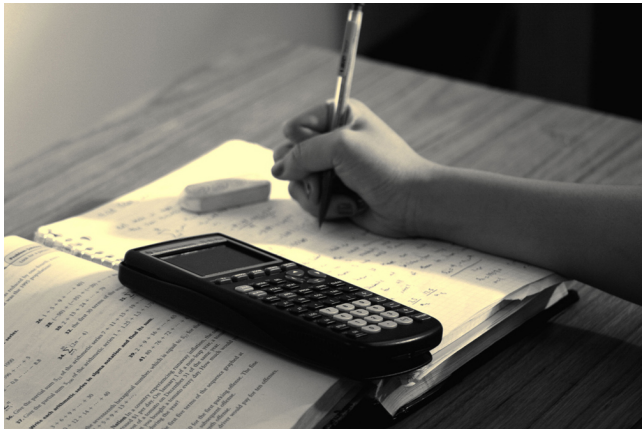



***Figure 1.*** *Problem-solving skills are essential to your success in Physics. (credit: scui3asteveo, Flickr).*

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

## Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

### Step 1

*Examine the situation to determine which physical principles are involved.* It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

## Step 2

*Make a list of what is given or can be inferred from the problem as stated (identify the knowns).* Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, "stopped" means velocity is zero, and we often can take initial time and position as zero.

## Step 3

*Identify exactly what needs to be determined in the problem (identify the unknowns).* In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

## Step 4

*Find an equation or set of equations that can help you solve the problem.* Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

## Step 5

*Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.* This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

## Step 6

*Check the answer to see if it is reasonable: Does it make sense?* This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin

to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

## Unreasonable Results

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at 0.40 $\text{m/s}^2$ for 100 s, his final speed will be 40 m/s (about 150 km/h)–clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving–it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

### Step 1

*Solve the problem using strategies as outlined and in the format followed in the worked examples in the text.* In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

$$\boldsymbol{v = v_0 + at = 0 + (0.40\ \text{m/s}^2)(100\ \text{s}) = 40\ \text{m/s}.}$$

### Step 2

*Check to see if the answer is reasonable.* Is it too large or too small, or does it have the wrong sign, improper units, …? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

$$\left(\frac{\mathbf{40\ m}}{\mathbf{s}}\right)\left(\frac{\mathbf{3.28\ ft}}{\mathbf{m}}\right)\left(\frac{\mathbf{1\ mi}}{\mathbf{5280\ ft}}\right)\left(\frac{\mathbf{60\ s}}{\mathbf{min}}\right)\left(\frac{\mathbf{60\ min}}{\mathbf{1\ h}}\right) = \mathbf{89\ mph}$$

This velocity is about four times greater than a person can run—so it is too large.

### Step 3

*If the answer is unreasonable, look for what specifically could cause the identified difficulty.* In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at 0.40 m/s$^2$, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of 0.40 m/s$^2$ for 100 s (almost two minutes).

## Section Summary

- *The six basic problem solving steps for physics are:*

*Step* 1. Examine the situation to determine which physical principles are involved.

*Step* 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).

*Step* 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

*Step* 4. Find an equation or set of equations that can help you solve the problem.

*Step* 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

*Step* 6. Check the answer to see if it is reasonable: Does it make sense?

## *Conceptual Questions*

**1:** What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

**2:** What is the last thing you should do when solving a problem? Explain.

# 14. 2.8 Falling Objects

*Summary*

- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

## Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration, independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones. You can watch the video of the hammer and feather drop on the Moon by visiting NASA

at https://www.history.nasa.gov/alsj/a15/video15.html#closeout3

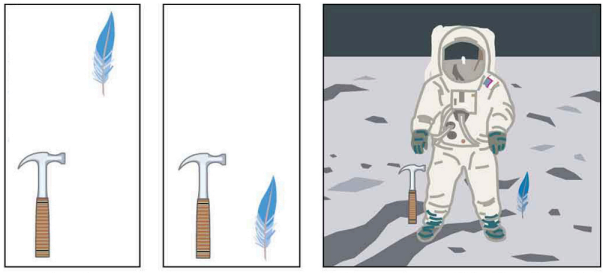



***Figure 1.*** *A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the acceleration due to gravity is only* $\mathbf{1.67\ m/s^2}$. *https://youtu.be/oYEgdZ3iEKA*



*A YouTube element has been excluded from this version of the*

*text. You can view it online here:*
*https://pressbooks.bccampus.ca/douglasphys1104summer2021/?p=157*

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free-fall**.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the **acceleration due to gravity**. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, ***g***. It is constant at any given location on Earth and has the average value

$$\mathbf{g = 9.80\ m/s^2}$$

Although ***g*** varies from $\mathbf{9.78\ m/s^2}$ to $\mathbf{9.83\ m/s^2}$, depending on latitude, altitude, underlying geological formations, and local topography, the average value of $\mathbf{9.80\ m/s^2}$ will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward* (*towards the center of Earth*). In fact, its direction *defines* what we call vertical. Note that whether the acceleration ***a*** in the kinematic equations has the value ***+g*** or ***−g***

depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.80\ \text{m/s}^2$, and if we define the downward direction as positive, then $a = g = 9.80\ \text{m/s}^2$.

## One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude $g$. We will also represent vertical displacement with the symbol $y$ and use $x$ for horizontal displacement.

KINEMATIC EQUATIONS FOR OBJECTS IN FREE FALL WHERE ACCELERATION = -G

$$v = v_0 - gt$$
$$y = y_0 + v_0 t - {}^1\!/_2\, g\, t^2$$
$$v^2 = {v_0}^2 - 2\, g\, (y - y_0)$$

## Example 1: Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

**Strategy**

Draw a sketch.

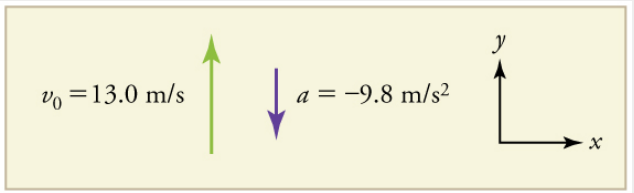



***Figure 2.***

We are asked to determine the position $y$ at various times. It is reasonable to take the initial position $y_0$ to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown

upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so $a$ is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as $y_1$ and $v_1$; $y_2$ and $v_2$; and $y_3$ and $v_3$.

**Solution for Position $y_1$**

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\ \text{m/s}$; $a = -g = -9.80\ \text{m/s}^2$; and $t = 1.00\ \text{s}$.

2. Identify the best equation to use. We will use $y = y_0 + v_0t - {}^1/_2\, g\, t^2$ because it includes only one unknown, $y$ (or $y_1$, here), *which is the value we want to find.*

3. Plug in the known values and solve for $y_1$.

$y_1 = 0 + (13.0\ \text{m/s})(1.00\ \text{s}) + (0.5)(-9.80\ \text{m/s}^2)(1.00\text{s})^2 = 8.10\ \text{m}$

**Discussion**

The rock is 8.10 m above its starting point at $t = 1.00\ \text{s}$, since $y_1 > y_0$. It could be *moving* up or down; the only way to tell is to calculate $v_1$ and find out if it is positive or negative.

**Solution for Velocity $v_1$**

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\ \text{m/s}$; $a = -g = -9.80\ \text{m/s}^2$; and $t = 1.00\ \text{s}$. We also know from the solution above that $y_1 = 8.10\ \text{m}$.

2. Identify the best equation to use. The most straightforward is $\mathbf{v = v_0 - gt}$ (from $\boldsymbol{v = v_0 + at}$, where $\boldsymbol{a}$ **= gravitational acceleration =** $\boldsymbol{-g}$).

3. Plug in the knowns and solve.

$$\boldsymbol{v = v_0 - gt = 13.0\ m/s - (9.80\ m/s^2)(1.00\ s) = +3.20\ m/s}$$

**Discussion**

The positive value for $\boldsymbol{v_1}$ means that the rock is still heading upward at $\boldsymbol{t = 1.00\ s}$. However, it has slowed from its original 13.0 m/s, as expected.

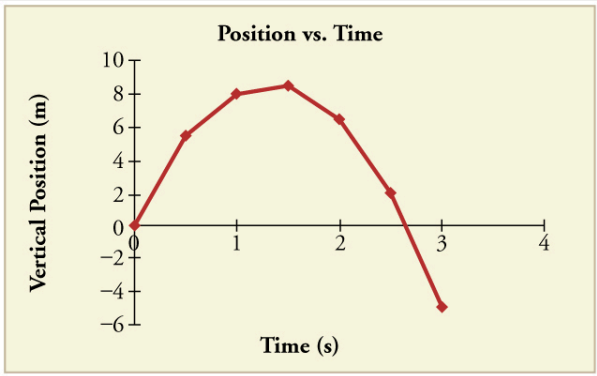
**Solution for Remaining Times**

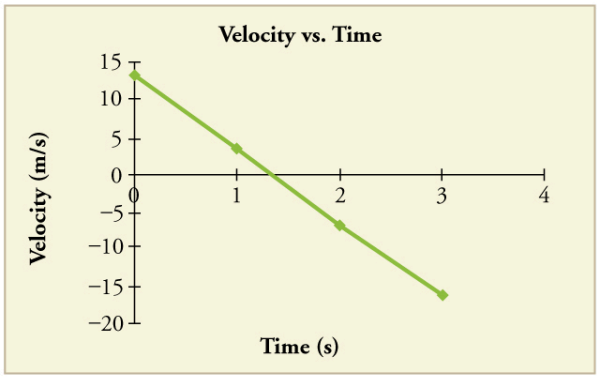
The procedures for calculating the position and velocity at $\boldsymbol{t = 2.00\ s}$ and **3.00 s** are the same as those above. The results are summarized in Table 1 and illustrated in Figure 3.

| **Time,** $\boldsymbol{t}$ | **Position,** $\boldsymbol{y}$ | **Velocity,** $\boldsymbol{v}$ | **Acceleration,** $\boldsymbol{a}$ |
|---|---|---|---|
| 1.00 s | 8.10 m | 3.20 m/s | –9.80 m/s2 |
| 2.00 s | 6.40 m | –6.60 m/s | –9.80 m/s2 |
| 3.00 s | –5.10 m | –16.4 m/s | –9.80 m/s2 |

**Table 1.** Results.

Graphing the data helps us understand it more clearly.


Position vs. Time
Vertical Position (m)
10
8
6
4
2
0
−2
−4
−6
0
1
2
3
4
Time (s)


Velocity vs. Time
Velocity (m/s)
15
10
5
0
−5
−10
−15
−20
0
1
2
3
4
Time (s)

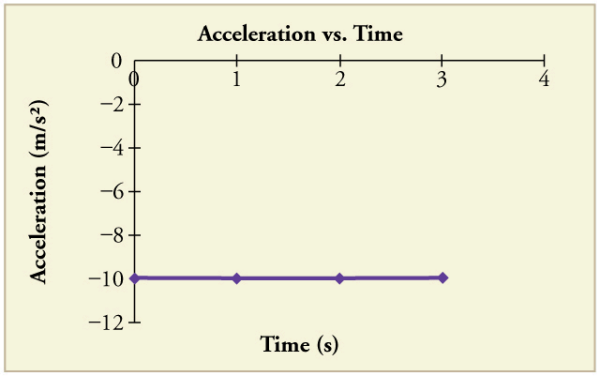

Acceleration vs. Time
Acceleration (m/s²)
0
−2
−4
−6
−8
−10
−12
0
1
2
3
4
Time (s)

***Figure 3.*** *Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. Misconception Alert! Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is time, not space. The actual path of the rock in space is straight up, and straight down.*

**Discussion**

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since $\mathbf{y_1}$ and $\mathbf{v_1}$ are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both $\mathbf{y_3}$ and $\mathbf{v_3}$ are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still $\mathbf{-9.80\ m/s^2}$. Its acceleration is $\mathbf{-9.80\ m/s^2}$ for the whole trip—while it is moving up and while it is moving down. Note that the values for $\mathbf{y}$ are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

## MAKING CONNECTIONS: TAKE HOME EXPERIMENT—REACTION TIME

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

## Example 2: Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it

is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

**Strategy**

Draw a sketch.

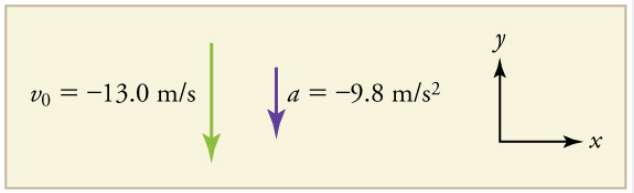



***Figure 4.***

Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $\mathbf{y_0 = 0}$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

**Solution**

1. Identify the knowns. $\mathbf{y_0 = 0}$; $\mathbf{y_1 = -5.10\ m}$; $\mathbf{v_0 = -13.0\ m/s}$; $\mathbf{a = -g = -9.80\ m/s^2}$.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $\mathbf{v^2 = v_0^2 + 2a(y - y_0)}$ works well because the only unknown in it is $\mathbf{v}$. (We will plug $\mathbf{y_1}$ in for $\mathbf{y}$.)

3. Enter the known values

$$v^2 = (-13.0\ \text{m/s})^2 + (2)(-9.80\ \text{m/s}^2)(-5.10\ \text{m} - 0\ \text{m}) = 268.96\ \text{m}^2/\text{s}^2$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

$$\boldsymbol{v = \pm 16.4\ \mathbf{m/s}}.$$

The negative root is chosen to indicate that the rock is still heading down. Thus,

v=−16.4 m/s

**Discussion**

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed*. (See Example 1 and Figure 5(a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from Example 1) when the initial velocity is 13.0 m/s straight up, a result of **±3.20 m/s** is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.

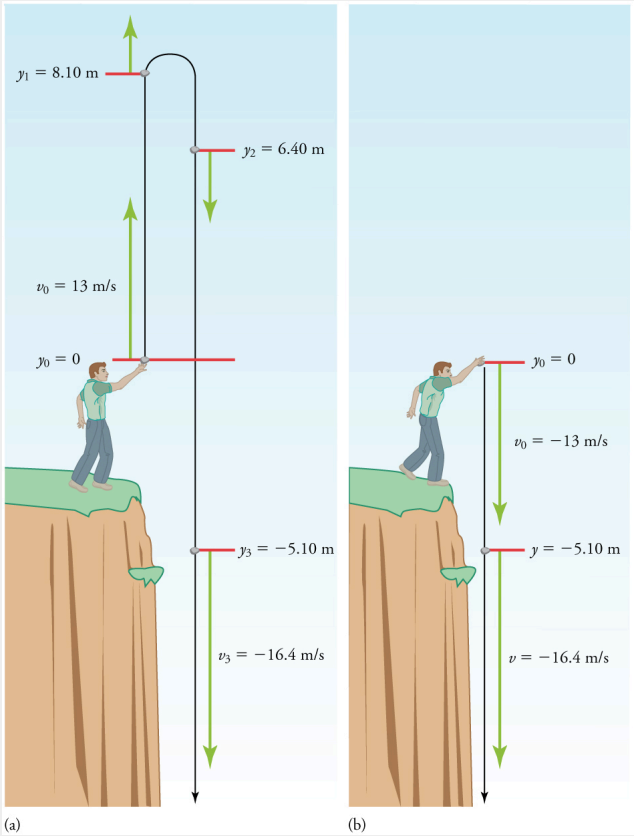



***Figure 5.*** *(a) A person throws a rock straight up, as explored in Example 1. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in Example 2. Note that at the same distance below the point of release, the rock has the same velocity in both cases.*

Another way to look at it is this: In Example 1, the rock

is thrown up with an initial velocity of **13.0 m/s**. It rises and then falls back down. When its position is $y = 0$ on its way back down, its velocity is **-13.0 m/s**. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y = -5.10$ **m** to be the same whether we have thrown it upwards at **+13.0 m/s** or thrown it downwards at **-13.0 m/s**. The velocity of the rock on its way down from $y = 0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

## Example 3: Find g from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See,

for example, Figure 6. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.

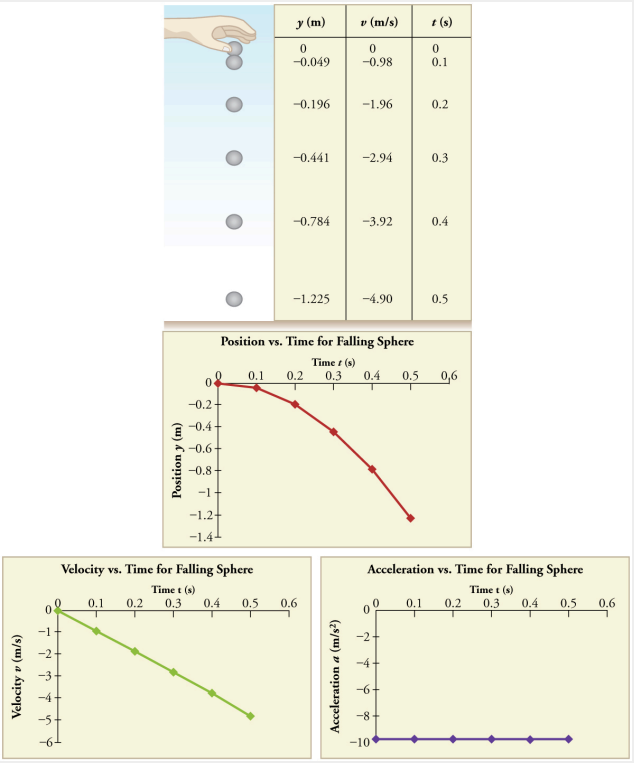



***Figure 6.*** *Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.*

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming

the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

**Strategy**

Draw a sketch.

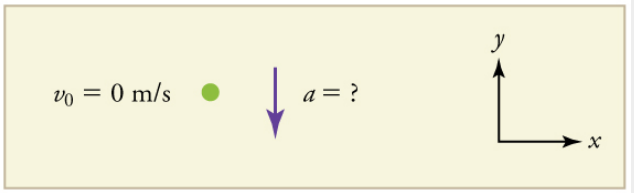



***Figure 7.***

We need to solve for acceleration $\boldsymbol{a}$. Note that in this case, displacement is downward and therefore negative, as is acceleration.

**Solution**

1. Identify the knowns. $\boldsymbol{y_0 = 0}$; $\boldsymbol{y = -1.0000\ m}$; $\boldsymbol{t = 0.45173\ s}$; $\boldsymbol{v_0 = 0}$.

2. Choose the equation that allows you to solve for $\boldsymbol{a}$ using the known values.

$$\boldsymbol{y = y_0 + v_0 t - {}^1\!/_2\, g\, t^2}$$

3. Substitute 0 for $\boldsymbol{v_0}$ and rearrange the equation to solve for $\boldsymbol{a}$. Substituting 0 for $\boldsymbol{v_0}$ yields

$$\boldsymbol{y = {}^1\!/_2\, a\, t^2}$$

Solving for $\boldsymbol{a}$ gives $\boldsymbol{a = 2\, y\, /\, t^2}$

4. Substitute known values yields

$$a = 2\ (1.0000\ m)\ /\ (0.45173\ s)^2 = -\ 9.8010\ \ m/s^2$$

so, because $a = -g$ with the directions we have chosen,

$$g = 9.8010\ m/s^2$$

**Discussion**

The negative value for $a$ indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of $9.80\ m/s^2$, so $9.8010\ m/s^2$ makes sense. Since the data going into the calculation are relatively precise, this value for $g$ is more precise than the average value of $9.80\ m/s^2$; it represents the local value for the acceleration due to gravity.

## *Check Your Understanding*

**1:** A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

## PHET EXPLORATIONS: Graphing,

## Moving Man

You need to be familiar with motion graphs. The earlier chapter covered this topic. You might want to review it. Here are the links to two simulations that you might find useful.

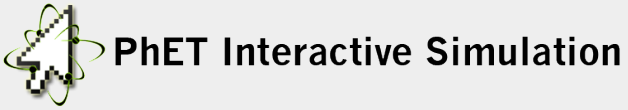



***Figure 8.*** *Moving Man*.

If you use the projectile motion simulation and point the cannon straight up or at 90.0 degrees to the horizontal, then you can see free fall motion.

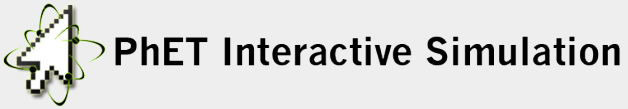



***Figure 8.*** *Projectile Motion*.

## Section Summary

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity $\boldsymbol{g}$, which averages
$$g=9.80 \text{ m/s}^2$$
- Whether the acceleration $a$ should be taken as $\boldsymbol{+g}$ or $\boldsymbol{-g}$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $\boldsymbol{a = -g = -9.80 \text{ m/s}^2}$ is negative. In the opposite case, $\boldsymbol{a = +g = 9.80 \text{ m/s}^2}$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $\boldsymbol{+g}$ or $\boldsymbol{-g}$ substituted for $\boldsymbol{a}$.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.
- The acceleration due to gravity is less on the Moon.

Click here to see a High Definition Youtube of the Apollo 11 Moon landing and watch people move in the lower gravity.
https://youtu.be/SMA0zbFK5jE



*A YouTube element has been excluded from this version of the text. You can view it online here: https://pressbooks.bccampus.ca/douglasphys1104summer2021/?p=157*

## *Conceptual Questions*

**1:** What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

**2:** An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the

acceleration due to gravity have the same sign on the way up as on the way down?

**3:** Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

**4:** If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

**5:** The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

**6:** How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of **g** on Earth)?

## *Problems & Exercises*

Assume air resistance is negligible unless otherwise stated.

**1:** Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$.

**2:** Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

**3:** A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

**4:** A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.4 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

**5:** A dolphin in an aquatic show jumps straight up out of

the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

**6:** A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool. (a) How long are her feet in the air? (b) What is her highest point above the board? (c) What is her velocity when her feet hit the water?

**7:** (a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?

**8:** A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

**9:** You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

**10:** A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?

**11:** Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can't see the rock right away but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?

**12:** An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance travelled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance travelled during the last second of motion before hitting the ground.

**13:** There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

**14:** A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 0.312 s to go past the window. What was the ball's initial velocity? Hint: First consider only the distance along the window, and solve for the ball's velocity at the bottom of the

window. Next, consider only the distance from the ground to the bottom of the window, and solve for the initial velocity using the velocity at the bottom of the window as the final velocity.

**15:** Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

**16:** A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms ($8.00 \times 10^{-5}$ s). (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

**17:** A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

**18:** A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with

the floor if that contact lasts 3.50 ms ($3.50 \times 10^{-3}$ s). (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

## Glossary

**free-fall**
the state of movement that results from gravitational force only

**acceleration due to gravity**
acceleration of an object as a result of gravity

## *Solutions*

**Check Your Understanding**

**1:** We know that initial position $y_0 = 0$, final position $y = -30.0\ \text{m}$, and $a = -g = -9.80\ \text{m/s}^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ to solve for $t$. Inserting $a = -g$, we obtain

$$y = 0 + 0 - \frac{1}{2}gt^2$$

$$t^2 = \frac{2y}{-g}$$

$$t = \pm\sqrt{\frac{2y}{-g}} = \pm\sqrt{\frac{2(-30.0\ \text{m}}{-9.80\ \text{m/s}^2}} = \pm\sqrt{6.12\ \text{s}^2} = 2.47\ \text{s} \approx 2.5\ \text{s}$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

**Conceptual Problems**

1. 9.80 m/s$^2$ down. Always
2. (a) at top (b) Yes (c) Yes, always the same at 9.80 m/s$^2$ down
3. speed is the same, velocity is different
4. Slower. Maximum height would be less
5. 6 times
6. 6 times

**Problems & Exercises**

**1:** (a) $y_1$=6.28 m; $v_1$=10.1 m/s (b) $y_2$=10.1 m; $v_2$=5.20 m/s (c) y_3=11.5 m; v_3=0.300 m/s (d) y_4=10.4 m; v_4=-4.60 m/s

**2**: assume up is positive (a) Δy = − 1.23 m or 1.23 m down (b) v = − 4.90 m/s or 4.90 m/s down (b) − 4.90 m and − 9.80 m/s (e) -30.6 m and = − 24.5 m/s

**3:** $v_0$=4.95 m/s

4: (a) acceleration **a= -9.80 m/s$^2$** , initial velocity $v_o$ = − 1.4 m/s and time t = 1.8 s (b) 18 m

**5:** (a) a = -9.80 m/s$^2$ ; $v_0$=13.0 m/s ; y_=0 m (b) v=0 m/s Unknown is distance **y** to top of trajectory, where velocity is zero. Use equation $v^2 = v_0^2 + 2a(y - y_0)$ because it contains all known values except for **y**.

$v^2 - v_0^2 \quad 2a(y - y_0)$

$\frac{v^2 - v_0}{2a} \quad y - y_0$

$$y \quad y_0 + \frac{v^2 - v_0^2}{2a} = 0\ \text{m} + \frac{(0\ \text{m/s})^2 - (13.0\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 8.62\ \text{m}$$

Dolphins measure about 2 meters long and can jump several times their length out of the water, so this is a reasonable result. (c) 2.65 s

**6:** a = − 9.80 m/s$^2$ $v_o$ = +4.00 m/s (a) v = $v_f$ = 0 m/s as it is top of the arc, so Δy= +0. 816 m Read the question carefully, it is not 1.8 m as that is how far they start ABOVE the water level.

(b) Δy = − 1.80 m or $y_o$ = 0 and y = $y_f$ = − 1.80 m so vf = − 7.16 m/s You take the negative root as you know the swimmer is moving down as they hit the water.

**7:** (a) 8.26 m (b) 0.717 s

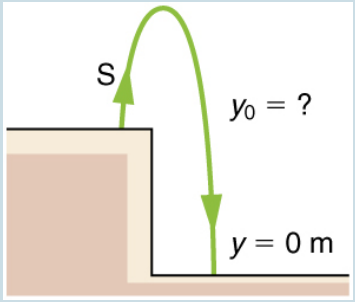



***Figure 9.***

8: a = − 9.80 m/s$^2$ $v_o$ = +11.00 m/s Δ y = (1.80 m − 2.20 m ) = − 0.40 m so t = 2.28 s note that v final = = 11.3 m/s

**9:** 1.91 s

10: $a = -9.80\ m/s^2$ $v_o = ?\ m/s$ $\Delta y = +2.50\ m$ when $v = 0\ m/s$ (a) $v_o =$ m/s (b) $t =$ s

**11:** (a) 94.0 m (b) 3.13 s

12: (a) $a = -9.80\ m/s^2$ $v_o = 0\ m/s$ $t = 1.00\ s$ so $\Delta y = -4.90\ m$ distance travelled = 4.90 m

(b) $a = -9.80\ m/s^2$ $v_o = 0\ m/s$ $t = ?\ s$ $\Delta y = -75.0\ m$ so $v_{final} = -38.3\ m/s$ $t = 3.91\ s$

(c) $a = -9.80\ m/s^2$ $v_o = 0\ m/s$ $t = = 3.91\ s = 1.00\ s = 2.91\ s$ $\Delta y = ?$ so $\Delta y = -41.4\ m$ this gives distance travelled in the last second to be ( 75.0 m − 41.4 m = 33.5 m)

**13:** (a) -70.0 m/s (downward) (b) 6.10 s

14: (a) $a = -9.80\ m/s^2$ $v_o = ?\ m/s$ $t = 0.312\ s$ $\Delta y = +2.00\ m$ $v\ final = ?\ m/s$ . Use $\boldsymbol{y = y_o + v_o t - {}^1\!/_2\, g\, t^2}$ to find$v_o = v_{bottom\ of\ window} = +7.94\ m/s$. Then $v_{final} = v_{top\ of\ window} = 4.88\ m/s$ You can do the rest.

**15:** (a ) 19.6 m (b) 18.5 m

**16:** (a) $a = -9.80\ m/s^2$ $v_o = ?0\ m/s$ $t = ?\ s$ $\Delta y = -1.50$ and $v\ final = ?\ m/s$ so v final =

(b) $a = -9.80\ m/s^2$ $v_o =$ m/s $t = ?\ s$ $\Delta y = +1.45\ m$ $v_{final} = +1.45\ m$ so vo =

(c) $a = (v_{final} - v_o) / t$ where $t = 8.00 \times 10^{-5}\ s$

(d) $v^2 = v_o^2 + 2\,a\,\Delta y$

**17:** (a) 305 m (b) 262 m -29.2 m/s (c) 8.91 s

**18:** (a) $a = -9.80\ m/s^2$ $v_o = ?0\ m/s$ $t = ?\ s$ $\Delta y = -1.50$ and $v\ final = ?\ m/s$ so v final =

(b) $a = -9.80\ m/s^2$ $v_o =$ m/s $t = ?\ s$ $\Delta y = +1.10\ m$ $v_{final} = +1.45\ m$ so vo =

(c) $a = (v_{final} - v_o) / t$ where $t = 3.50 \times 10^{-3}\ s$

PART IV

# CHAPTER 3 TWO-DIMENSIONAL KINEMATICS



***Figure 1.*** *Everyday motion that we experience is, thankfully, rarely as tortuous as a rollercoaster ride like this—the Dragon Khan in Spain's Universal Port Aventura Amusement Park. However, most motion is in curved, rather than straight-line, paths. Motion along a curved path is two- or three-dimensional motion, and can be described in a similar fashion to one-dimensional motion. (credit: Boris23/Wikimedia Commons).*

The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved

paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Motion not confined to a plane, such as a car following a winding mountain road, is described by three-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight-line motion in the previous chapter. This simple extension will allow us to apply physics to many more situations, and it will also yield unexpected insights about nature.

# 15. 3.1 Kinematics in Two Dimensions: An Introduction

*Summary*

- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.



***Figure 1.*** *Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers).*

# Two-Dimensional Motion: Walking in a City

Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in Figure 2.

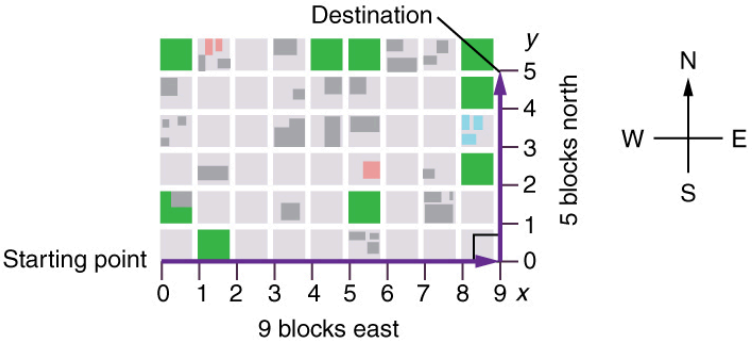



***Figure 2.*** *A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.*

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem, $a^2+b^2=c^2$, can be used to find the straight-line distance.

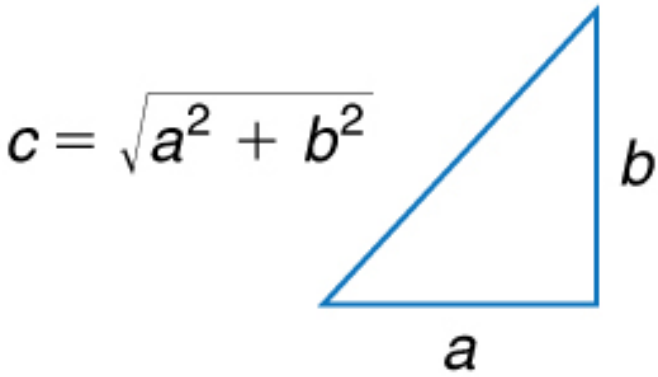



***Figure 3.*** *The Pythagorean theorem relates the length of the legs of a right triangle, labeled* ***a*** *and* ***b****, with the hypotenuse, labeled* ***c****. The relationship is given by:* $a^2+b^2=c^2$*. This can be rewritten, solving for* ***c****:* $c = \sqrt{(a^2+b^2)}$*.*

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is

$$\sqrt{(\mathbf{9\ blocks})^2 + (\mathbf{5\ blocks})^2} = \mathbf{10.3\ blocks},$$

considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that "9" and "5" have only one significant digit, they are discrete numbers. In this case "9 blocks" is the same as "9.0 or 9.00 blocks." We have decided to use three significant figures in the answer in order to show the result more precisely.)

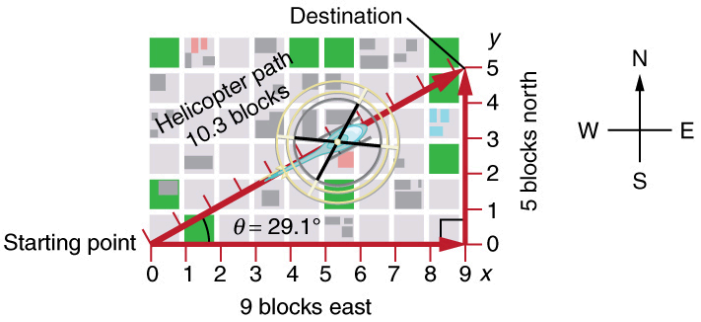



***Figure 4.*** *The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.*

The fact that the straight-line distance (10.3 blocks) in Figure 4 is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that **vectors** are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector's magnitude. The arrow's length is indicated by hash marks in Figure 2 and Figure 4. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in Figure 4. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This

means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in Chapter 3.2 Vector Addition and Subtraction: Graphical Methods and Chapter 3.3 Vector Addition and Subtraction: Analytical Methods.)

## The Independence of Perpendicular Motions

The person taking the path shown in Figure 4 walks east and then north (two perpendicular directions). How far he or she walks east is only affected by his or her motion eastward. Similarly, how far he or she walks north is only affected by his or her motion northward.

### INDEPENDENCE OF MOTION

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let's

compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.

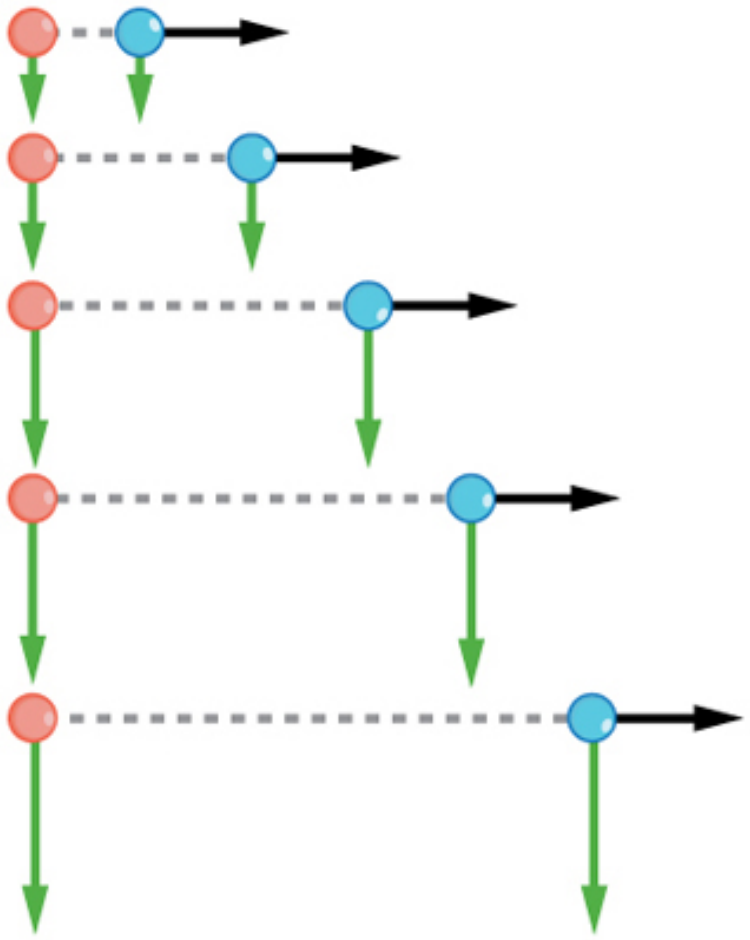


***Figure 5.*** *This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.*

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in Chapter 3.2 Vector Addition and Subtraction: Graphical Methods and Chapter 3.3 Vector Addition and Subtraction: Analytical Methods. We will find such techniques to be useful in many areas of physics.

## PHET EXPLORATIONS: LADYBUG MOTION IN 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position,

velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.




***Figure 6.*** *Ladybug Motion 2D*

## Summary

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

## Glossary

**vector**
: a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

# 16. 3.2 Vector Addition and Subtraction: Graphical Methods

*Summary*

- Understand the rules of vector addition, subtraction, and multiplication.
- Apply graphical methods of vector addition and subtraction to determine the displacement of moving objects.

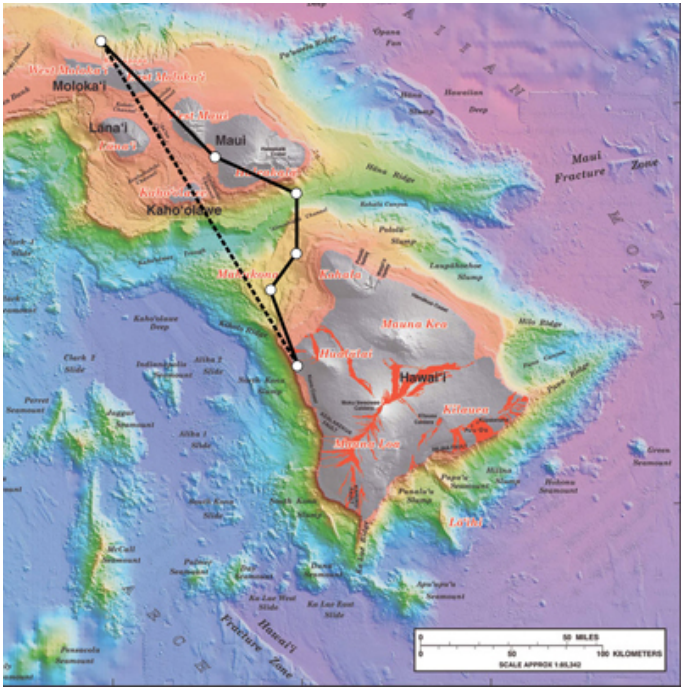



***Figure 1.*** *Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. (credit: US Geological Survey).*

## Vectors in Two Dimensions

A **vector** is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an

arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

Figure 2 shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in Chapter 3.1 Kinematics in Two Dimensions: An Introduction. We shall use the notation that a symbol with an arrow over it, such as $\vec{\mathrm{D}}$, stands for a vector. Its magnitude is represented by the symbol in italics, **D**, and its direction by $\boldsymbol{\theta}$.

### VECTORS IN THIS TEXT

In this text, we will represent a vector with an arrow over a symbol. For example, we will represent the quantity force with the vector $\vec{\mathrm{F}}$, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as **F**, and the direction of the variable will be given by an angle $\boldsymbol{\theta}$.

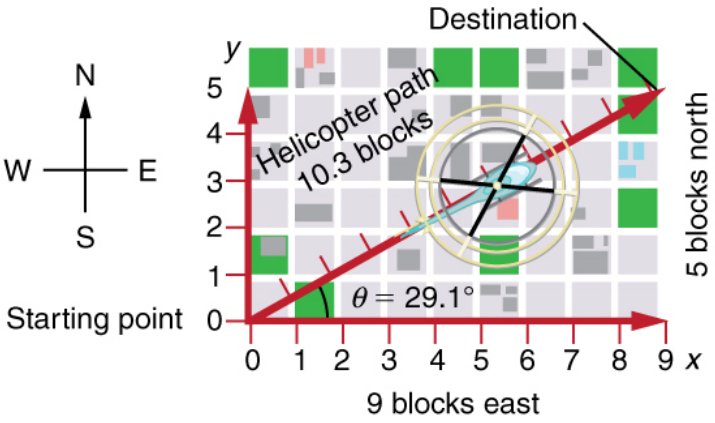



***Figure 2.*** *A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle* **29.1°** *north of east.*

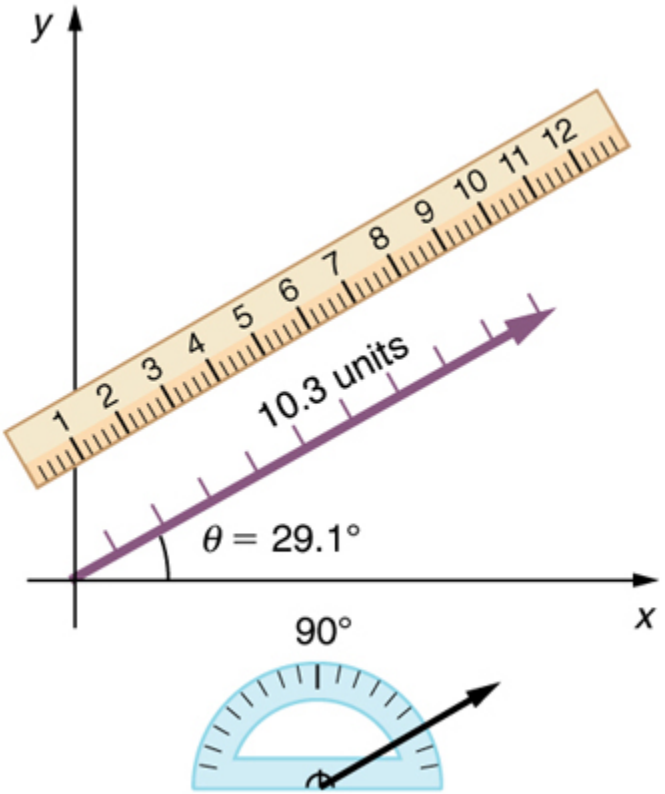



***Figure 3.*** *To describe the resultant vector for the person walking in a city considered in Figure 2 graphically, draw an arrow to represent the total displacement vector* **D**. *Using a protractor, draw a line at an angle* $\boldsymbol{\theta}$ *relative to the east-west axis. The length* **D** *of the arrow is proportional to the vector's magnitude and is measured along the line with a ruler. In this example, the magnitude* **D** *of the vector is 10.3 units, and the direction* $\boldsymbol{\theta}$ *is* $\mathbf{29.1^{o}}$ *north of east.*

## Vector Addition: Head-to-Tail Method

The **head-to-tail method** is a graphical way to add vectors, described in Figure 4 below and in the steps following. The **tail** of

the vector is the starting point of the vector, and the **head** (or tip) of a vector is the final, pointed end of the arrow.

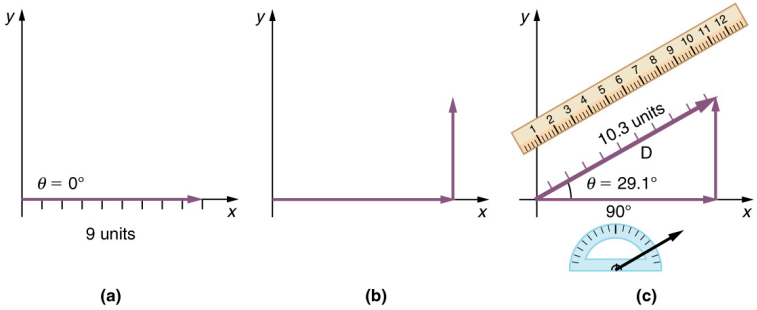



***Figure 4.*** *Head-to-Tail Method: The head-to-tail method of graphically adding vectors is illustrated for the two displacements of the person walking in a city considered in Figure 2. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector. (c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or* ***resultant vector D****. The length of the arrow* **D** *is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis)* $\boldsymbol{\theta}$ *is measured with a protractor to be* $\mathbf{29.1^0}$.

***Step 1.*** *Draw an arrow to represent the first vector (9 blocks to the east) using a ruler and protractor.*

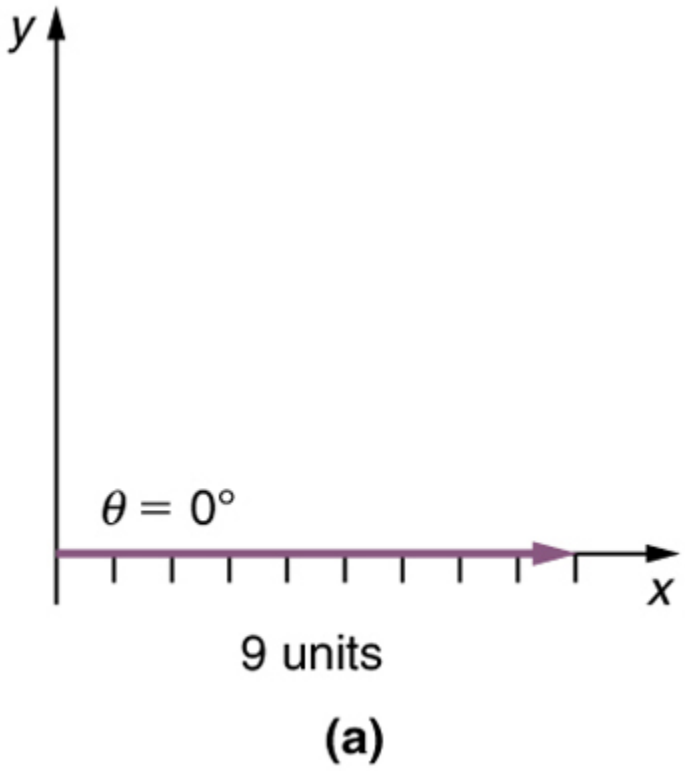



***Figure 5.***

***Step 2.*** Now draw an arrow to represent the second vector (5 blocks to the north). *Place the tail of the second vector at the head of the first vector*.

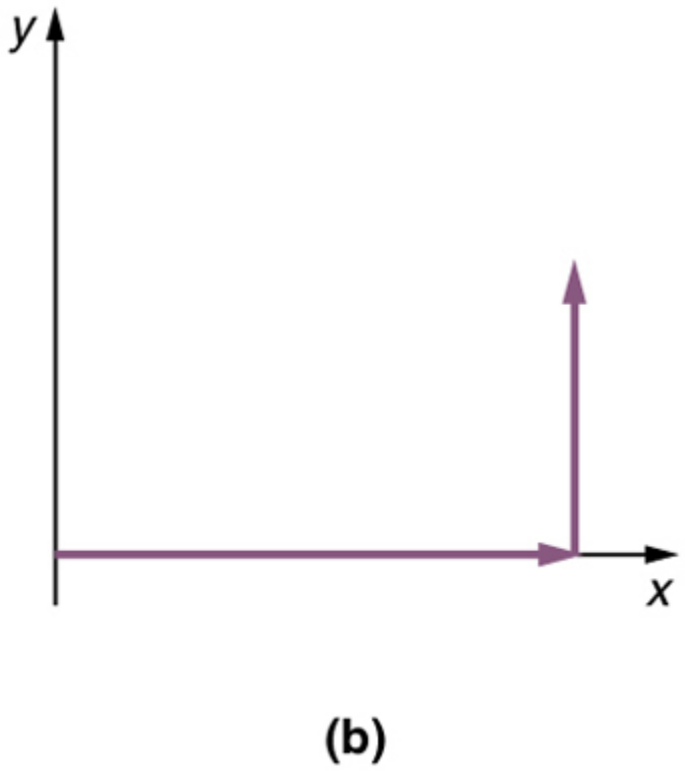



***Figure 6.***

***Step 3.*** *If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail.*

***Step 4.*** *Draw an arrow from the tail of the first vector to the head of the last vector.* This is the **resultant**, or the sum, of the other vectors.

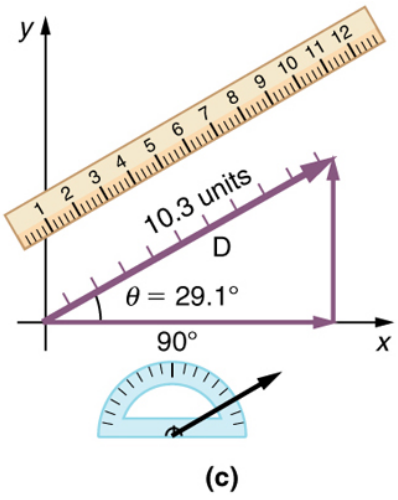



***Figure 7.***

**Step 5.** To get the **magnitude** of the resultant, *measure its length with a ruler. (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)*

**Step 6.** To get the **direction** of the resultant, *measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this angle.)*

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

## Example 1: Adding Vectors Graphically Using the Head-to-Tail Method: A Women Takes a Walk

Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction **49.0°** north of east. Then, she walks 23.0 m heading **15.0°** north of east. Finally, she turns and walks 32.0 m in a direction 68.0° south of east.

**Strategy**

Represent each displacement vector graphically with an arrow, labeling the first $\vec{A}$, the second $\vec{B}$, and the third $\vec{C}$, making the lengths proportional to the distance and the directions as specified relative to an east-west line. The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted $\vec{R}$.

**Solution**

(1) Draw the three displacement vectors.

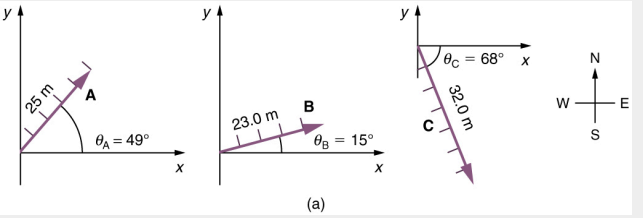



***Figure 8.***

(2) Place the vectors head to tail retaining both their initial magnitude and direction.

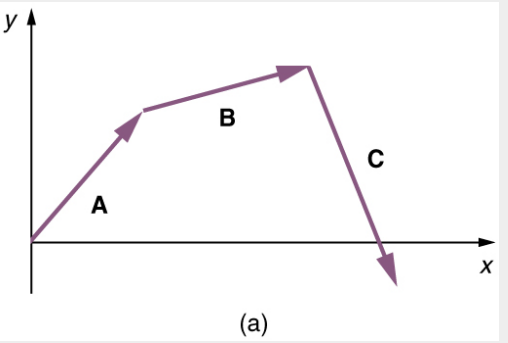



***Figure 9.***

(3) Draw the resultant vector, $\vec{\mathrm{R}}$.

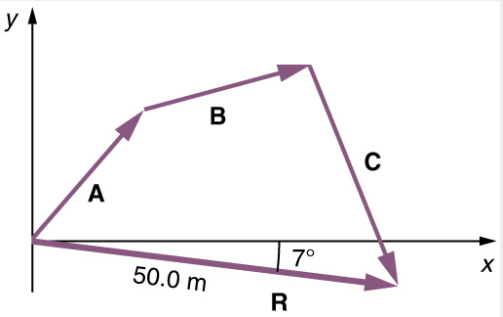



***Figure 10.***

(4) Use a ruler to measure the magnitude of $\vec{\mathrm{R}}$ and a protractor to measure the direction of $\vec{\mathrm{R}}$. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.

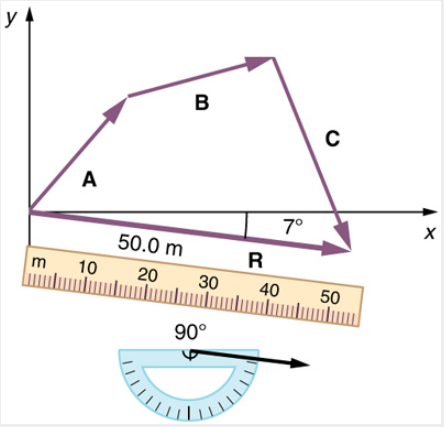



***Figure 11.***

In this case, the total displacement **R** is seen to have a magnitude of 50.0 m and to lie in a direction **7.0°** south of east. By using its magnitude and direction, this vector can be expressed as **R = 50.0 m** and $\boldsymbol{\theta}$ **= 7.0°** south of east.

**Discussion**

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in Figure 12 and we will still get the same solution.

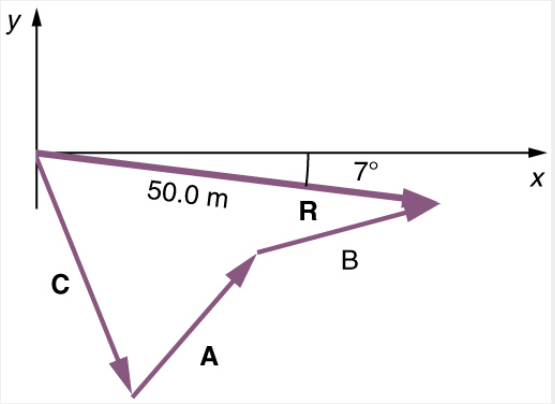



***Figure 12.***

Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is **commutative**. Vectors can be added in any order.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}.$$

(This is true for the addition of ordinary numbers as well—you get the same result whether you add **2+3** or **3+2**, for example).

## Vector Subtraction

Vector subtraction is a straightforward extension of vector addition.

To define subtraction (say we want to subtract $\vec{\mathrm{B}}$ from $vec\mathrm{A}$, written $\vec{\mathrm{A}} - \vec{\mathrm{B}}$, we must first define what we mean by subtraction. The *negative* of a vector $\vec{\mathrm{B}}$ is defined to be $-\vec{\mathrm{B}}$; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in Figure 13. In other words, $\vec{\mathrm{B}}$ has the same length as $-\vec{\mathrm{B}}$, but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.

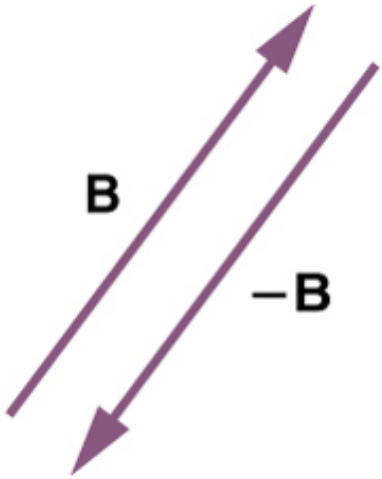



***Figure 13.*** *The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So* **B** *is the negative of* **–B**; *it has the same length but opposite direction.*

The *subtraction* of vector $\vec{\mathrm{B}}$ from vector $\vec{\mathrm{A}}$ is then simply defined to be the addition of $-\vec{\mathrm{B}}$ to $\vec{\mathrm{A}}$. Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

This is analogous to the subtraction of scalars (where, for example, **5-2=5+(-2)**). Again, the result is independent of the order in which the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

### Example 2: Subtracting Vectors Graphically: A Women Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction **66.0°** north of east from her current location, and then travel 30.0 m in a direction **112°** north of east (or **22.0°** west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.

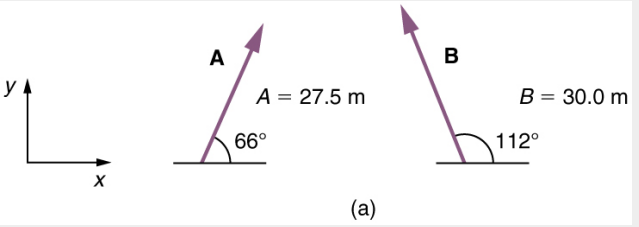



***Figure 14.***

**Strategy**

We can represent the first leg of the trip with a vector $\vec{A}$, and the second leg of the trip with a vector $\vec{B}$. The dock is located at a location $\vec{A} + \vec{B}$. If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance $\vec{B}$ (30.0 m) in the direction **180°-112°=68°** south of east. We represent this as $-\vec{B}$, as shown below. The vector $-\vec{B}$ has the same magnitude as $\vec{B}$ but is in the opposite direction. Thus, she will end up at a location $\vec{A} + (-\vec{B})$, or $\vec{A} - \vec{B}$.

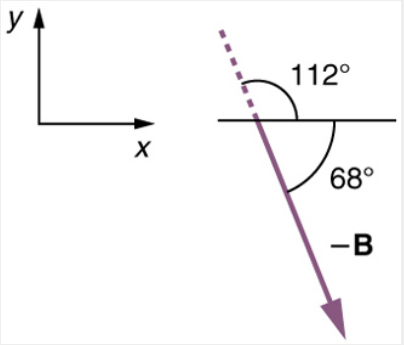



***Figure 15.***

We will perform vector addition to compare the location of the dock, $\vec{A} + \vec{B}$, with the location at which the woman mistakenly arrives, $\vec{A} + (-\vec{B})$.

**Solution**

(1) To determine the location at which the woman arrives by accident, draw vectors $\vec{A}\$\$and[latex] - \vec{B}$.

(2) Place the vectors head to tail.

(3) Draw the resultant vector $\vec{R}$.

(4) Use a ruler and protractor to measure the magnitude and direction of $\vec{R}$.

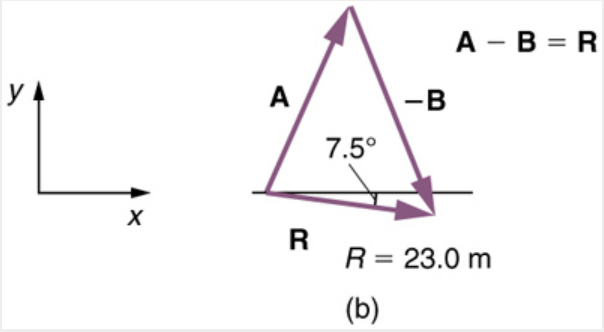



***Figure 16.***

In this case, **R=23.0 m** and $\boldsymbol{\theta}$**=7.5°** south of east.

(5) To determine the location of the dock, we repeat this method to add vectors $\vec{\mathrm{A}}$ and $\vec{\mathrm{B}}$. We obtain the resultant vector $\vec{\mathrm{R}}'$:

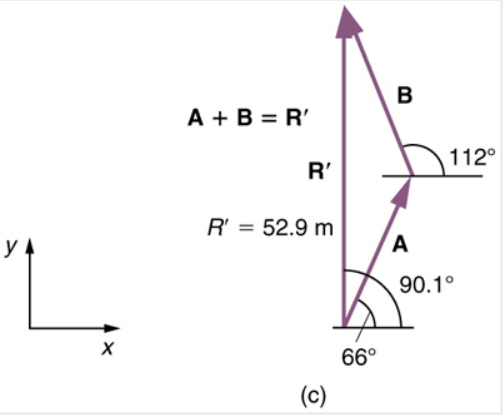



***Figure 17.***

In this case **R=52.9 m** and **θ=90.1°** north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

**Discussion**

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

## Multiplication of Vectors and Scalars

If we decided to walk three times as far on the first leg of the trip

considered in the preceding example, then we would walk **3 × 27.5 m**, or 82.5 m, in a direction **66.0°** north of east. This is an example of multiplying a vector by a positive **scalar**. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the *opposite* direction. For example, if you multiply by −2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector $\vec{A}$ is multiplied by a scalar **c**,

- the magnitude of the vector becomes the absolute value of **c** $\vec{A}$ ,
- if **c** is positive, the direction of the vector does not change,
- if **c** is negative, the direction is reversed.

In our case, **c=3** and **A=27.5 m**. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

## Resolving a Vector into Components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular **components** of a single vector, for example the $x$- *and* $y$-components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction **29.0°** north of east

and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in Chapter 3.4 Projectile Motion, and much more when we cover **forces** in Chapter 4 Dynamics: Newton's Laws of Motion. Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in Chapter 3.3 Vector Addition and Subtraction: Analytical Methods are ideal for finding vector components.

### PHET EXPLORATIONS: MAZE GAME

Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.



PhET Interactive Simulation

***Figure 18.*** *Maze Game*

## Summary

- The **graphical method of adding vectors** $\vec{A}$ and $\vec{B}$ involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector $\vec{R}$ is defined such that $\vec{A} + \vec{B} = \vec{R}$. The magnitude and direction of $\vec{R}$ are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector** $\vec{B}$ from $\vec{A}$ involves adding the opposite of vector $\vec{B}$, which is defined as $-\vec{B}$. In this case, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = \vec{R}$. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector $\vec{R}$.
- Addition of vectors is **commutative** such that $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.
- The **head-to-tail method** of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector $\vec{A}$ is multiplied by a scalar quantity **c**, the magnitude of the product is given by $\mathbf{c}\vec{A}$. If **c** is positive, the direction of the product points in the same direction as $\vec{A}$; if **c** is negative, the direction of the product points in the opposite direction as $\vec{A}$.

## *Conceptual Questions*

**1:** Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?

**2:** Give a specific example of a vector, stating its magnitude, units, and direction.

**3:** What do vectors and scalars have in common? How do they differ?

**4:** Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?

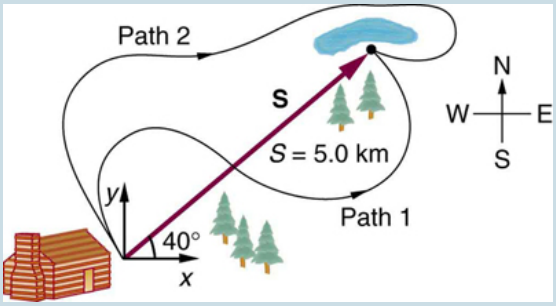



***Figure 19.***

**5:** If an airplane pilot is told to fly 123 km in a straight line

to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in Figure 20. What other information would he need to get to Sacramento?




***Figure 20.***

**6:** Suppose you take two steps A and B (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give

zero? Is the maximum distance you can end up from the starting point A+B the sum of the lengths of the two steps?

**7:** Explain why it is not possible to add a scalar to a vector.

**8:** If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

## *Problems & Exercises*

**Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.**

**1:** Find the following for path A in Figure 21: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

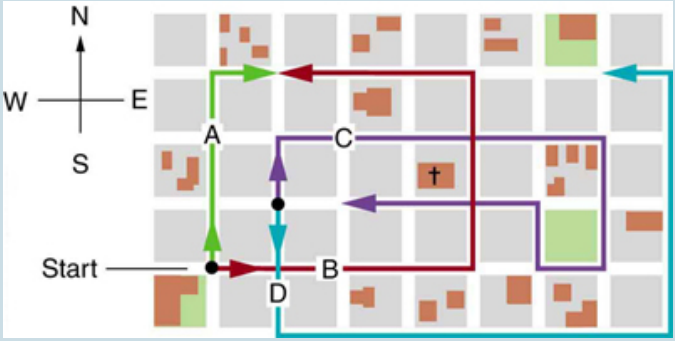



***Figure 21.*** *The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.*

**2:** Find the following for path B in Figure 21: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

**3:** Find the north and east components of the displacement for the hikers shown in Figure 19.

**4:** Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\overrightarrow{\mathrm{A}}$ and $\vec{\mathrm{B}}$, as in Figure 22, then this problem asks you to find their sum $\vec{\mathrm{R}} = \vec{\mathrm{A}} + \vec{\mathrm{B}}$.)

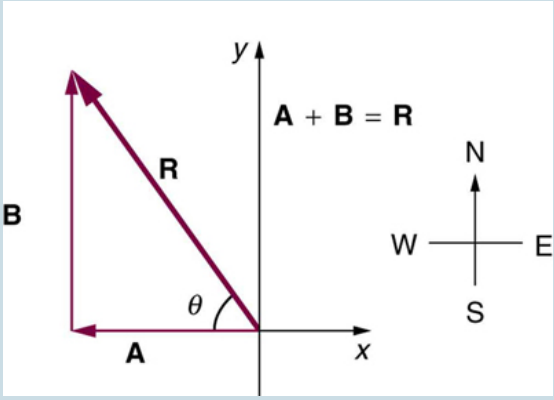



***Figure 22.*** *The two displacements* **A** *and* **B** *add to give a total displacement* **R** *having magnitude* **R** *and direction* $\boldsymbol{\theta}$.

**5:** Suppose you first walk 12.0 m in a direction 20° west of north and then 20.0 m in a direction 40.0° south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\vec{\mathrm{A}}$ and $\vec{\mathrm{B}}$, as in Figure 23, then this problem finds their sum $\vec{\mathrm{R}} = \vec{\mathrm{A}} + \vec{\mathrm{B}}$.)

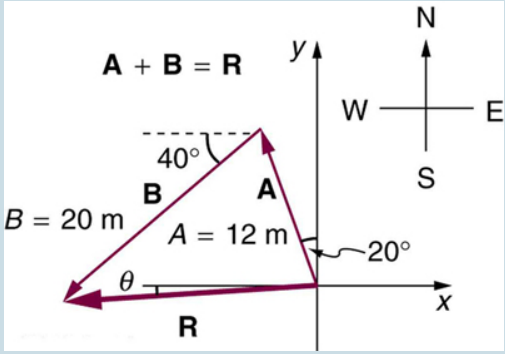



***Figure 23.***

**6:** Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg $\vec{\mathrm{B}}$, which is 20.0 m in a direction exactly 40° south of west, and then leg $\vec{\mathrm{A}}$, which is 12.0 m in a direction exactly 20° west of north. (This problem shows that $\vec{\mathrm{A}} + \vec{\mathrm{B}} = \vec{\mathrm{B}} + \vec{\mathrm{A}}$.)

**7:** (a) Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction 40.0° north of east (which is equivalent to subtracting $\vec{\mathrm{B}}$ from $\vec{\mathrm{A}}$ –that is, to finding $\vec{\mathrm{R}}' = \vec{\mathrm{A}} - \vec{\mathrm{B}}$). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction 40.0° south of west and then 12.0 m in a direction 20.0° east of south (which is equivalent to subtracting $\vec{\mathrm{A}}$ from $\vec{\mathrm{B}}$ –that is, to finding $\vec{\mathrm{R}}'' = \vec{\mathrm{B}} - \vec{\mathrm{A}} = -\vec{\mathrm{R}}'$). Show that this is the case.

**8:** Show that the *order* of addition of three vectors does not affect their sum. Show this property by choosing any three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$, all having different lengths and directions. Find the sum $\vec{A} + \vec{B} + \vec{C}$ then find their sum when added in a different order and show the result is the same. (There are five other orders in which $\vec{A}$, $\vec{B}$, and $\vec{C}$ can be added; choose only one.)

**9:** Show that the sum of the vectors discussed in Example 2 gives the result shown in Figure 17.

**10:** Find the magnitudes of velocities $\mathbf{v_A}$ and $\mathbf{v_B}$ in Figure 24

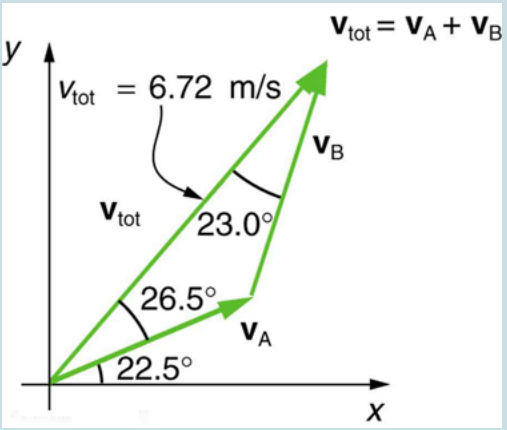



***Figure 24.*** *The two velocities* $\mathbf{v_A}$ *and* $\mathbf{v_B}$ *add to give a total* $\mathbf{v_{tot}}$.

**11:** Find the components of $\mathbf{v_{tot}}$ along the $x$- and $y$-axes in Figure 24.

**12:** Find the components of $\boldsymbol{v_{tot}}$ along a set of perpendicular axes rotated 30° counterclockwise relative to those in Figure 24.

## Glossary

**component (of a 2-d vector)**
: a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

**commutative**
: refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum

**direction (of a vector)**
: the orientation of a vector in space

**head (of a vector)**
: the end point of a vector; the location of the tip of the vector's arrowhead; also referred to as the "tip"

**head-to-tail method**
: a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

**magnitude (of a vector)**
: the length or size of a vector; magnitude is a scalar quantity

**resultant**
: the sum of two or more vectors

**resultant vector**
: the vector sum of two or more vectors

**scalar**
: a quantity with magnitude but no direction

**tail**

the start point of a vector; opposite to the head or tip of the arrow

## *Solutions*

**Problems & Exercises**

**1:** (a) $\mathbf{480\ m}$ (b) $\mathbf{379\ m, 18.4^o}$ east of north

**3:** north component $\mathbf{3.21\ km}$, east component $\mathbf{3.83\ km}$

**5:** $\mathbf{19.5\ m, 4.65^o}$ south of west

**7:** (a) $\mathbf{26.6\ m, 65.1^o}$ north of east (b) $\mathbf{26.6\ m, 65.1^o}$ south of west

**9:** $\mathbf{52.9\ m, 90.1^o}$ with respect to the $x$-axis.

**11:** $x$-component $\mathbf{4.41\ m/s}$, $y$-component $\mathbf{5.07\ m/s}$

# 17. 3.3 Vector Addition and Subtraction: Analytical Methods

*Summary*

- Understand the rules of vector addition and subtraction using analytical methods.
- Apply analytical methods to determine vertical and horizontal component vectors.
- Apply analytical methods to determine the magnitude and direction of a resultant vector.

**Analytical methods** of vector addition and subtraction employ geometry and simple trigonometry rather than the ruler and protractor of graphical methods. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization. However, analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Analytical methods are limited only by the accuracy and precision with which physical quantities are known.

## Resolving a Vector into Perpendicular Components

Analytical techniques and right triangles go hand-in-hand in physics because (among other things) motions along perpendicular directions are independent. We very often need to separate a vector into perpendicular components. For example, given a vector like $\vec{\mathrm{A}}$ in Figure 1, we may wish to find which two perpendicular vectors, $\mathrm{A}_x$ and $\mathrm{A}_y$, add to produce it. Notice these perpendicular component vectors will not have vector arrows in this text.

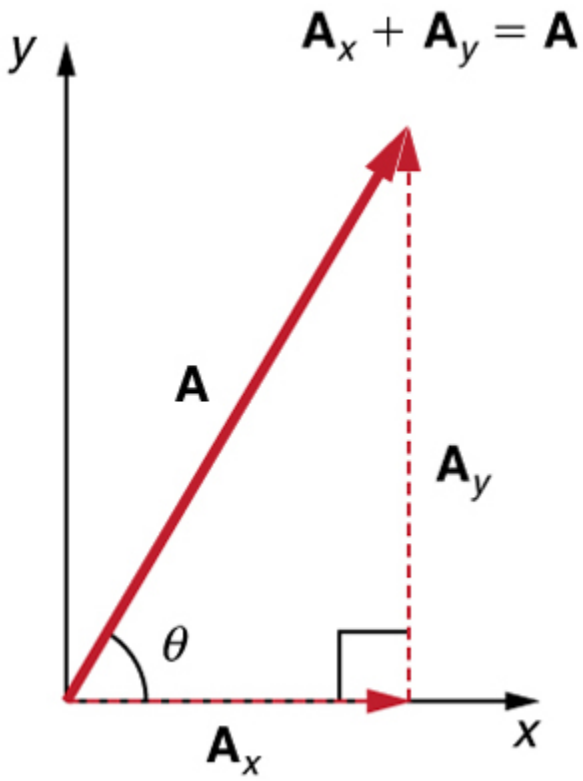



***Figure 1.*** *The vector* **A***, with its tail at the origin of an x, y-coordinate system, is shown together with its x- and y-components,* $\mathbf{A_x}$ *and* $\mathbf{A_y}$*. These vectors form a right triangle. The analytical relationships among these vectors are summarized below.*

$\mathrm{A}_x$ and $\mathrm{A}_y$ are defined to be the components of $\vec{\mathrm{A}}$ along the x- and y-axes. The three vectors $\vec{\mathrm{A}}$, $\mathrm{A}_x$, and $\mathrm{A}_y$ form a right triangle:

$$\mathbf{A_x} + \mathbf{A_y} = \vec{\mathbf{A}}.$$

Note that this relationship between vector components and the resultant vector holds only for vector quantities (which include both

magnitude and direction). The relationship does not apply for the magnitudes alone. For example, if $\mathrm{A}_x = 3\ \mathrm{m}$ east, $\mathrm{A}_y = 4\ \mathrm{m}$ north, and $\vec{\mathrm{A}} = 5\ \mathrm{m}$ north-east, then it is true that the vectors $\mathrm{A}_x + \mathrm{A}_y = \vec{\mathrm{A}}$. However, it is *not* true that the sum of the magnitudes of the vectors is also equal. That is,

$$\mathbf{3\ m + 4\ m \neq 5\ m}$$

Thus,

$$\mathbf{A_x + A_y \neq \vec{A}}$$

If the vector $\vec{\mathrm{A}}$ is known, then its magnitude **A** and its angle $\boldsymbol{\theta}$ (its direction) are known. To find $\mathrm{A}_x$ and $\mathrm{A}_y$, its x- and y-components, we use the following relationships for a right triangle.

$$\mathbf{A_x = A \cos\ \theta}$$

and

$$\mathbf{A_y = A \sin\ \theta.}$$

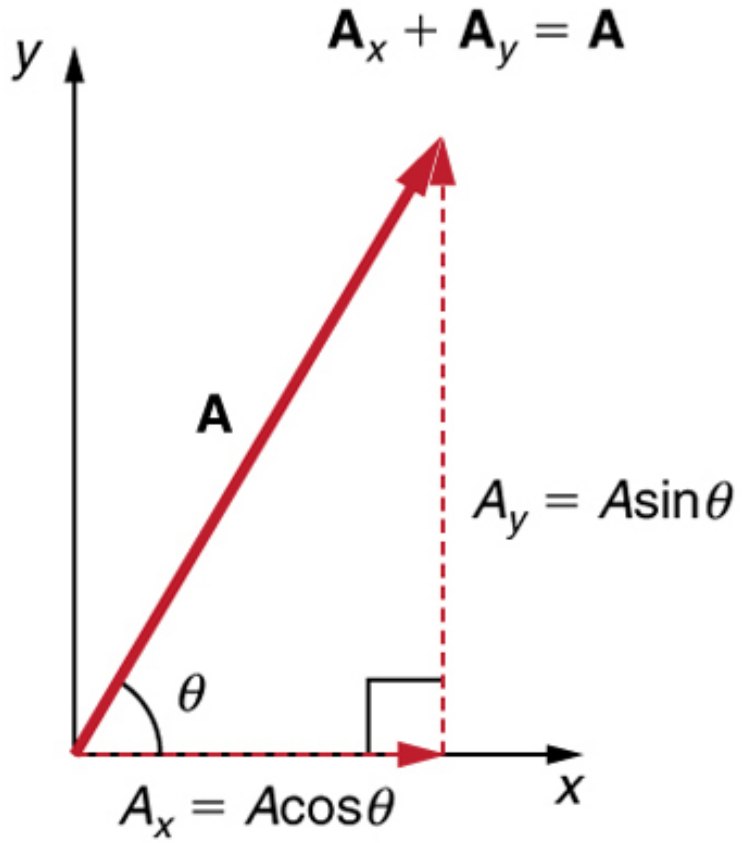



***Figure 2.*** *The magnitudes of the vector components* $\mathbf{A_x}$ *and* $\mathbf{A_y}$ *can be related to the resultant vector* **A** *and the angle* $\boldsymbol{\theta}$ *with trigonometric identities. Here we see that* $\mathbf{A_x = A\cos\theta}$ *and* $\mathbf{A_y = A\sin\theta}$.

Suppose, for example, that $\vec{\mathrm{A}}$ is the vector representing the total displacement of the person walking in a city considered in Chapter 3.1 Kinematics in Two Dimensions: An Introduction and Chapter 3.2 Vector Addition and Subtraction: Graphical Methods.

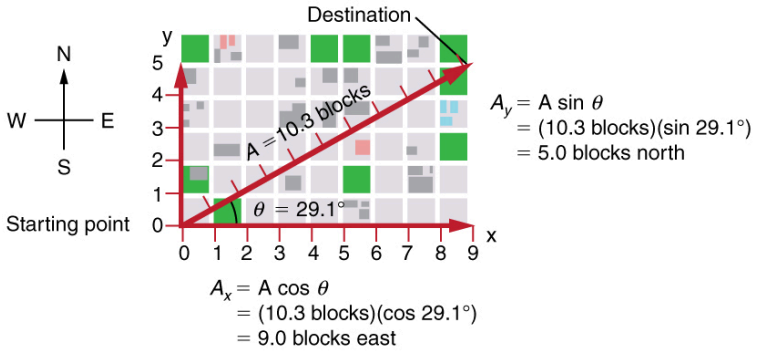



***Figure 3.*** *We can use the relationships* $\mathbf{A_x = A\cos\theta}$ *and* $\mathbf{A_y = A\sin\theta}$ *to determine the magnitude of the horizontal and vertical component vectors in this example.*

Then **A=10.3** blocks and $\boldsymbol{\theta}$**=29.1°**, so that

$$\mathbf{A}_x = \mathbf{A}\cos\boldsymbol{\theta} = (\mathbf{10.3\ blocks})(\cos 29.1^o) = \mathbf{9.0\ blocks}$$
$$\mathbf{A}_y = \mathbf{A}\sin\boldsymbol{\theta} = (\mathbf{10.3\ blocks})(\sin 29.1^o) = \mathbf{5.0\ blocks.}$$

## Calculating a Resultant Vector

If the perpendicular components $\mathrm{A}_x$ and $\mathrm{A}_y$ of a vector $\vec{\mathrm{A}}$ are known, then $\vec{\mathrm{A}}$ can also be found analytically. To find the magnitude A and direction θ of a vector from its perpendicular components $\mathrm{A}_x$ and $\mathrm{A}_y$, we use the following relationships:

$$\boldsymbol{A = \sqrt{\mathrm{A}_x{}^2 + \mathrm{A}_y{}^2}}$$
$$\boldsymbol{\theta = \tan^{-1}(\mathrm{A}_y / \mathrm{A}_x).}$$

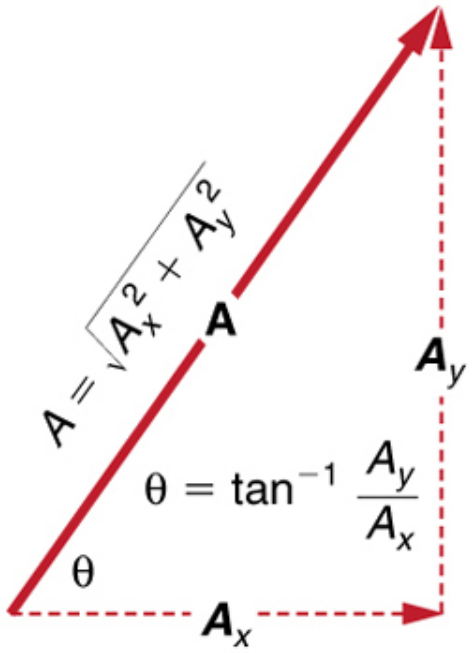



***Figure 4.*** *The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components* $\mathbf{A_x}$ *and* $\mathbf{A_y}$ *have been determined.*

Note that the equation $A = \sqrt{{\mathrm{A}_x}^2 + {\mathrm{A}_y}^2}$ is just the Pythagorean theorem relating the legs of a right triangle to the length of the hypotenuse. For example, if $\mathrm{A}_x$ and $\mathrm{A}_y$ are 9 and 5 blocks, respectively, then $A = \sqrt{9^2 + 5^2} = 10.3$ blocks, again consistent with the example of the person walking in a city. Finally, the direction is $\theta = \tan^{-1}(5/9) = 29.1^0$, as before.

### DETERMINING VECTORS AND VECTOR COMPONENTS WITH ANALYTICAL METHODS

Equations $\mathbf{A}_x = A\cos\theta$ and $\mathbf{A}_y = A\sin\theta$ are used to find the perpendicular components of a vector—that is, to go from A and θ to $\mathbf{A}_x$ and $\mathbf{A}_y$. Equations $A = \sqrt{{\mathbf{A}_x}^2 + {\mathbf{A}_y}^2}$ and $\theta = \tan^{-1}(\mathbf{A}_y / \mathbf{A}_x)$ are used to find a vector from its perpendicular components—that is, to go from $\mathbf{A}_x$ and $\mathbf{A}_y$ to A and θ. Both processes are crucial to analytical methods of vector addition and subtraction.

## Adding Vectors Using Analytical Methods

To see how to add vectors using perpendicular components, consider Figure 5, in which the vectors $\vec{\mathrm{A}}$ and $\vec{\mathrm{B}}$ are added to produce the resultant $\vec{\mathrm{R}}$.

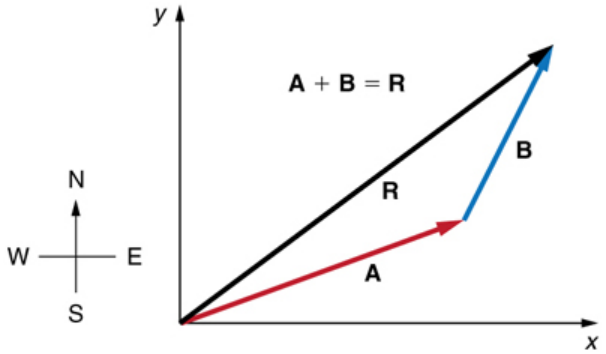



***Figure 5.*** *Vectors* **A** *and* **B** *are two legs of a walk, and* **R** *is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of* **R**.

If $\vec{\mathrm{A}}$ and $\vec{\mathrm{B}}$ represent two legs of a walk (two displacements), then $\vec{\mathrm{R}}$ is the total displacement. The person taking the walk ends up at the tip of $\vec{\mathrm{R}}$. There are many ways to arrive at the same point. In particular, the person could have walked first in the x-direction and then in the y-direction. Those paths are the x- and y-components of the resultant, $\mathrm{R}_x$ and $\mathrm{R}_y$. If we know $\mathrm{R}_x$ and $\mathrm{R}_y$, we can find R and θ using the equations $\boldsymbol{A = \sqrt{\mathrm{A}_x{}^2 + \mathrm{A}_y{}^2}}$ and $\boldsymbol{\theta = \tan^{-1}(\mathrm{A}_y \,/\, \mathrm{A}_x)}$. When you use the analytical method of vector addition, you can determine the components or the magnitude and direction of a vector.

***Step 1.*** *Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen perpendicular axes.* Use the equations $\mathrm{A}_x = A\cos\theta$ and $\mathrm{A}_y = A\sin\theta$ to find the components. In Figure 6, these components are $\mathrm{A}_x$, $\mathrm{A}_y$, $\mathrm{B}_x$, and $\mathrm{B}_y$. The angles that vectors $\vec{\mathrm{A}}$ and $\vec{\mathrm{B}}$ make with the *x*-axis are $\boldsymbol{\theta_{\mathrm{A}}}$ and $\boldsymbol{\theta_{\mathrm{B}}}$, respectively.

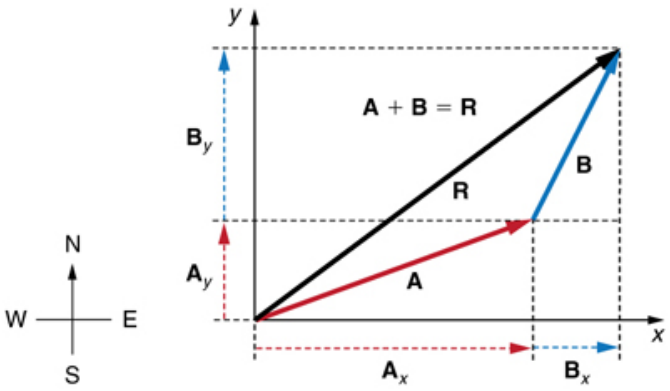



***Figure 6.*** *To add vectors* **A** *and* **B**, *first determine the horizontal and vertical components of each vector. These are the dotted vectors* $\mathbf{A_x}$, $\mathbf{A_y}$, $\mathbf{B_x}$*and* $\mathbf{B_y}$ *shown in the image.*

***Step 2.*** *Find the components of the resultant along each axis by adding the components of the individual vectors along that axis.* That is, as shown in Figure 7,

$$\mathbf{R}_x = \mathbf{A}_x + \mathbf{B}_x$$

and

$$\mathbf{R}_y = \mathbf{A}_y + \mathbf{B}_y.$$

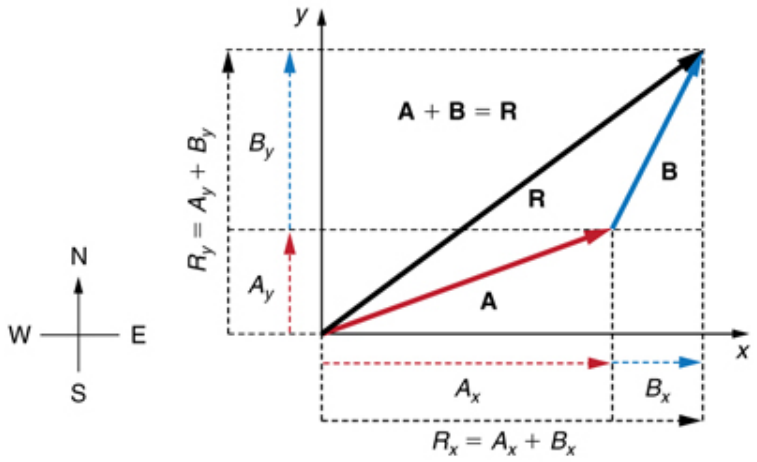



***Figure 7.*** *The magnitude of the vectors* $\mathbf{A_x}$ *and* $\mathbf{B_x}$ *add to give the magnitude* $\mathbf{R_x}$ *of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors* $\mathbf{A_y}$ *and* $\mathbf{B_y}$*add to give the magnitude* $\mathbf{R_y}$ *of the resultant vector in the vertical direction.*

Components along the same axis, say the $x$-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the $y$-axis. (For example, a 9-block eastward walk could be taken in two legs, the first 3 blocks east and the second 6 blocks east, for a total of 9, because they are along the same direction.) So resolving vectors into components along common axes makes it easier to add them. Now that the components of $\vec{\mathrm{R}}$ are known, its magnitude and direction can be found.

***Step 3.*** *To get the magnitude* **R** *of the resultant, use the Pythagorean theorem:*

$$\mathbf{R} = \sqrt{\mathbf{R}_x{}^2 + \mathbf{R}_y{}^2}.$$

***Step 4.*** *To get the direction of the resultant:*

$$\boldsymbol{\theta} = \mathbf{tan}^{-1}(\mathbf{R}_y \,/\, \mathbf{R}_x).$$

The following example illustrates this technique for adding vectors using perpendicular components.

### Example 1: Adding Vectors Using Analytical Methods

Add the vector $\vec{\mathbf{A}}$ to the vector $\vec{\mathbf{B}}$ shown in Figure 8, using perpendicular components along the $x$– and $y$-axes. The $x$– and $y$-axes are along the east–west and north–south directions, respectively. Vector $\vec{\mathbf{A}}$ represents the first leg of a walk in which a person walks **53.0 m** in a direction **20.0°** north of east. Vector $\vec{\mathbf{B}}$ represents the second leg, a displacement of **34.0 m** in a direction **63.0°** north of east.

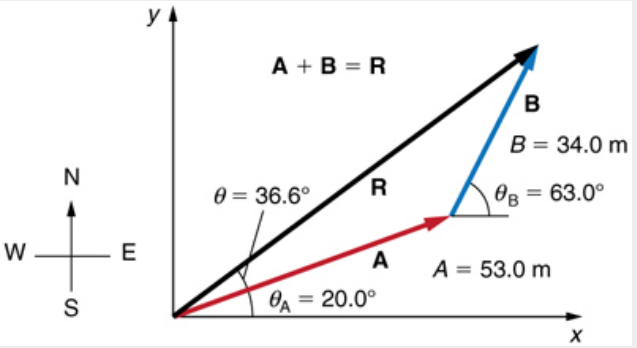



***Figure 8.*** *Vector* **A** *has magnitude* **53.0 m** *and direction* **20.0**$^0$ *north of the x-axis. Vector* **B** *has magnitude* **34.0 m** *and direction* **63.0**$^0$ *north of the x-axis. You can use analytical methods to determine the magnitude and direction of* **R**.

**Strategy**

The components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ along the $x$– and $y$-axes represent walking due east and due north to get to the same ending point. Once found, they are combined to produce the resultant.

**Solution**

Following the method outlined above, we first find the components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ along the $x$– and $y$-axes. Note that **A=53.0 m**, $\boldsymbol{\theta}_\mathbf{A}$**=20.0°**, **B=34.0 m**, and $\boldsymbol{\theta}_\mathbf{B}$**=63.0°**. We find the $x$-components by using $\mathbf{A}_x = A\cos\theta$, which gives

$$\mathbf{A}_x = A\cos\,\theta_A = (\mathbf{53.0\ m})(\cos 20.0^o) = (\mathbf{53.0\ m})(\mathbf{0.940}) = \mathbf{49.8\ m}$$

and

$$\mathbf{B}_x = B\cos\,\theta_B = (34.0\text{ m})(\cos 63.0^o)$$
$$= (34.0\text{ m})(0.454) = 15.4\text{ m}.$$

Similarly, the $y$-components are found using $\mathbf{A}_y = A\sin\theta$:

$$\mathbf{A}_y = A\sin\,\theta_A = (53.0\text{ m})(\sin 20.0^o)$$
$$= (53.0\text{ m})(0.342) = 18.1\text{ m}$$

and

$$\mathbf{B}_y = B\sin\,\theta_B = (34.0\text{ m})(\sin 63.0^o)$$
$$= (34.0\text{ m})(0.891) = 30.3\text{ m}.$$

The $x$- and $y$-components of the resultant are thus

$$\mathbf{R}_x = \mathbf{A}_x + \mathbf{B}_x = 49.8\text{ m} + 15.4\text{ m} = 65.2\text{ m}$$

and

$$\mathbf{R}_y = \mathbf{A}_y + \mathbf{B}_y = 18.1\text{ m} + 30.3\text{ m} = 48.4\text{ m}.$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem:

$$R = \sqrt{\mathbf{R}_x^2 + \mathbf{R}_y^2} = \sqrt{(65.2)^2 + (48.4)^2\text{ m}}$$

so that

$$R = 81.2\text{ m}.$$

Finally, we find the direction of the resultant:

$$\theta = \tan^{-1}(\mathbf{R}_y\,/\,\mathbf{R}_x) = +\tan^{-1}(48.4/65.2).$$

Thus,

$$\theta = \tan^{-1}(0.742) = 36.6^o.$$

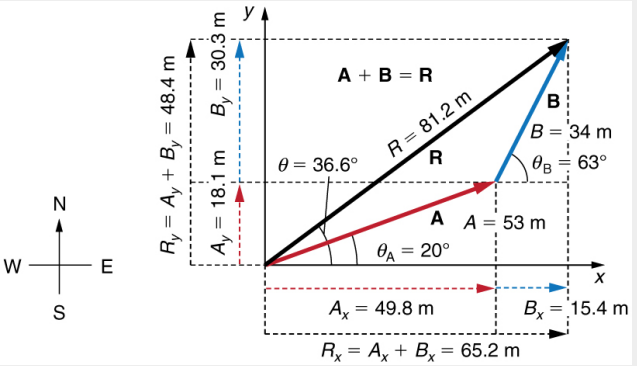



***Figure 9.*** *Using analytical methods, we see that the magnitude of* **R** *is* ***81.2 m*** *and its direction is* $\mathbf{36.6^0}$ *north of east.*

**Discussion**

This example illustrates the addition of vectors using perpendicular components. Vector subtraction using perpendicular components is very similar—it is just the addition of a negative vector.

Subtraction of vectors is accomplished by the addition of a negative vector. That is, $\vec{\mathrm{A}} - \vec{\mathrm{B}} \equiv \vec{\mathrm{A}} + (-\vec{\mathrm{B}})$. Thus, *the method for the subtraction of vectors using perpendicular components is identical to that for addition*. The components of $-\vec{\mathrm{B}}$ are the negatives of the components of $\vec{\mathrm{B}}$. The $x$- and $y$-components of the resultant $\vec{\mathrm{A}} - \vec{\mathrm{B}} = \vec{\mathrm{R}}$ are thus

$$\mathbf{R}_x = \mathbf{A}_x + (-\mathbf{B}_x)$$

and

$$\mathbf{R}_y = \mathbf{A}_y + (-\mathbf{B}_y)$$

and the rest of the method outlined above is identical to that for addition. (See Figure 10.)

Analyzing vectors using perpendicular components is very useful in many areas of physics, because perpendicular quantities are often independent of one another. The next module, Chapter 3.4 Projectile Motion, is one of many in which using perpendicular components helps make the picture clear and simplifies the physics.

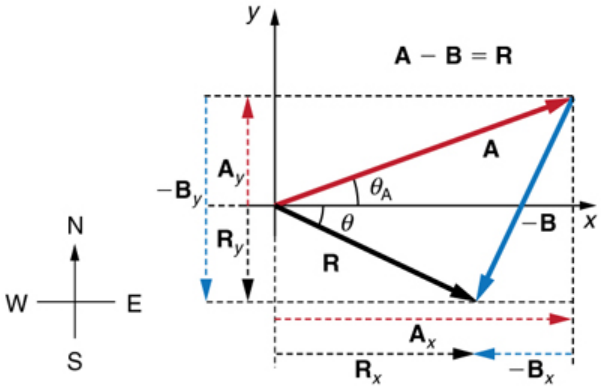



***Figure 10.*** *The subtraction of the two vectors shown in Figure 5. The components of* **–B** *are the negatives of the components of* **B**. *The method of subtraction is the same as that for addition.*

PHET EXPLORATIONS: VECTOR ADDITION

Learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.



PhET Interactive Simulation

***Figure 11.*** *Vector Addition*

## Summary

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ using the analytical method are as follows:

  Step 1: Determine the coordinate system for the vectors. Then,

determine the horizontal and vertical components of each vector using the equations

$$\mathbf{A}_x = \boldsymbol{A}\cos\,\boldsymbol{\theta}$$
$$\mathbf{B}_x = \boldsymbol{B}\cos\,\boldsymbol{\theta}$$

and

$$\mathbf{A}_y = \boldsymbol{A}\sin\,\boldsymbol{\theta}$$
$$\mathbf{B}_y = \boldsymbol{B}\sin\,\boldsymbol{\theta}.$$

Step 2: Add the horizontal and vertical components of each vector to determine the components $\mathbf{R}_x$ and $\mathbf{R}_y$ of the resultant vector, $\vec{\mathbf{R}}$:

$$\mathbf{R}_x = \mathbf{A}_x + \mathbf{B}_x$$

and

$$\mathbf{R}_y = \mathbf{A}_y + \mathbf{B}_y.$$

Step 3: Use the Pythagorean theorem to determine the magnitude, **R**, of the resultant vector $\vec{\mathbf{R}}$:

$$\boldsymbol{R} = \sqrt{\mathbf{R}_x{}^2 + \mathbf{R}_y{}^2}.$$

Step 4: Use a trigonometric identity to determine the direction, $\boldsymbol{\theta}$, of **R**:

$$\boldsymbol{\theta} = \tan^{-1}(\mathbf{R}_y / \mathbf{R}_x).$$

## *Conceptual Questions*

**1:** Suppose you add two vectors $\vec{\mathrm{A}}$ and $\vec{\mathrm{B}}$. What relative direction between them produces the resultant with the

greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?

**2:** Give an example of a nonzero vector that has a component of zero.

**3:** Explain why a vector cannot have a component greater than its own magnitude.

**4:** If the vectors $\vec{A}$ and $\vec{B}$ are perpendicular, what is the component of $\vec{A}$ along the direction of $\vec{B}$? What is the component of $\vec{B}$ along the direction of $\vec{A}$?

## *Problems & Exercises*

**1:** Find the following for path C in Figure 12: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

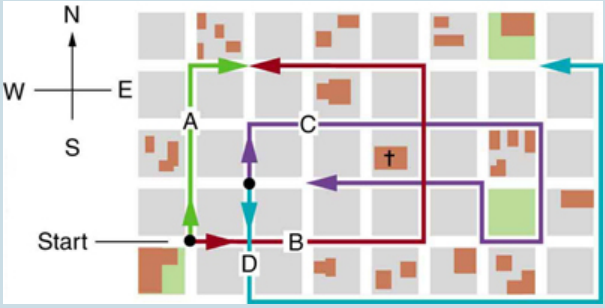



***Figure 12.*** *The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.*

**2:** Find the following for path D in Figure 12: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

**3:** Find the north and east components of the displacement from San Francisco to Sacramento shown in Figure 13.




***Figure 13.***

**4:** Solve the following problem using analytical techniques: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $vec\mathrm{A}$ and $\vec{\mathrm{B}}$, as in Figure 14, then this problem asks you to find their sum $\vec{\mathrm{R}} = \vec{\mathrm{A}} + \vec{\mathrm{B}}$.)

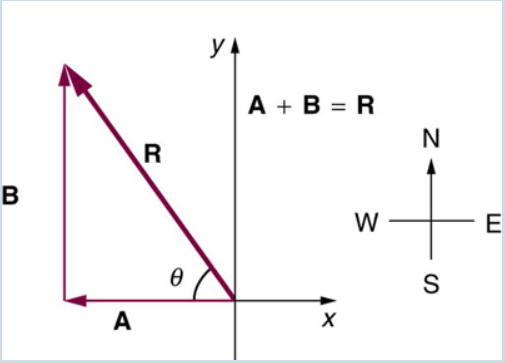



***Figure 14.*** *The two displacements* **A** *and* **B** *add to give a total displacement* **R** *having magnitude* **R** *and direction* $\boldsymbol{\theta}$.

Note that you can also solve this graphically. Discuss why the analytical technique for solving this problem is potentially more accurate than the graphical technique.

**5:** Repeat Exercise 4 using analytical techniques, but reverse the order of the two legs of the walk and show that you get the same final result. (This problem shows that adding them in reverse order gives the same result–that is, $\vec{\mathrm{B}} + \vec{\mathrm{A}} = \vec{\mathrm{A}} + \vec{\mathrm{B}}$.) Discuss how taking another path to reach the same point might help to overcome an obstacle blocking you other path.

**6:** You drive 7.50 km in a straight line in a direction 15° east of north. (a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (This determination is equivalent to find the components of the displacement along the east and north

directions.) (b) Show that you still arrive at the same point if the east and north legs are reversed in order.

**7:** Do Exercise 4 again using analytical techniques and change the second leg of the walk to 25.0 m straight south. (This is equivalent to subtracting $\vec{B}$ from $\vec{A}$ –that is, finding $\vec{R}' = \vec{A} - \vec{B}$) (b) Repeat again, but now you first walk 25.0 m north and then 18.0 m east. (This is equivalent to subtract $\vec{A}$ from $\vec{B}$ –that is, to find $\vec{A} = \vec{B} + \vec{C}$. Is that consistent with your result?)

**8:** A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be 80.0 m long and the next to be 105 m. These sides are represented as displacement vectors $\vec{A}$ from $\vec{B}$ in Figure 15. She then correctly calculates the length and orientation of the third side $\vec{C}$. What is her result?

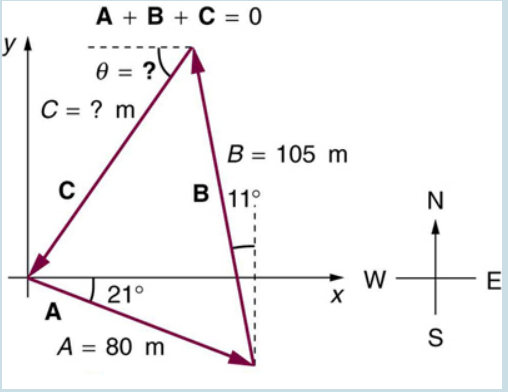



***Figure 15.***

**9:** You fly 32.0 km in a straight line in still air in the direction 35.0° south of west. (a) Find the distances you would have to fly straight south and then straight west to arrive at the same point. (This determination is equivalent to finding the components of the displacement along the south and west directions.) (b) Find the distances you would have to fly first in a direction 45.0° south of west and then in a direction 45.0° west of north. These are the components of the displacement along a different set of axes—one rotated 45°.

**10:** A farmer wants to fence off his four-sided plot of flat land. He measures the first three sides, shown as $\vec{\mathrm{A}}$, $\vec{\mathrm{B}}$, and $\vec{\mathrm{C}}$ in Figure 16, and then correctly calculates the length and orientation of the fourth side $\vec{\mathrm{D}}$. What is his result?

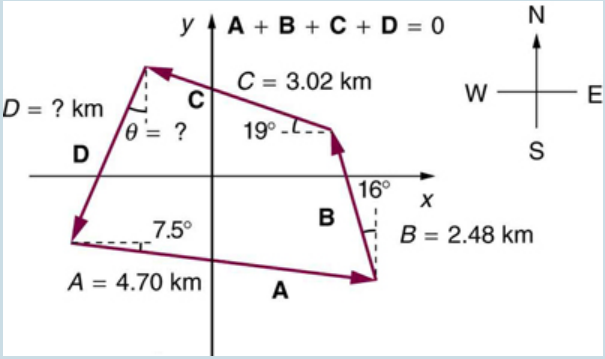



***Figure 16.***

**11:** In an attempt to escape his island, Gilligan builds a raft and sets to sea. The wind shifts a great deal during the day, and he is blown along the following straight lines: 2.50 km 45.0° north of west; then 4.70 km 60.0° south of east; then 1.30 km 25.0° south of west; then 5.10 km straight east; then 1.70 km 5.00° east of north; then 7.20 km 55.0° south of west; and finally 2.80 km 10.0° north of east. What is his final position relative to the island?

**12:** Suppose a pilot flies 40.0 km in a direction 60° north of east and then flies 30.0 km in a direction 15° north of east as shown in Figure 17. Find her total distance **R** from the starting point and the direction θ of the straight-line path to the final position. Discuss qualitatively how this flight would be altered by a wind from the north and how the effect of the wind would depend on both wind speed and the speed of the plane relative to the air mass.

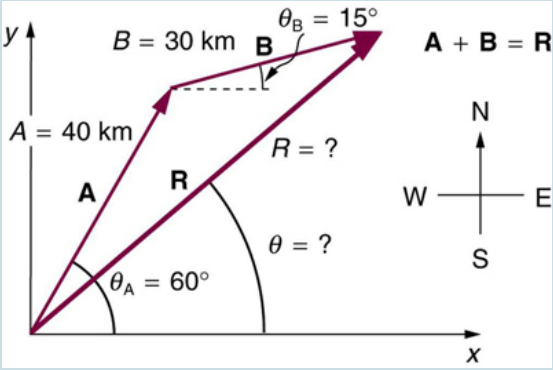



***Figure 17.***

## Glossary

**analytical method**
: the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

## *Solutions*

**Problems & Exercises**

**1:** (a) 1.56 km (b) 120 m east

**3:** North-component 87.0 km, east-component 87.0 km

**5:** 30.8 m, 35.8 degrees west of north

**7:** (a) $\mathbf{30.8\ m, 54.2^o}$ south of west (b) $\mathbf{30.8\ m, 54.2^o}$ north of east

**9:** (a) 18.4 km south, then 26.2 km west (b) 31.5 km at $\mathbf{45.0^o}$ south of west, then 5.56 km at $\mathbf{45.0^o}$ west of north

**11:** $\mathbf{7.34\ km, 63.5^o}$ south of east

# 18. 3.4 Projectile Motion

*Summary*

- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

**Projectile motion** is the **motion** of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a **projectile**, and its path is called its **trajectory**. The motion of falling objects, as covered in Chapter 2.6 Problem-Solving Basics for One-Dimensional Kinematics, is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which **air resistance** *is negligible*.

## PHET EXPLORATIONS: PROJECTILE MOTION

Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target.




***Figure 7.*** *Projectile Motion*

*An interactive or media element has been excluded from this version of the text. You can view it online here:*
*https://pressbooks.bccampus.ca/douglasphys1104summer2021/?p=222*

The most important fact to remember here is that *motions along perpendicular axes are independent* and thus can be analyzed separately. This fact was discussed in Chapter 3.1 Kinematics in Two Dimensions: An Introduction, where vertical and horizontal motions

were seen to be independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the $x$-axis and the vertical axis the $y$-axis. Figure 1 illustrates the notation for displacement, where $\vec{\mathbf{s}}$ is defined to be the total displacement and **x** and **y** are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are **s**, **x**, and **y**. (Note that in the last section we used the notation $\vec{\mathbf{A}}$ to represent a vector with components $\mathbf{A_x}$ and $\mathbf{A_y}$. If we continued this format, we would call displacement $\vec{\mathbf{s}}$ with components $\mathbf{s_x}$ and $\mathbf{s_y}$. However, to simplify the notation, we will simply represent the component vectors as **x** and **y**.)

Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the $x$– and $y$-axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple: $\mathbf{a_y = -g = -9.80\ m/s^2}$. (Note that this definition assumes that the upwards direction is defined as the positive direction. If you arrange the coordinate system instead such that the downwards direction is positive, then acceleration due to gravity takes a positive value.) Because gravity is vertical, $\mathbf{a_x = 0}$. Both accelerations are constant, so the kinematic equations can be used.

REVIEW OF KINEMATIC

## EQUATIONS (CONSTANT $\alpha$)

$$x = x_0 + \bar{v}t$$
$$\bar{v} = \frac{v_0 + v}{2}$$
$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v_2 = v_0^2 + 2a(x - x_0).$$

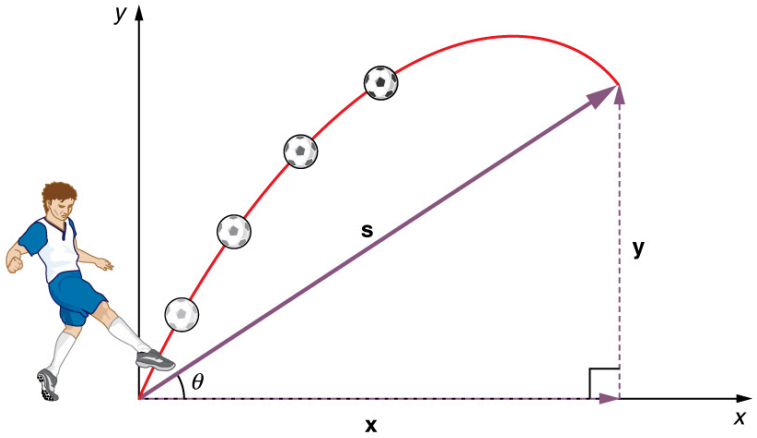



***Figure 1.*** *The total displacement* **s** *of a soccer ball at a point along its path. The vector* **s** *has components* **x** *and* **y** *along the horizontal and vertical axes. Its magnitude is* **s**, *and it makes an angle* $\boldsymbol{\theta}$ *with the horizontal.*

Given these assumptions, the following steps are then used to analyze projectile motion:

**Step 1.** *Resolve or break the motion into horizontal and vertical components along the x- and y-axes.* These axes are perpendicular, so $A_x = A\cos\theta$ and $A_y = A\sin\theta$ are used. The magnitude of the components of displacement $\vec{\mathbf{s}}$ along these axes are $\boldsymbol{x}$ and $\mathbf{y}$. The magnitudes of the components of the velocity $\vec{\mathbf{v}}$ are $\boldsymbol{v_x = v\cos\theta}$ and $\boldsymbol{v_y = v\sin\theta}$, where $\boldsymbol{v}$ is the magnitude of the velocity and $\boldsymbol{\theta}$ is its direction, as shown in Figure 2. Initial values are denoted with a subscript 0, as usual.

**Step 2.** *Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical.* The kinematic equations for horizontal and vertical motion take the following forms:

$$\textbf{Horizontal Motion}(a_x = 0)$$

$$\mathbf{x = x_0 + v_x t}$$

$$\mathbf{v_x = v_{0x} = v_x = \text{velocity is a constant.}}$$

$$\textbf{Vertical Motion(assuming positive is up } a_y = -g = -9.80\ \text{m/s}^2\textbf{)}$$

$$\mathbf{y = y_0 + \frac{1}{2}(v_{0y} + v_y)t}$$

$$\mathbf{v_y = v_{0y} - gt}$$

$$\mathbf{y = y_0 + v_{0y}t - \frac{1}{2}gt^2}$$

$$\mathbf{v_y^2 = v_{0y}^2 - 2g(y - y_0).}$$

**Step 3.** *Solve for the unknowns in the two separate motions—one horizontal and one vertical.* Note that the only common variable between the motions is time $\mathbf{t}$. The problem solving procedures here are the same as for one-dimensional **kinematics** and are illustrated in the solved examples below.

**Step 4.** *Recombine the two motions to find the total displacement* $\vec{\mathbf{s}}$ *and velocity* $\vec{\mathbf{v}}$. Because the $x$ – and $y$ -motions are perpendicular, we determine these vectors by using the techniques outlined in the

Chapter 3.3 Vector Addition and Subtraction: Analytical Methods

and employing $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}(A_y / A_x)$ in the following form, where $\theta_d$ is the direction of the displacement $\vec{s}$ and $\theta$ is the direction of the velocity $\vec{v}$:

$$s = \sqrt{x^2 + y^2}$$
$$\theta_d = \tan^{-1}(y / x)$$
$$v = \sqrt{v_x^2 + v_y^2}$$
$$\theta = \tan^{-1}(v_y / v_x).$$

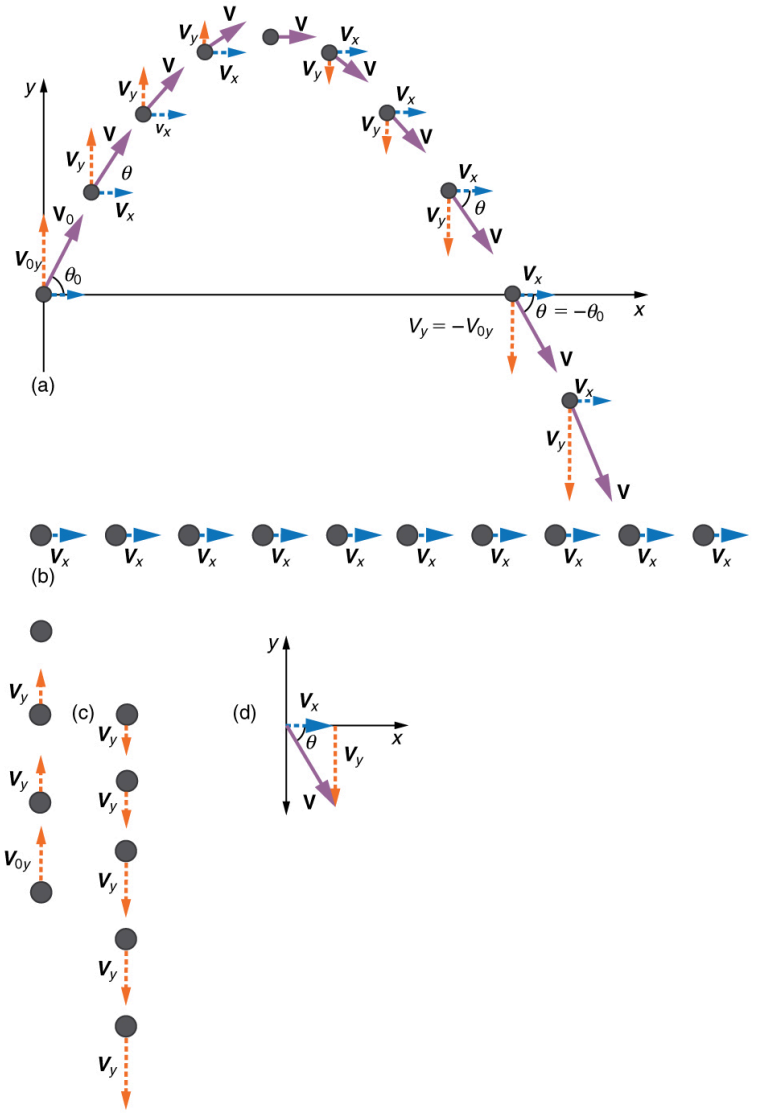



***Figure 2.*** *(a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because* $\mathbf{a_x=0}$ *and* $\mathbf{v_x}$ *is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The x – and y*

*–motions are recombined to give the total velocity at any given point on the trajectory.*

## Example 1: A Fireworks Projectile Explodes High and Away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75.0° above the horizontal, as illustrated in Figure 3. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?

**Strategy**

Because air resistance is negligible for the unexploded shell, the analysis method outlined above can be used. The motion can be broken into horizontal and vertical motions in which $\mathbf{a_x=0}$ and $\mathbf{a_y=-g}$. We can then define $\mathbf{x_0}$ and $\mathbf{y_0}$ to be zero and solve for the desired quantities.

**Solution for (a)**

By "height" we mean the altitude or vertical position $\mathbf{y}$ above the starting point. The highest point in any trajectory, called the apex, is reached when $\mathbf{v_y = 0}$. Since we know the initial and final velocities as well as the

initial position, we use the following equation to find $y$:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

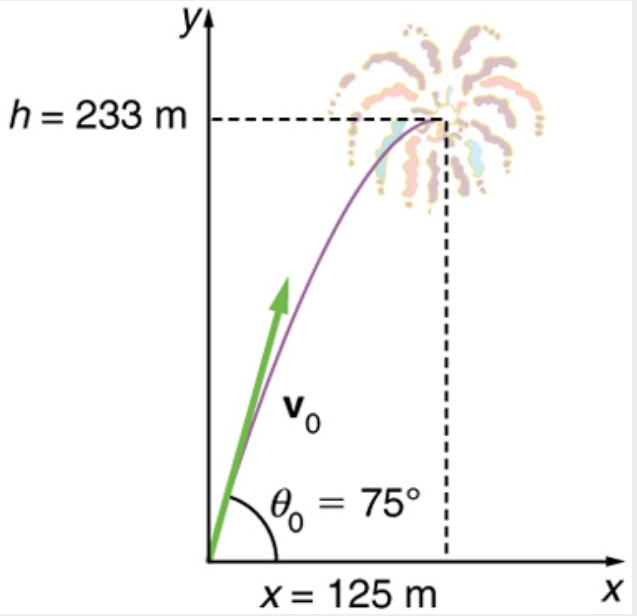



***Figure 3.*** *The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.*

Because $y_0$ and $v_y$ are both zero, the equation simplifies to

$$0 = v_{0y}^2 - 2gy.$$

Solving for $y$ gives

$$y = \frac{v_{0y}^2}{2g}.$$

Now we must find $v_{0y}$, the component of the initial velocity in the $y$-direction. It is given by $v_{0y} = v_0 \sin \theta$, where $v_0$ is the initial velocity of **70.0 m/s**, and $\theta_0$=75.0° is the initial angle. Thus,

$$v_{0y} = v_0 \sin \theta_0 = (70.0 \text{ m/s})(\sin 75^o) = 67.6 \text{ m/s}.$$

and $y$ is

$$y = \frac{(67.6 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)},$$

so that

$$y = 233 \text{ m}.$$

**Discussion for (a)**

Note that because up is positive, the initial velocity is positive, as is the maximum height, but the acceleration due to gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6 m/s initial vertical component of velocity will reach a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, and so the initial velocity would have to be somewhat larger than that given to reach the same height.

**Solution for (b)**

As in many physics problems, there is more than one way to solve for the time to the highest point. In this case, the easiest method is to use $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$. Because $y_0$ is zero, this equation reduces to simply

$$y = \frac{1}{2}(v_{0y} + v_y)t.$$

Note that the final vertical velocity, $v_y$, at the highest point is zero. Thus,

$$t = \frac{2y}{(v_{0y} + v_y)} = \frac{2(233\ \text{m})}{(67.6\ \text{m/s})} = 6.90\ \text{s}.$$

**Discussion for (b)**

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. (Another way of finding the time is by using $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, and solving the quadratic equation for $t$.)

**Solution for (c)**

Because air resistance is negligible, $a_x = 0$ and the horizontal velocity is constant, as discussed above. The horizontal displacement is horizontal velocity multiplied by time as given by $x = x_0 + v_x t$, where $x_0$ is equal to zero:

$$x = v_x t,$$

where $\mathbf{v_x}$ is the $x$-component of the velocity, which is given by $\mathbf{v_x = v_0 \cos \theta_0}$. Now,

$$v_x = v_0 \cos\theta_0 = (70.0 \text{ m/s})(\cos 75.0^o) = 18.1 \text{ m/s}.$$

The time $\mathbf{t}$ for both motions is the same, and so $\mathbf{x}$ is

$$x = (18.1 \text{ m/s})(6.90 \text{ s}) = 125 \text{ m}.$$

**Discussion for (c)**

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below.

In solving part (a) of the preceding example, the expression we found for $\mathbf{y}$ is valid for any projectile motion where air resistance is negligible. Call the maximum height $\mathbf{y = h}$; then,

$$h = \frac{v_{0y}^2}{2g}.$$

This equation defines the *maximum height of a projectile* and depends only on the vertical component of the initial velocity.

## DEFINING A COORDINATE

## SYSTEM

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the $x$ and $y$ positions. Often, it is convenient to choose the initial position of the object as the origin such that $x_0 = 0$ and $y_0 = 0$. It is also important to define the positive and negative directions in the $x$ and $y$ directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration, $g$, takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, $g$ takes a positive value.

## Example 2: Calculating Projectile

## Motion: Hot Rock Projectile

Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle 35.0° above the horizontal, as shown in Figure 4. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact?

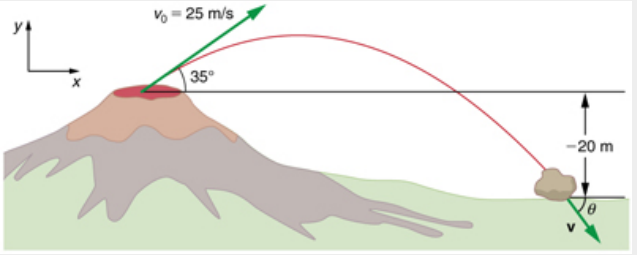



***Figure 4.*** *The trajectory of a rock ejected from the Kilauea volcano.*

**Strategy**

Again, resolving this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the desired quantities. The time a projectile is

in the air is governed by its vertical motion alone. We will solve for $t$ first. While the rock is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, the vertical and horizontal results will be recombined to obtain $v$ and $\theta_v$ at the final time $t$ determined in the first part of the example.

**Solution for (a)**

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2.$$

If we take the initial position $y_0$ to be zero, then the final position is $y = -20.0\ \text{m}$. Now the initial vertical velocity is the vertical component of the initial velocity, found from $v_{0y} = v_0 \sin \theta_0 = (25.0\ \text{m/s})(\sin 35.0°) = 14.3\ \text{m/s}$. Substituting known values yields

$$-20.0\ \text{m} = (14.3\ \text{m/s})t - 4.90\ \text{m/s}^2t^2.$$

Rearranging terms gives a quadratic equation in $t$:

$$4.90\ \text{m/s}^2t^2 - 14.3\ \text{m/s}t - 20.0\ \text{m} = 0.$$

This expression is a quadratic equation of the form $at^2+bt+c = 0$, where the constants are $a = 4.90$, $b = -14.3$, and $c=-20.0$. Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This equation yields two solutions: $t = 3.96$ and $t =$

**-1.03**. (It is left as an exercise for the reader to verify these solutions.) The time is **t = 3.96 s** or **-1.03 s**. The negative value of time implies an event before the start of motion, and so we discard it. Thus,

$$t = 3.96 \text{ s}.$$

**Discussion for (a)**

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3 m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

**Solution for (b)**

From the information now in hand, we can find the final horizontal and vertical velocities $v_x$ and $v_y$ and combine them to find the total velocity $v$ and the angle $\theta_0$ it makes with the horizontal. Of course, $v_x$ is constant so we can solve for it at any horizontal location. In this case, we chose the starting point since we know both the initial velocity and initial angle. Therefore:

$$v_x = v_0 \cos \theta_0 = (25.0 \text{ m/s})(\cos 35^o) = 20.5 \text{ m/s}.$$

The final vertical velocity is given by the following equation:

$$v_y = v_{0y} - gt,$$

where $v_{0y}$ was found in part (a) to be **14.3 m/s**. Thus,

$$v_y = 14.3 \text{ m/s} - (9.80 \text{ m/s}^2)(3.96 \text{ s})$$

so that

$$v_y = -24.5 \text{ m/s}.$$

To find the magnitude of the final velocity $v$ we

combine its perpendicular components, using the following equation:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20.5 \text{ m/s})^2 + (-24.5 \text{ m/s})^2},$$

which gives

$$v = 31.9 \text{ m/s}.$$

The direction $\theta_v$ is found from the equation:

$$\theta_v = \tan^{-1}(v_y / v_x)$$

so that

$$\theta_v = \tan^{-1}(-24.5/20.5) = \tan^{-1}(-1.19).$$

Thus,

$$\theta_v = -50.1^o.$$

**Discussion for (b)**

The negative angle means that the velocity is **50.1°** below the horizontal. This result is consistent with the fact that the final vertical velocity is negative and hence downward—as you would expect because the final altitude is 20.0 m lower than the initial altitude. (See Figure 4.)

One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. Galileo was the first person to fully comprehend this characteristic. He used it to predict the range of a projectile. On level ground, we define **range** to be the horizontal distance **R** traveled by a projectile. Galileo and many others were interested in the range of projectiles primarily for military purposes—such as aiming cannons. However, investigating the range of projectiles can shed light on other

interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.

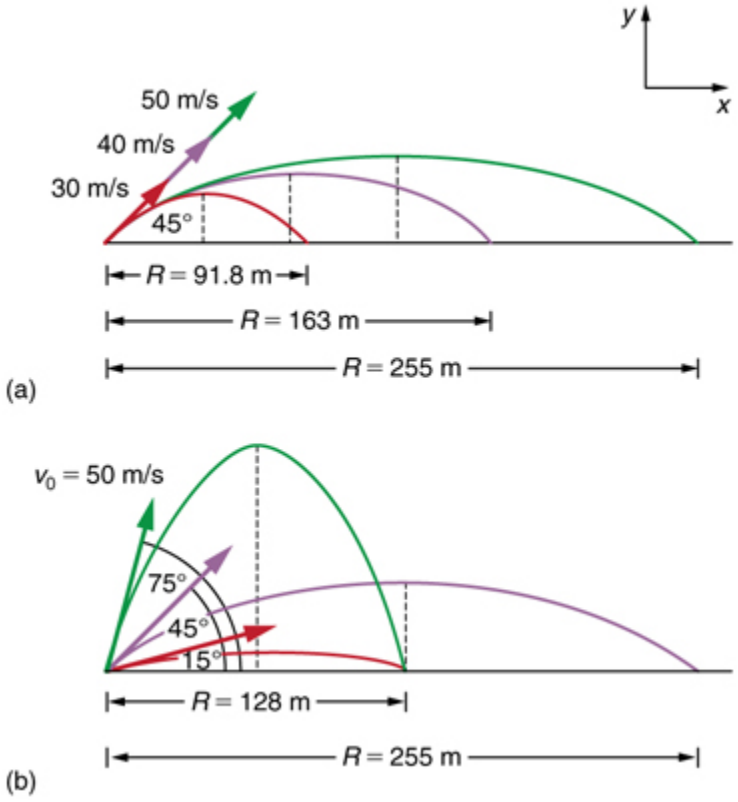



***Figure 5.*** *Trajectories of projectiles on level ground. (a) The greater the initial speed* $\boldsymbol{v_0}$*, the greater the range for a given initial angle. (b) The effect of initial angle* $\boldsymbol{\theta_0}$ *on the range of a projectile with a given initial speed. Note that the range is the same for* $\mathbf{15^o}$ *and* $\mathbf{75^o}$*, although the maximum heights of those paths are different.*

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed $\boldsymbol{v_0}$, the greater the range, as shown in Figure 5(a). The initial angle $\boldsymbol{\theta_0}$ also has a dramatic effect on the range, as illustrated in Figure 5(b). For a fixed initial speed,

such as might be produced by a cannon, the maximum range is obtained with $\boldsymbol{\theta_0 = 45°}$. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately **38°**. Interestingly, for every initial angle except **45°** there are two angles that give the same range—the sum of those angles is **90°**. The range also depends on the value of the acceleration of gravity $\boldsymbol{g}$. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range **R** of a projectile on *level ground* for which air resistance is negligible is given by

$$\boldsymbol{R = \frac{v_0^2 \sin 2\theta_0}{g}},$$

where $\boldsymbol{v_0}$ is the initial speed and $\boldsymbol{\theta_0}$ is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that **R** is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See Figure 6.) If the initial speed is great enough, the projectile goes into orbit. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.

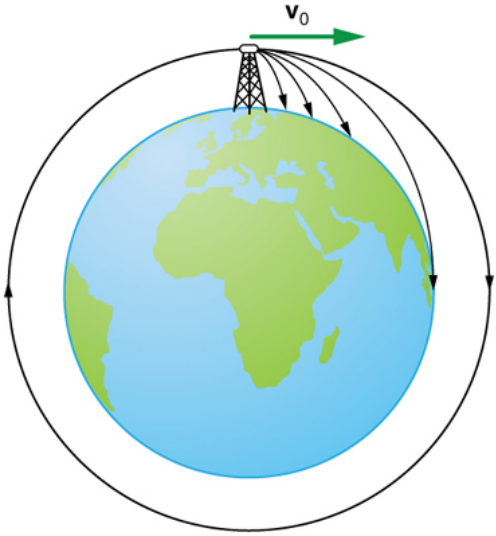



***Figure 6.*** *Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.*

## PHET EXPLORATIONS: PROJECTILE MOTION

Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial

speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target.



PhET Interactive Simulation

***Figure 7.*** Projectile Motion

## Summary

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:

1. Determine a coordinate system. Then, resolve the position and/or velocity of the object in the horizontal and vertical components. The components of position $\vec{\mathbf{s}}$ are given by the quantities $\mathbf{x}$ and $\mathbf{y}$, and the components of the velocity $\vec{\mathbf{v}}$ are given by $\mathbf{v_x = v \cos \theta}$ and $\mathbf{v_y = v \sin \theta}$, where $\mathbf{v}$ is the magnitude of the velocity and $\boldsymbol{\theta}$ is its direction.
2. Analyze the motion of the projectile in the horizontal direction using the following equations:

$$\textbf{Horizontal motion}(a_x = 0)$$
$$x = x_0 + v_x t$$
$$v_x = v_{0x} = \mathrm{v}_x = \textbf{velocity is a constant.}$$

3. Analyze the motion of the projectile in the vertical direction using the following equations:

**Vertical motion (Assuming positive direction is up; $a_y = -g = -9.80\ \text{m/s}^2$)**

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$
$$v_y = v_{0y} - gt$$
$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$
$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

4. Recombine the horizontal and vertical components of location and/or velocity using the following equations:

$$s = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}(y/x)$$
$$v = \sqrt{v_x^2 + v_y^2}$$
$$\theta_v = \tan^{-1}(v_y / v_x).$$

- The maximum height $h$ of a projectile launched with initial vertical velocity $v_{0y}$ is given by

$$h = \frac{v_{0y}^2}{2g}.$$

- The maximum horizontal distance traveled by a projectile is called the **range**. The range **R** of a projectile on level ground launched at an angle $\theta_0$ above the horizontal with initial speed $v_0$ is given by

$$\mathbf{R} = \frac{v_0^2 \sin 2\theta_0}{g}.$$

## *Conceptual Questions*

**1:** Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than at $t = 0$? (d) Can the speed ever be the same as the initial speed at a time other than at $t = 0$?

**2:** Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the acceleration ever zero? (b) Is the acceleration ever in the same direction as a component of velocity? (c) Is the acceleration ever opposite in direction to a component of velocity?

**3:** For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?

**4:** During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two

coins, in particular discussing whether they hit the floor at the same time.

## *Problems & Exercises*

**1:** A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of 30.0° above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the $x$ and $y$ distances from where the projectile was launched to where it lands?

**2:** A ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction. (a) At what speed does the ball hit the ground? (b) For how long does the ball remain in the air? (c)What maximum height is attained by the ball?

**3:** A ball is thrown horizontally from the top of a 60.0-m building and lands 100.0 m from the base of the building. Ignore air resistance. (a) How long is the ball in the air? (b) What must have been the initial horizontal component of the velocity? (c) What is the vertical component of the velocity just before the ball hits the ground? (d) What is the velocity (including both the horizontal and vertical components) of the ball just before it hits the ground?

**4:** (a) A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up 32° ramp at a speed of 40.0 m/s (144 km/h). How many buses

can he clear if the top of the takeoff ramp is at the same height as the bus tops and the buses are 20.0 m long? (b) Discuss what your answer implies about the margin of error in this act—that is, consider how much greater the range is than the horizontal distance he must travel to miss the end of the last bus. (Neglect air resistance.)

**5:** An archer shoots an arrow at a 75.0 m distant target; the bull's-eye of the target is at same height as the release height of the arrow. (a) At what angle must the arrow be released to hit the bull's-eye if its initial speed is 35.0 m/s? In this part of the problem, explicitly show how you follow the steps involved in solving projectile motion problems. (b) There is a large tree halfway between the archer and the target with an overhanging horizontal branch 3.50 m above the release height of the arrow. Will the arrow go over or under the branch?

**6:** A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand. (a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used? (b) What other angle gives the same range, and why would it not be used? (c) How long did this pass take?

**7:** Verify the ranges for the projectiles in Figure 5(a) for $\theta = 45°$ and the given initial velocities.

**8:** Verify the ranges shown for the projectiles in Figure 5(b) for an initial velocity of 50 m/s at the given initial angles.

**9:** The cannon on a battleship can fire a shell a maximum distance of 32.0 km. (a) Calculate the initial velocity of the shell. (b) What maximum height does it reach? (At its highest, the shell is above 60% of the atmosphere—but air resistance is not really negligible as assumed to make this problem easier.) (c) The ocean is not flat, because the Earth is curved. Assume that the radius of the Earth is $6.37 \times 10^3$ km. How many meters lower will its surface be 32.0 km from the ship along a horizontal line parallel to the surface at the ship? Does your answer imply that error introduced by the assumption of a flat Earth in projectile motion is significant here?

**10:** An arrow is shot from a height of 1.5 m toward a cliff of height H. It is shot with a velocity of 30 m/s at an angle of 60° above the horizontal. It lands on the top edge of the cliff 4.0 s later. (a) What is the height of the cliff? (b) What is the maximum height reached by the arrow along its trajectory? (c) What is the arrow's impact speed just before hitting the cliff?

**11:** In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.600 m and the acceleration achieved from this position is 1.25 times the acceleration due to gravity, $g$. How far can they jump? State your assumptions. (Increased range can be achieved by swinging the arms in the direction of the jump.)

**12:** The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum

range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.

**13:** Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle θ below the horizontal. The service line is 11.9 m from the net, which is 0.91 m high. What is the angle θ such that the ball just crosses the net? Will the ball land in the service box, whose out line is 6.40 m from the net?

**14:** A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. (a) If the ball is thrown at an angle of 25° relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground? (b) How long does it take to get to the receiver? (c) What is its maximum height above its point of release?

**15:** Gun sights are adjusted to aim high to compensate for the effect of gravity, effectively making the gun accurate only for a specific range. (a) If a gun is sighted to hit targets that are at the same height as the gun and 100.0 m away, how low will the bullet hit if aimed directly at a target 150.0 m away? The muzzle velocity of the bullet is 275 m/s. (b) Discuss qualitatively how a larger muzzle velocity would affect this problem and what would be the effect of air resistance.

**16:** An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.

**17:** An owl is carrying a mouse to the chicks in its nest. Its position at that time is 4.00 m west and 12.0 m above the center of the 30.0 cm diameter nest. The owl is flying east at 3.50 m/s at an angle 30.0° below the horizontal when it accidentally drops the mouse. Is the owl lucky enough to have the mouse hit the nest? To answer this question, calculate the horizontal position of the mouse when it has fallen 12.0 m.

**18:** Suppose a soccer player kicks the ball from a distance 30 m toward the goal. Find the initial speed of the ball if it just passes over the goal, 2.4 m above the ground, given the initial direction to be 40° above the horizontal.

**19:** Can a goalkeeper at her/ his goal kick a soccer ball into the opponent's goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s.

**20:** The free throw line in basketball is 4.57 m (15 ft) from the basket, which is 3.05 m (10 ft) above the floor. A player standing on the free throw line throws the ball with an initial speed of 7.15 m/s, releasing it at a height of 2.44 m (8 ft) above the floor. At what angle above the horizontal must the ball be thrown to exactly hit the basket? Note that most players will use a large initial angle rather than a flat shot because it allows for a larger margin of error. Explicitly show how you follow the steps involved in solving projectile motion problems.

**21:** In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw

it at an angle of 38.0° above the horizontal? (Although the maximum distance for a projectile on level ground is achieved at 45° when air resistance is neglected, the actual angle to achieve maximum range is smaller; thus, 38° will give a longer range than 45° in the shot put.)

**22:** A basketball player is running at 5.00 m/s directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise 0.750 m above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?

**23:** A football player punts the ball at a 45.0° angle. Without an effect from the wind, the ball would travel 60.0 m horizontally. (a) What is the initial speed of the ball? (b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?

**24:** Prove that the trajectory of a projectile is parabolic, having the form $y = ax + bx^2$. To obtain this expression, solve the equation $x = v_{0x}t$ for $t$ and substitute it into the expression for $y = v_{0y}t - (1/2)gt^2$ (These equations describe the $x$ and $y$ positions of a projectile that starts at the origin.) You should obtain an equation of the form $y = ax + bx^2$ where $a$ and $b$ are constants.

**25:** Derive $\mathrm{R} = \dfrac{v_0^2 \sin 2\theta_0}{g}$ for the range of a projectile on level ground by finding the time $t$ at which $y$

becomes zero and substituting this value of $t$ into the expression for $x - x_0$, noting that $R = x - x_0$.

**26: Unreasonable Results** (a) Find the maximum range of a super cannon that has a muzzle velocity of 4.0 km/s. (b) What is unreasonable about the range you found? (c) Is the premise unreasonable or is the available equation inapplicable? Explain your answer. (d) If such a muzzle velocity could be obtained, discuss the effects of air resistance, thinning air with altitude, and the curvature of the Earth on the range of the super cannon.

**27: Construct Your Own Problem** Consider a ball tossed over a fence. Construct a problem in which you calculate the ball's needed initial velocity to just clear the fence. Among the things to determine are; the height of the fence, the distance to the fence from the point of release of the ball, and the height at which the ball is released. You should also consider whether it is possible to choose the initial speed for the ball and just calculate the angle at which it is thrown. Also examine the possibility of multiple solutions given the distances and heights you have chosen.

## Glossary

**air resistance**
a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

**kinematics**
the study of motion without regard to mass or force

**motion**
displacement of an object as a function of time

**projectile**
an object that travels through the air and experiences only acceleration due to gravity

**projectile motion**
the motion of an object that is subject only to the acceleration of gravity

**range**
the maximum horizontal distance that a projectile travels

**trajectory**
the path of a projectile through the air

## *Solutions*

**Problems & Exercises**

1: $x = 1.30\ \text{m} \times 10^2 \quad y = 30.9\ \text{m}.$

3: (a) $3.50\ \text{s}$ (b) $28.6\ \text{m/s}$ (c) $34.3\ \text{m/s}$ (d) $44.7\ \text{m/s}$, $50.2^0$ below horizontal

5: (a) $18.4^0$ (b) The arrow will go over the branch.

7: $\text{R} = \dfrac{v_0^2}{\sin 2\theta_0 g}$

For $\theta = 45^0$, $\text{R} = \dfrac{v_0^2}{g}$ $\text{R} = 91.8\ \text{m}$ for $v_0 = 30\ \text{m/s}$; $\text{R} = 163\ \text{m}$ for

$v_0 = 40\ \text{m/s};\ R = 255\ \text{m}$ for $v_0 = 50\ \text{m/s}$.

**9:** (a) $560\ \text{m/s}$ (b) $8.00 \times 10^3\ \text{m}$ (c) $80.0\ \text{m}$. This error is not significant because it is only 1% of the answer in part (b).

**11:** $1.50\ \text{m}$, assuming launch angle of $45^0$

**13:** $\theta = 6.1^0$ yes, the ball lands at 5.3 m from the net

**15:** (a) $-0.486\ \text{m}$ (b) The larger the muzzle velocity, the smaller the deviation in the vertical direction, because the time of flight would be smaller. Air resistance would have the effect of decreasing the time of flight, therefore increasing the vertical deviation.

**17:** $4.23\ \text{m}$. No, the owl is not lucky; he misses the nest.

**19:** No, the maximum range (neglecting air resistance) is about $92\ \text{m}$.

**21:** $15.0\ \text{m/s}$

**23:** (a) $24.2\ \text{m/s}$ (b) The ball travels a total of $57.4\ \text{m}$ with the brief gust of wind.

**25:**

$$y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta)t - \frac{1}{2}gt^2,$$

so that $t = \dfrac{2(v_0 \sin\theta)}{g}$

$x - x_0 = v_{0x}t = (v_0 \cos\theta)t = R$, and substituting for $t$ gives:

$$R = v_0 \cos\theta\left(\frac{2v_0 \sin\theta}{g}\right) = \left(\frac{2v_0^2 \sin\theta\cos\theta}{g}\right)$$

since $2\sin\theta\cos\theta = \sin 2\theta$, the range is:

$$\mathbf{R} = \frac{v_0^2 \sin 2\theta}{g}.$$

PART V

# CHAPTER 4 DYNAMICS: FORCE AND NEWTON'S LAWS OF MOTION

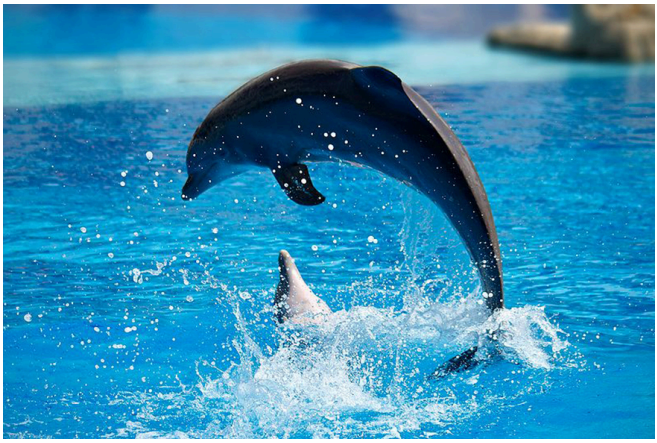
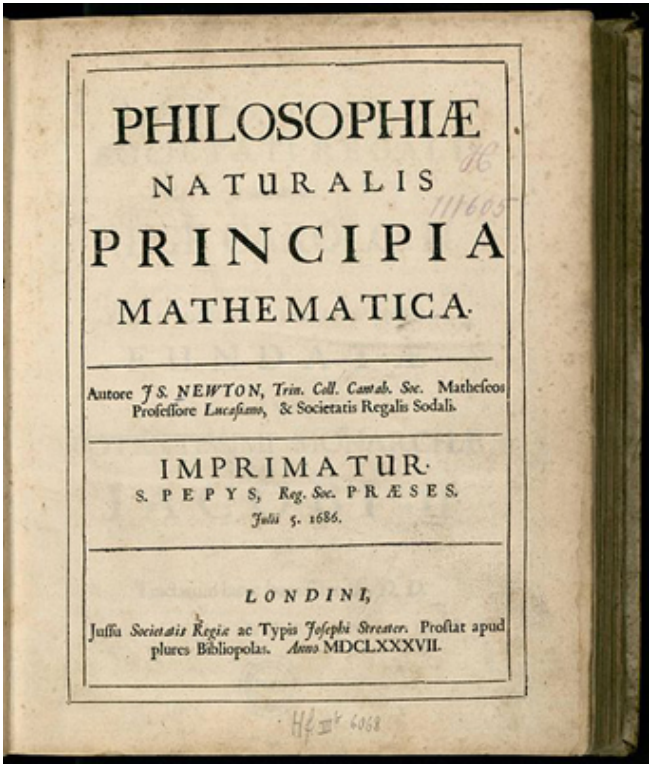


***Figure 1.*** *Newton's laws of motion describe the motion of the dolphin's path. (credit: Jin Jang)*

Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a dolphin jumping out of the water, or a pole vaulter, or the flight of a bird, or the orbit of a satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under

which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

Isaac Newton's (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance into the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. For many centuries natural philosophers had debated the nature of the universe based largely on certain rules of logic with great weight given to the thoughts of earlier classical philosophers such as Aristotle (384–322 BC). Among the many great thinkers who contributed to this change were Newton and Galileo.


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *J S. NEWTON, Trin. Coll. Cantab. Soc.* Matheseos Professore *Lucasiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
*Julii* 5. 1686.

*LONDINI,*
Jussu *Societatis Regiæ* ac Typis *Josephi Streater.* Prostat apud plures Bibliopolas. *Anno* MDCLXXXVII.

***Figure 2.*** *Isaac Newton's monumental work, Philosophiae Naturalis Principia Mathematica, was published in 1687. It proposed scientific laws that are still used today to describe the motion of objects. (credit: Service commun de la documentation de l'Université de Strasbourg)*

Galileo was instrumental in establishing *observation* as the absolute determinant of truth, rather than "logical" argument. Galileo's use of the telescope was his most notable achievement in demonstrating the importance of observation. He discovered moons orbiting Jupiter and made other observations that were inconsistent with certain ancient ideas and religious dogma. For this reason, and because of the manner in which he dealt with those in authority, Galileo was tried by the Inquisition and punished. He spent the

final years of his life under a form of house arrest. Because others before Galileo had also made discoveries by *observing* the nature of the universe, and because repeated observations verified those of Galileo, his work could not be suppressed or denied. After his death, his work was verified by others, and his ideas were eventually accepted by the church and scientific communities.

Galileo also contributed to the formation of what is now called Newton's first law of motion. Newton made use of the work of his predecessors, which enabled him to develop laws of motion, discover the law of gravity, invent calculus, and make great contributions to the theories of light and color. It is amazing that many of these developments were made with Newton working alone, without the benefit of the usual interactions that take place among scientists today.

It was not until the advent of modern physics early in the 20th century that it was discovered that Newton's laws of motion produce a good approximation to motion only when the objects are moving at speeds much, much less than the speed of light and when those objects are larger than the size of most molecules (about $\mathbf{10^{-9}\ m}$ in diameter). These constraints define the realm of classical mechanics, as discussed in Chapter 1 Introduction to the Nature of Science and Physics. At the beginning of the $20^{\text{th}}$ century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, developed quantum theory. This theory does not have the constraints present in classical physics. All of the situations we consider in this chapter are in the realm of classical physics.

## MAKING CONNECTIONS: PAST AND PRESENT PHILOSOPHY

*The importance of observation* and the concept of *cause and effect* were not always so entrenched in human thinking. This realization was a part of the evolution of modern physics from natural philosophy. The achievements of Galileo, Newton, Einstein, and others were key milestones in the history of scientific thought. Most of the scientific theories that are described in this book descended from the work of these scientists.

# 19. 4.1 Development of Force Concept

*Summary*

- Define force.

**Dynamics** is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of **force**—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, as illustrated in Figure 1, we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors, as illustrated in Figure 2(a) for two ice skaters. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or by trigonometric methods. These ideas were developed in Chapter 3 Two-Dimensional Kinematics.

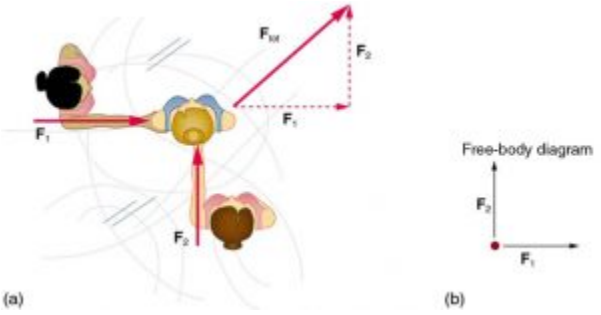



***Figure 1.*** *Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater.*

Figure 1(b) is our first example of a **free-body diagram**, which is a technique used to illustrate all the **external forces** acting on a body. The body is represented by a single isolated point (or free body), and only those forces acting *on* the body from the outside (external forces) are shown. (These forces are the only ones shown, because only external forces acting on the body affect its motion. We can ignore any internal forces within the body.) Free-body diagrams are very useful in analyzing forces acting on a system and are employed extensively in the study and application of Newton's laws of motion.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in Figure 2, and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force. Many other possibilities exist for standard forces. (One that you will encounter later is the magnetic force between two wires carrying electric current.) Some alternative definitions of force will be given later in this chapter.

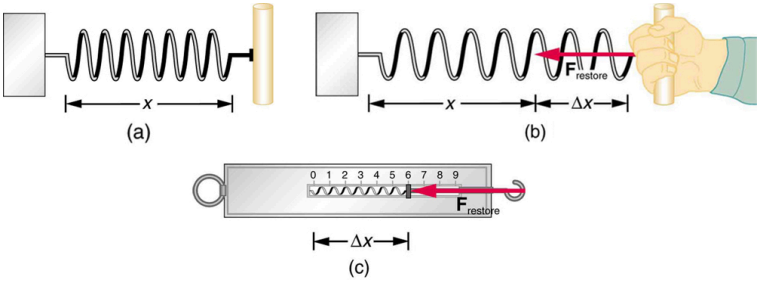



***Figure 2.*** *The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length* $\mathbf{x}$ *when undistorted. (b) When stretched a distance* $\mathbf{\Delta x}$*, the spring exerts a restoring force,* $\mathbf{F_{restore}}$*, which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force* $\mathbf{F_{restore}}$ *is exerted on whatever is attached to the hook. Here* $\mathbf{F_{restore}}$ *has a magnitude of 6 units in the force standard being employed.*

## TAKE-HOME EXPERIMENT: FORCE STANDARDS

To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would

you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

## Section Summary

- **Dynamics** is the study of how forces affect the motion of objects.
- **Force** is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.
- **External forces** are any outside forces that act on a body. A **free-body diagram** is a drawing of all external forces acting on a body.

### *Conceptual Questions*

**1:** Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.

**2:** What properties do forces have that allow us to classify them as vectors?

## Glossary

**dynamics**
: the study of how forces affect the motion of objects and systems

**external force**
: a force acting on an object or system that originates outside of the object or system

**free-body diagram**
: a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

**force**
: a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

# 20. 4.2 Newton's First Law of Motion: Inertia

*Summary*

- Define mass and inertia.
- Understand Newton's first law of motion.

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What **Newton's first law of motion** states, however, is the following:

### NEWTON'S FIRST LAW OF MOTION

A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb "remains." We can think of this law as preserving the status quo of motion.

Rather than contradicting our experience, **Newton's first law of motion** states that there must be a *cause* (which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)*. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface, we can imagine the object sliding in a straight line indefinitely. Friction is thus the *cause* of the slowing (consistent with Newton's first law). The object would not slow down at all if friction were completely eliminated. Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object will slow down. Friction is an external force.

Newton's first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important not only here—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law, and Newton, who clarified it, was to ask the fundamental question, "What is the cause?" Thinking in terms of cause and effect is a

worldview fundamentally different from the typical ancient Greek approach when questions such as "Why does a tiger have stripes?" would have been answered in Aristotelian fashion, "That is the nature of the beast." True perhaps, but not a useful insight.

## Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called **inertia**. Newton's first law is often called the **law of inertia**. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its **mass**. Roughly speaking, mass is a measure of the amount of "stuff" (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

### *Check Your Understanding*

**1:** Which has more mass: a kilogram of cotton balls or a kilogram of gold?

## Section Summary

- **Newton's first law of motion** states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the **law of inertia**.
- **Inertia** is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- **Mass** is the quantity of matter in a substance.

### *Conceptual Questions*

**1:** How are inertia and mass related?

**2:** What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

## Glossary

**inertia**
: the tendency of an object to remain at rest or remain in motion

**law of inertia**
: see Newton's first law of motion

**mass**
: the quantity of matter in a substance; measured in kilograms

**Newton's first law of motion**
: a body at rest remains at rest, or, if in motion, remains in

motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

## *Solutions*

**Check Your Understanding**

**1:** They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

# 21. 4.3 Newton's Second Law of Motion: Concept of a System

*Summary*

- Define net force, external force, and system.
- Understand Newton's second law of motion.
- Apply Newton's second law to determine the weight of an object.

**Newton's second law of motion** is closely related to Newton's first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton's second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton's second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an **acceleration**. Newton's first law says that a net external force causes a change in motion; thus, we see that a *net external force causes acceleration*.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct—an **external force** acts from outside the **system** of interest. For

example, in Figure 1(a) the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again looking at Figure 1(a), the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton's first law. (The internal forces actually cancel, as we shall see in the next section.) *You must define the boundaries of the system before you can determine which forces are external.* Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton's laws. This concept will be revisited many times on our journey through physics.

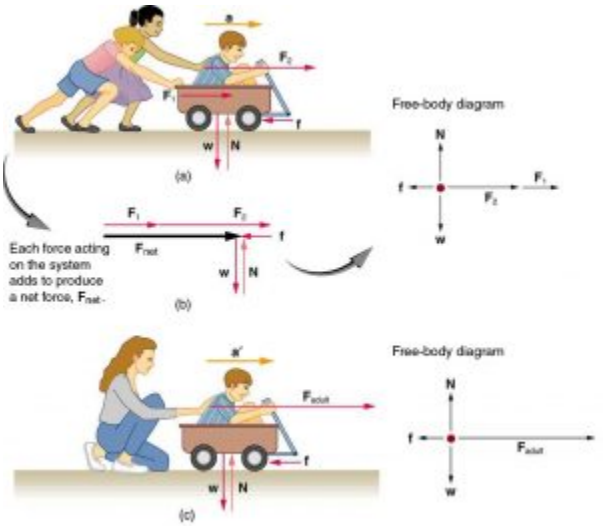



***Figure 1.*** *Different forces exerted on the same mass produce different accelerations. (a) Two children push a wagon with a child in it. Arrows representing all external forces are shown. The system of interest is the wagon and its rider. The weight* ***w*** *of the system and the support of the ground* **N** *are also shown for completeness and are assumed to cancel. The vector* ***f*** *represents the friction acting on the wagon, and it acts to the left, opposing the motion of the wagon. (b) All of the external forces acting on the system add together to produce a net force,* $\mathbf{F_{net}}$*. The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, we draw the vectors collinearly. (c) A larger net external force produces a larger acceleration (****a'>a****) when an adult pushes the child.*

Now, it seems reasonable that acceleration should be directly proportional to and in the same direction as the net (total) external force acting on a system. This assumption has been verified experimentally and is illustrated in Figure 1. In part (a), a smaller force causes a smaller acceleration than the larger force illustrated in part (c). For completeness, the vertical forces are also shown; they are assumed to cancel since there is no acceleration in the vertical

direction. The vertical forces are the weight $\boldsymbol{w}$ and the support of the ground $\mathbf{N}$, and the horizontal force $\boldsymbol{f}$ represents the force of friction. These will be discussed in more detail in later sections. For now, we will define friction as a force that opposes the motion past each other of objects that are touching. Figure 1(b) shows how vectors representing the external forces add together to produce a net force, $\mathbf{F_{net}}$.

To obtain an equation for Newton's second law, we first write the relationship of acceleration and net external force as the proportionality

$$\mathbf{a} \propto \boldsymbol{F}_{\mathbf{net}},$$

where the symbol $\propto$ means "proportional to," and $\mathbf{F_{net}}$ is the net external force. (The net external force is the vector sum of all external forces and can be determined graphically, using the head-to-tail method, or analytically, using components. The techniques are the same as for the addition of other vectors, and are covered in Chapter 3 Two-Dimensional Kinematics.) This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*. Once the system of interest is chosen, it is important to identify the external forces and ignore the internal ones. It is a tremendous simplification not to have to consider the numerous internal forces acting between objects within the system, such as muscular forces within the child's body, let alone the myriad of forces between atoms in the objects, but by doing so, we can easily solve some very complex problems with only minimal error due to our simplification

Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. And indeed, as illustrated in Figure 2, the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

$$\mathbf{a} \propto \frac{\mathbf{1}}{\boldsymbol{m}}$$

where ***m*** is the mass of the system. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is exactly linearly proportional to the net external force.

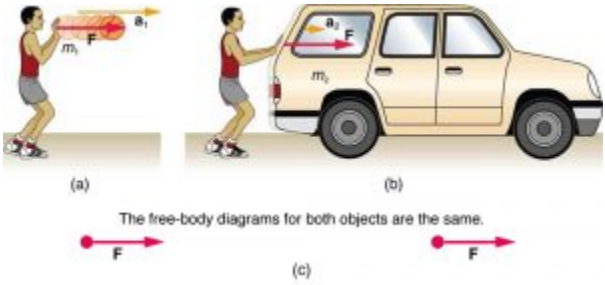



***Figure 2.*** *The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems.*

It has been found that the acceleration of an object depends *only* on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

## NEWTON'S SECOND LAW OF MOTION

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

In equation form, Newton's second law of motion is

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}_{\mathbf{net}}}{m}.$$

This is often written in the more familiar form

$$\vec{\mathbf{F}}_{\mathbf{net}} = m\vec{\mathbf{a}}.$$

When only the magnitude of force and acceleration are considered, this equation is simply

$$F_{\mathbf{net}} = ma.$$

Although these last two equations are really the same, the first gives more insight into what Newton's second law means. The law is a *cause and effect relationship* among three quantities that is not simply based on their definitions. The validity of the second law is completely based on experimental verification.

## Units of Force

$\vec{\mathbf{F}}_{\mathbf{net}} = m\vec{\mathbf{a}}$ is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the newton (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of $\mathbf{1\ m/s^2}$. That is, from $\vec{\mathbf{F}}_{\mathbf{net}} = m\vec{\mathbf{a}}$ we can see that,

$$\mathbf{1\ N = 1\ kg \cdot m/s^2}.$$

While almost the entire world uses the newton for the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

## Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its **weight** $\boldsymbol{w}$. Weight can be denoted as a vector $\vec{\mathbf{w}}$ because it has a direction; *down* is, by definition, the direction of gravity, and hence weight is a downward force. The magnitude of weight is denoted as $\boldsymbol{w}$. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration $\boldsymbol{g}$. Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass $\boldsymbol{m}$ falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude $\boldsymbol{w}$. Newton's second law states that the magnitude of the net external force on an object is $\mathbf{F_{net} = ma}$.

Since the object experiences only the downward force of gravity, $\mathbf{F_{net} = w}$. We know that the acceleration of an object due to gravity is $\boldsymbol{g}$, or $\boldsymbol{a = g}$. Substituting these into Newton's second law gives

## WEIGHT

This is the equation for *weight*—the gravitational force on a mass $\boldsymbol{m}$:

$$\boldsymbol{w = mg.}$$

Since $\boldsymbol{g}$ = **9.80 m/s$^2$** on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:

$$\boldsymbol{w = mg = (1.0\ \text{kg})(9.80\ \text{m/s}^2) = 9.8\ \text{N}.}$$

Recall that $\boldsymbol{g}$ can take a positive or negative value, depending on the positive direction in the coordinate system. Be sure to take this into consideration when solving problems with weight.

When the net external force on an object is its weight, we say that it is in **free-fall**. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity $\boldsymbol{g}$ varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth's surface. On the Moon, for example, the acceleration due to gravity is only **1.67 m/s$^2$**. A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that *the weight of an object is the gravitational force on it from the nearest large body*, such as Earth, the Moon, the Sun, and so on. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of "weightlessness" and "microgravity," they are really referring to the phenomenon we call "free-fall" in physics. We shall use the above definition of weight, and we will make careful distinctions between free-fall and actual weightlessness.

It is important to be aware that weight and mass are very different physical quantities, although they are closely related. Mass is the quantity of matter (how much "stuff") and does not vary in classical physics, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our "weight" in kilograms, but never in the correct units of newtons.

## COMMON MISCONCEPTIONS: MASS VS. WEIGHT

Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram (or the "slug" in English units).

Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object (***m***) multiplied by the acceleration due to gravity (***g***). Like any other force, weight is measured in terms of newtons (or pounds in English units).

Assuming the mass of an object is kept intact, it will remain the same, regardless of its location. However, because weight depends on the acceleration due to gravity, the weight of an object *can change* when the object enters into a region with stronger or weaker gravity. For example, the acceleration due to gravity on the Moon is **1.67 m/s$^2$** (which is much less than the acceleration due to gravity on Earth, **9.80 m/s$^2$**). If you measured your weight on Earth and then measured your weight on the Moon, you would find that you "weigh" much less, even though you do not look any skinnier. This is because the force of gravity is weaker on the Moon. In fact, when people say that they are "losing weight," they really mean that they are losing "mass" (which in turn causes them to weigh less).

## TAKE-HOME EXPERIMENT: MASS AND WEIGHT

What do bathroom scales measure? When you stand

on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same "mass" on Earth as on the Moon?

## Example 1: What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb) parallel to the ground. The mass of the mower is 24 kg. What is its acceleration**?**




***Figure 3.*** *The net force on a lawn mower is 51 N to the right. At what rate does the lawn mower accelerate to the right?*

**Strategy**

Since $\mathbf{F_{net}}$ and $\boldsymbol{m}$ are given, the acceleration can be calculated directly from Newton's second law as stated in $\mathbf{F_{net}} = \boldsymbol{m}\,\boldsymbol{a}$.

**Solution**

The magnitude of the acceleration $\boldsymbol{a}$ is $\boldsymbol{a} = \dfrac{\boldsymbol{F}_{\mathbf{net}}}{\boldsymbol{m}}$.

Entering known values gives

$$\boldsymbol{a} = \frac{\mathbf{51\ N}}{\mathbf{24\ kg}}$$

Substituting the units $\mathbf{kg \cdot m/s^2}$ for N yields

$$\boldsymbol{a} = \frac{\mathbf{51\ kg \cdot m/s^2}}{\mathbf{24\ kg}} = \mathbf{2.1\ m/s^2}.$$

**Discussion**

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moves forward), and the vertical forces must cancel if there is to be no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long because the person's top speed would soon be reached.

## Example 2: What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its

thrust **T**, for the four-rocket propulsion system shown in Figure 4. The sled's initial acceleration is **49 m/s$^2$**, the mass of the system is 2100 kg, and the force of friction opposing the motion is known to be 650 N.

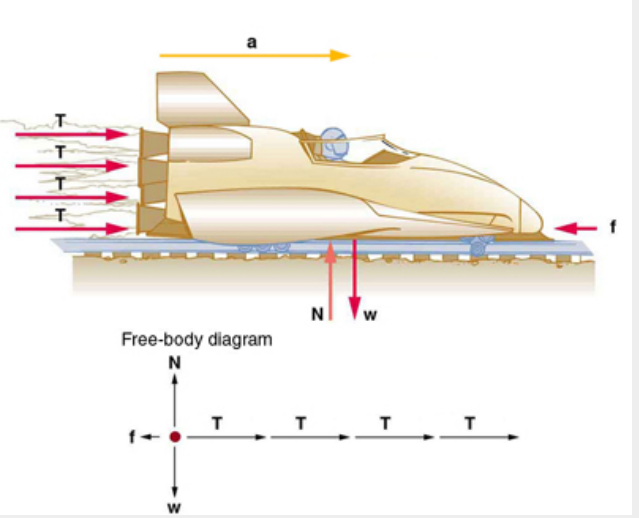



***Figure 4.*** *A sled experiences a rocket thrust that accelerates it to the right. Each rocket creates an identical thrust* **T**. *As in other situations where there is only horizontal acceleration, the vertical forces cancel. The ground exerts an upward force* **N** *on the system that is equal in magnitude and opposite in direction to its weight,* ***w***. *The system here is the sled, its rockets, and rider, so none of the forces between these objects are considered. The arrow representing friction (****f****) is drawn larger than scale.*

### Strategy

Although there are forces acting vertically and horizontally, we assume the vertical forces cancel since there is no vertical acceleration. This leaves us with only

horizontal forces and a simpler one-dimensional problem. Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in the figure.

**Solution**

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines. Since we have defined the direction of the force and acceleration as acting "to the right," we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

$$\boldsymbol{F_{\text{net}} = ma},$$

where $\boldsymbol{F_{\text{net}}}$ is the net force along the horizontal direction. We can see from Figure 4 that the engine thrusts add, while friction opposes the thrust. In equation form, the net external force is

$$\boldsymbol{F_{\text{net}} = 4T - f}.$$

Substituting this into Newton's second law gives

$$\boldsymbol{F_{\text{net}} = ma = 4T - f}.$$

Using a little algebra, we solve for the total thrust 4T:

$$\boldsymbol{4\,T = ma + f}.$$

Substituting known values yields

$$\boldsymbol{4\,T = ma + f = (2100\ \text{kg})(49\ \text{m/s}^2) + 650\ \text{N}}.$$

So the total thrust is

$$\boldsymbol{4\,T = 1.0 \times 10^5\ \text{N}},$$

and the individual thrusts are

$$T = \frac{1.0 \times 10^5 \text{ N}}{4} = 2.6 \times 10^4 \text{ N}.$$

**Discussion**

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and the setup designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 $g$'s. (Recall that $g$, the acceleration due to gravity, is $9.80 \text{ m/s}^2$.When we say that an acceleration is 45 $g$'s, it is $45 \times 9.80 \text{ m/s}^2$, which is approximately $440 \text{ m/s}^2$.) While living subjects are not used any more, land speeds of 10,000 km/h have been obtained with rocket sleds. In this example, as in the preceding one, the system of interest is obvious. We will see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.

Newton's second law of motion is more than a definition; it is a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature. The next section introduces the third and final law of motion.

## Section Summary

- Acceleration, $a$, is defined as a change in velocity, meaning a

change in its magnitude or direction, or both.

- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is $\mathbf{a} = \frac{\boldsymbol{F}_{\mathbf{net}}}{\boldsymbol{m}}$.
- This is often written in the more familiar form: $\boldsymbol{F}_{\mathbf{net}} = \boldsymbol{m}\,\boldsymbol{a}$.
- The weight $\boldsymbol{w}$ of an object is defined as the force of gravity acting on an object of mass $\boldsymbol{m}$. The object experiences an acceleration due to gravity $\boldsymbol{g}$:

$$\boldsymbol{w} = \boldsymbol{m}\,\boldsymbol{g}.$$

- If the only force acting on an object is due to gravity, the object is in free fall.
- Friction is a force that opposes the motion past each other of objects that are touching.

## *Conceptual Questions*

**1:** Which statement is correct? (a) Net force causes motion. (b) Net force causes change in motion. Explain your answer and give an example.

**2:** Why can we neglect forces such as those holding a body together when we apply Newton's second law of motion?

**3:** Explain how the choice of the "system of interest" affects which forces must be considered when applying Newton's second law of motion.

**4:** Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.

**5:** A system can have a nonzero velocity while the net external force on it *is* zero. Describe such a situation.

**6:** A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory?

**7:** (a) Give an example of different net external forces acting on the same system to produce different accelerations. (b) Give an example of the same net external force acting on systems of different masses, producing different accelerations. (c) What law accurately describes both effects? State it in words and as an equation.

**8:** If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.

**9:** If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?

**10:** The gravitational force on the basketball in Figure 2 is ignored. When gravity *is* taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?

## *Problems & Exercises*

**You may assume data taken from illustrations is accurate to three digits.**

**1:** A 63.0-kg sprinter starts a race with an acceleration of 4.20 $m/s^2$.What is the net external force on him?

**2:** If the sprinter from the previous problem accelerates at that rate for 20 m, and then maintains that velocity for the remainder of the 100-m dash, what will be his time for the race?

**3:** A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of its acceleration.

**4:** Since astronauts in orbit are apparently weightless, a clever method of measuring their masses is needed to monitor their mass gains or losses to adjust diets. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted and the astronaut's acceleration is measured to be 0.893 $m/s^2$. (a) Calculate her mass. (b) By exerting a force on the astronaut, the vehicle in which they orbit experiences an equal and opposite force. Discuss how this would affect the measurement of the astronaut's acceleration. Propose a method in which recoil of the vehicle is avoided.

**5:** In Figure 3, the net external force on the 24-kg mower is stated to be 51 N. If the force of friction opposing the

motion is 24 N, what force F (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force F is removed. How far will the mower go before stopping?

**6:** The same rocket sled drawn in Figure 5 is decelerated at a rate of 196 $m/s^2$. What force is necessary to produce this deceleration? Assume that the rockets are off. The mass of the system is 2100 kg.

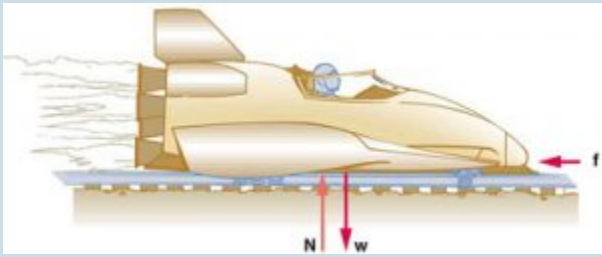



***Figure 5.***

**7:** (a) If the rocket sled shown in Figure 6 starts with only one rocket burning, what is the magnitude of its acceleration? Assume that the mass of the system is 2100 kg, the thrust T is $2.4 \times 10^4$ N, and the force of friction opposing the motion is known to be 650 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?

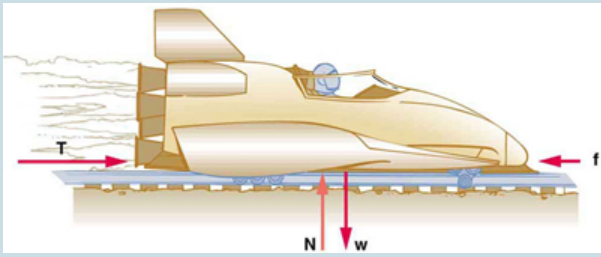



***Figure 6.***

**8:** What is the deceleration of the rocket sled if it comes to rest in 1.1 s from a speed of 1000 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)

**9:** Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg. (a) What is the system of interest if the acceleration of the child in the wagon is to be calculated? (b) Draw a free-body diagram, including all forces acting on the system. (c) Calculate the acceleration. (d) What would the acceleration be if friction were 15.0 N?

**10:** A powerful motorcycle can produce an acceleration of 3.50 $m/s^2$ while traveling at 90.0 km/h. At that speed the forces resisting motion, including friction and air resistance, total 400 N. (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force the motorcycle exerts backward

on the ground to produce its acceleration if the mass of the motorcycle with rider is 245 kg?

**11:** The rocket sled shown in Figure 7 accelerates at a rate of 49.0 $m/s^2$. Its passenger has a mass of 75.0 kg. (a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight by using a ratio. (b) Calculate the direction and magnitude of the total force the seat exerts against his body.

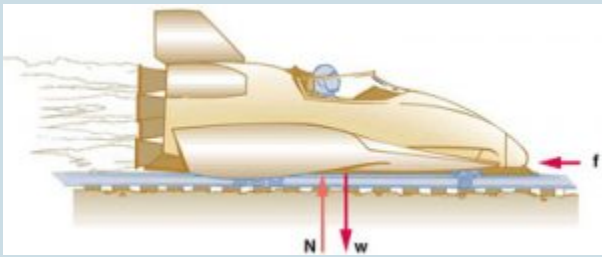



***Figure 7.***

**12:** Repeat the previous problem for the situation in which the rocket sled decelerates at a rate of 201 $m/s^2$. In this problem, the forces are exerted by the seat and restraining belts.

**13:** The weight of an astronaut plus his space suit on the Moon is only 250 N. How much do they weigh on Earth? What is the mass on the Moon? On Earth?

**14:** Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 10,000 kg. The thrust

of its engines is 30,000 N. (a) Calculate its the magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.

## Glossary

**acceleration**
: the rate at which an object's velocity changes over a period of time

**free-fall**
: a situation in which the only force acting on an object is the force due to gravity

**friction**
: a force past each other of objects that are touching; examples include rough surfaces and air resistance

**net external force**
: the vector sum of all external forces acting on an object or system; causes a mass to accelerate

**Newton's second law of motion**
: the net external force $F_{\text{net}}$ on an object with mass $m$ is proportional to and in the same direction as the acceleration of the object, $a$, and inversely proportional to the mass; defined mathematically as $\mathbf{a} = \dfrac{\boldsymbol{F}_{\mathbf{net}}}{\boldsymbol{m}}$

**system**
: defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces

**weight**

the force $w$ due to gravity acting on an object of mass $m$; defined mathematically as: $w = mg$, where $g$ is the magnitude and direction of the acceleration due to gravity

## *Solutions*

**Problems & Exercises**

**1:** 265 N

**3:** $\mathbf{13.3\ m/s}$

**7:** (a) $\mathbf{12\ m/s^2}$. (b) The acceleration is not one-fourth of what it was with all rockets burning because the frictional force is still as large as it was with all rockets burning.

**9:** (a) The system is the child in the wagon plus the wagon.

(b)

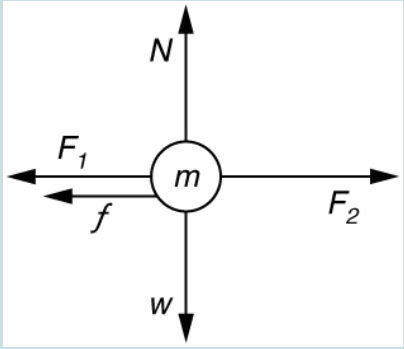



***Figure 8.***

(c) $\boldsymbol{a = 0.130\ \mathrm{m/s^2}}$ in the direction of the second child's push. (d) $\boldsymbol{a = 0.00\ \mathrm{m/s^2}}$

**11:** (a) $\boldsymbol{3.68 \times 10^3\ \mathrm{N}}$. This force is 5.00 times greater than his weight. (b)

**$\boldsymbol{3750\ \mathrm{N};\ 11.3^0}$ above horizontal**

**13:** $\boldsymbol{1.5 \times 10^3\ \mathrm{N},\ 150\ \mathrm{kg},\ 150\ \mathrm{kg}}$

# 22. 4.4 Newton's Third Law of Motion: Symmetry in Forces

*Summary*

- Understand Newton's third law of motion.
- Apply Newton's third law to define systems and solve problems of motion.

There is a passage in the musical *Man of la Mancha* that relates to Newton's third law of motion. Sancho, in describing a fight with his wife to Don Quixote, says, "Of course I hit her back, Your Grace, but she's a lot harder than me and you know what they say, 'Whether the stone hits the pitcher or the pitcher hits the stone, it's going to be bad for the pitcher.'" This is exactly what happens whenever one body exerts a force on another—the first also experiences a force (equal in magnitude and opposite in direction). Numerous common experiences, such as stubbing a toe or throwing a ball, confirm this. It is precisely stated in **Newton's third law of motion**.

NEWTON'S THIRD LAW OF

## MOTION

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.

This law represents a certain *symmetry in nature*: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as "action-reaction," where the force exerted is the action and the force experienced as a consequence is the reaction. Newton's third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton's third law at work by taking a look at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in Figure 1. She pushes against the pool wall with her feet and accelerates in the direction *opposite* to that of her push. The wall has exerted an equal and opposite force back on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different systems*. In this case, there are two systems that we could investigate: the swimmer or the wall. If we select the swimmer to be the system of interest, as in the figure, then $\mathbf{F}_{\textbf{wall on feet}}$ is an external force on this system and affects its motion. The swimmer moves in the direction of $\mathbf{F}_{\textbf{wall on feet}}$. In contrast, the force $\mathbf{F}_{\textbf{feet on wall}}$ acts on the wall and not on our system of interest. Thus $\mathbf{F}_{\textbf{feet on wall}}$ does not directly affect the motion of the system and does not cancel $\mathbf{F}_{\textbf{wall on feet}}$. Note that the swimmer pushes in the direction opposite to that in

which she wishes to move. The reaction to her push is thus in the desired direction.

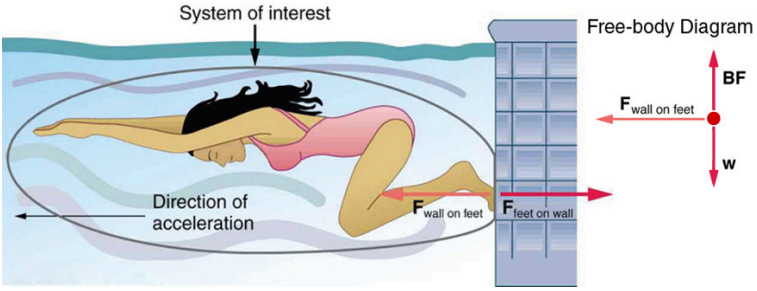



***Figure 1.*** *When the swimmer exerts a force* $\mathbf{F_{feet\ on\ wall}}$ *on the wall, she accelerates in the direction opposite to that of her push. This means the net external force on her is in the direction opposite to* $\mathbf{F_{feet\ on\ wall}}$*. This opposition occurs because, in accordance with Newton's third law of motion, the wall exerts a force* $\mathbf{F_{wall\ on\ feet}}$ *on her, equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Note that* $\mathbf{F_{feet\ on\ wall}}$ *does not act on this system (the swimmer) and, thus, does not cancel* $\mathbf{F_{wall\ on\ feet}}$*. Thus the free-body diagram shows only* $F_{\mathbf{wall\ on\ feet}}$*,* $\boldsymbol{w}$*, the gravitational force, and* **BF***, the buoyant force of the water supporting the swimmer's weight. The vertical forces* $\boldsymbol{w}$ *and* **BF** *cancel since there is no vertical motion.*

Other examples of Newton's third law are easy to find. As a professor paces in front of a whiteboard, she exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes her to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the drive wheels in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward. Helicopters create lift by pushing air down, thereby experiencing an upward reaction force. Birds and airplanes also fly by exerting force on air in a direction opposite to that of whatever force they need. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself in the water by ejecting water through a funnel from its body, similar to a

jet ski. In a situation similar to Sancho's, professional cage fighters experience reaction forces when they punch, sometimes breaking their hand by hitting an opponent's body.

In another example, rockets move forward by expelling gas backward at high velocity. Rockets range in size from fireworks so small that ordinary people use them to immense Saturn Vs that once propelled massive payloads toward the Moon. The propulsion of all rockets, jet engines, deflating balloons, and even squids and octopuses is explained by the same physical principle—Newton's third law of motion. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber, and the gas therefore exerts a large reaction force forward on the rocket, as shown in Figure 2. This reaction force is called **thrust**. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can more readily expel the exhaust gases.

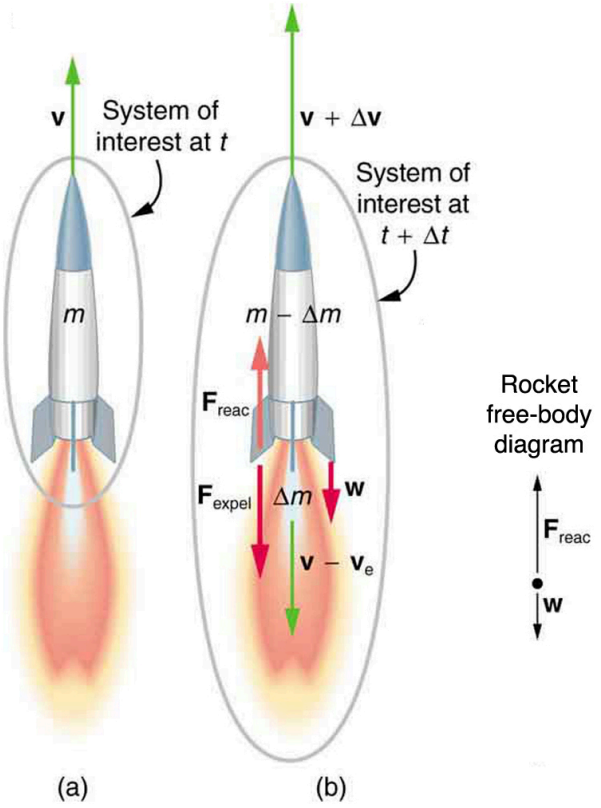



***Figure 2.*** *(a) This rocket has a mass* ***m*** *and an upward velocity* ***v****. The net external force on the system is* ***−mg****, if air resistance is neglected. (b) A time* ***Δt*** *later the system has two main parts, the ejected gas and the remainder of the rocket. The reaction force on the rocket is what overcomes the gravitational force and accelerates it upward.*

## Example 1: Getting Up To Speed: Choosing the Correct System

A physics professor pushes a cart of demonstration equipment to a lecture hall, as seen in Figure 3. Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.

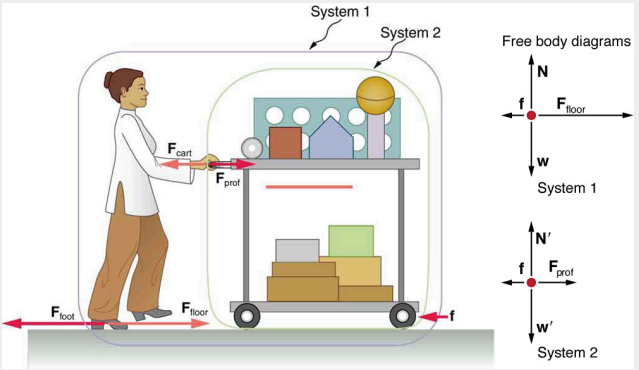



***Figure 3.*** *A professor pushes a cart of demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for* ***f****, since it is too small to draw to scale). Different questions are asked in each example; thus, the system of interest must be defined differently for each. System 1 is appropriate for Example 2, since it asks for the acceleration of the entire group of objects. Only* $\mathbf{F_{floor}}$ *and* ***f*** *are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for this example so that* $\mathbf{F_{prof}}$ *will be an external force and enter into Newton's second law. Note that the free-body diagrams, which allow us to apply Newton's second law, vary with the system chosen.*

### Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in Figure 3. The professor pushes backward with a force $\mathbf{F_{foot}}$ of 150 N. According to Newton's third law, the floor exerts a forward reaction force $\mathbf{F_{floor}}$ of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction. The problem is therefore one-dimensional along the

horizontal direction. As noted, $\boldsymbol{f}$ opposes the motion and is thus in the opposite direction of $\mathbf{F}_{\textbf{floor}}$. Note that we do not include the forces $\mathbf{F}_{\textbf{prof}}$ or $\mathbf{F}_{\textbf{cart}}$ because these are internal forces, and we do not include $\mathbf{F}_{\textbf{foot}}$ because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

**Solution**

Newton's second law is given by

$$\boldsymbol{a} = \frac{\boldsymbol{F}_{\textbf{net}}}{\boldsymbol{m}}.$$

The net external force on System 1 is deduced from Figure 3 and the discussion above to be

$$\mathbf{F}_{\textbf{net}} = \mathbf{F}_{\textbf{floor}} - \mathbf{f} = \mathbf{150\ N} - \mathbf{24.0\ N} = \mathbf{126\ N}.$$

The mass of System 1 is

$$\boldsymbol{m} = \mathbf{(65.0 + 12.0 + 7.0)\ kg} = \mathbf{84\ kg}.$$

These values of $\mathbf{F}_{\textbf{net}}$ and $\boldsymbol{m}$ produce an acceleration of

$$\boldsymbol{a} \quad \frac{\boldsymbol{F}_{\textbf{net}}}{\boldsymbol{m}}$$

$$\boldsymbol{a} \quad \frac{\mathbf{126\ N}}{\mathbf{84\ kg}} = \mathbf{1.5\ m/\ s^2}$$

**Discussion**

None of the forces between components of System 1, such as between the professor's hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is to

note that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the professor on the cart results in an equal and opposite force back on her. In this case both forces act on the same system and, therefore, cancel. Thus internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

## Example 2: Force of the Cart—Choosing a New System

Calculate the force the professor exerts on the cart in Figure 3 using data from the previous example if needed.

**Strategy**

If we now define the system of interest to be the cart plus equipment (System 2 in Figure 3), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart, $\mathbf{F_{prof}}$, is an external force acting on System 2. $\mathbf{F_{prof}}$ was internal to System 1, but it is external to

System 2 and will enter Newton's second law for System 2.

**Solution**

Newton's second law can be used to find $\mathbf{F_{prof}}$. Starting with

$$\boldsymbol{a} = \frac{\boldsymbol{F_{net}}}{\boldsymbol{m}}$$

and noting that the magnitude of the net external force on System 2 is

$$\boldsymbol{F_{net}} = \boldsymbol{F_{prof}} - \boldsymbol{f},$$

we solve for $\mathbf{F_{prof}}$, the desired quantity:

$$\boldsymbol{F_{prof}} = \boldsymbol{F_{net}} + \boldsymbol{f}.$$

The value of $\boldsymbol{f}$ is given, so we must calculate net $\mathbf{F_{net}}$. That can be done since both the acceleration and mass of System 2 are known. Using Newton's second law we see that

$$\boldsymbol{F_{net}} = \boldsymbol{ma},$$

where the mass of System 2 is 19.0 kg ($\boldsymbol{m}$= 12.0 kg + 7.0 kg) and its acceleration was found to be $\boldsymbol{a}$ **= 1.5 m/s$^2$** in the previous example. Thus,

$$\boldsymbol{F_{net}} = \boldsymbol{ma},$$
$$\boldsymbol{F_{net}} = \mathbf{(19.0\ kg)(1.5\ m/s^2) = 29\ N}.$$

Now we can find the desired force:

$$\boldsymbol{F_{prof}} = \boldsymbol{F_{net}} + \boldsymbol{f},$$
$$\boldsymbol{F_{prof}} = \mathbf{29\ N + 24.0\ N = 53\ N}.$$

**Discussion**

It is interesting that this force is significantly less than

the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor.

The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which is not necessarily the same thing).

## PHET EXPLORATIONS: GRAVITY FORCE LAB

Visualize the gravitational force that two objects exert on each other. Change properties of the objects in order to see how it changes the gravity force.

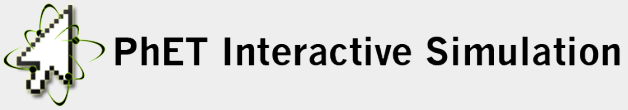



***Figure 3.*** *Gravity Force Lab*

## Section Summary

- **Newton's third law of motion** represents a basic symmetry in nature. It states: Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts.
- A **thrust** is a reaction force that pushes a body forward in response to a backward force. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.

### *Conceptual Questions*

**1:** When you take off in a jet aircraft, there is a sensation of being pushed back into the seat. Explain why you move backward in the seat—is there really a force backward on you? (The same reasoning explains whiplash injuries, in which the head is apparently thrown backward.)

**2:** A device used since the 1940s to measure the kick or recoil of the body due to heart beats is the "ballistocardiograph." What physics principle(s) are involved here to measure the force of cardiac contraction? How might we construct such a device?

**3:** Describe a situation in which one system exerts a force on another and, as a consequence, experiences a force that is equal in magnitude and opposite in direction. Which of Newton's laws of motion apply?

**4:** Why does an ordinary rifle recoil (kick backward) when

fired? The barrel of a recoilless rifle is open at both ends. Describe how Newton's third law applies when one is fired. Can you safely stand close behind one when it is fired?

**5:** An American football lineman reasons that it is senseless to try to out-push the opposing player, since no matter how hard he pushes he will experience an equal and opposite force from the other player. Use Newton's laws and draw a free-body diagram of an appropriate system to explain how he can still out-push the opposition if he is strong enough.

**6:** Newton's third law of motion tells us that forces always occur in pairs of equal and opposite magnitude. Explain how the choice of the "system of interest" affects whether one such pair of forces cancels.

## *Problems & Exercises*

**1:** What net external force is exerted on a 1100-kg artillery shell fired from a battleship if the shell is accelerated at $2.40 \times 10^4$ m/s$^2$? What is the magnitude of the force exerted on the ship by the artillery shell?

**2:** A brave but inadequate rugby player is being pushed backward by an opposing player who is exerting a force of 800 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating at 1.20 m/s$^2$ backward. (a) What is the force of friction between the losing player's feet

and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is 110 kg? (c) Draw a sketch of the situation showing the system of interest used to solve each part. For this situation, draw a free-body diagram and **write the net force equation.**

## Glossary

**Newton's third law of motion**
: whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts

**thrust**
: a reaction force that pushes a body forward in response to a backward force; rockets, airplanes, and cars are pushed forward by a thrust reaction force

## *Solutions*

**Problems & Exercises**

**1:** Force on shell: $\mathbf{2.64 \times 10^7\ N}$ Force exerted on ship = $\mathbf{-2.64 \times 10^7\ N}$, by Newton's third law

2: net force on the losing player = m a = (90.0 kg)(1.20 m/$s^2$) = 108 N. The pushing backwards force is 800 N but the

net force is only 108 Ns so that means friction = (800-108) = 692 N. Without friction the net force would be 800 N and the player would accelerate very quickly backwards. b) net force on the winning player = m a = (110.0 kg)(1.20 $m/s^2$) = 132 N. He is exerting a force of 800 N so the friction force = ?

# 23. 4.5 Normal, Tension, and Spring Forces

*Summary*

- Define normal, tension and spring forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.
- Use trigonometric identities to resolve weight into components.

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. Several of these categories (normal forces, tensions forces, and spring forces) are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this chapter.

## Normal Forces

**Weight** (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in Figure 1(a). But how do inanimate objects like a table support

the weight of a mass placed on them, such as shown in Figure 1(b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.

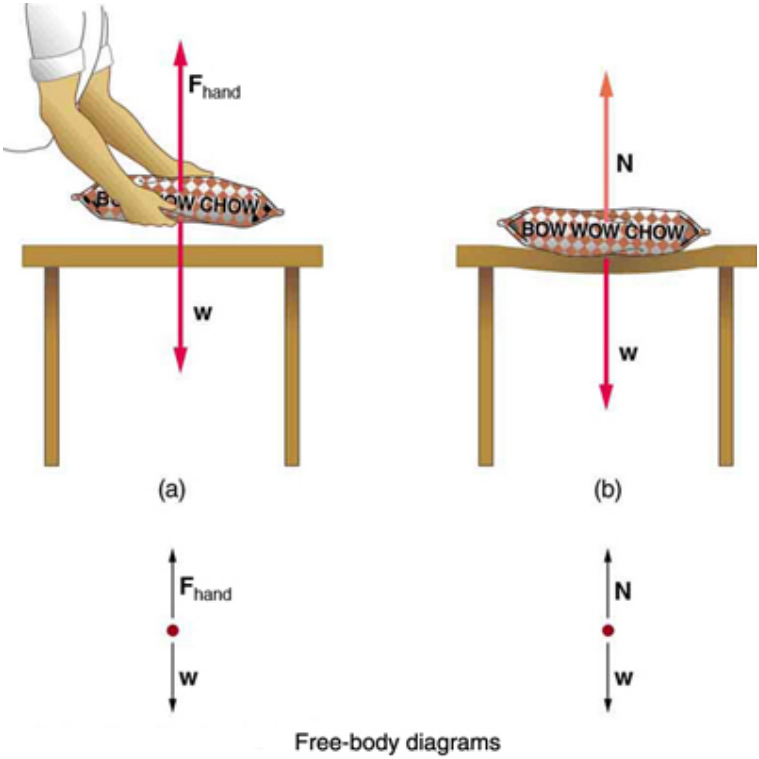



***Figure 1.*** *(a) The person holding the bag of dog food must supply an upward force* $\mathbf{F_{hand}}$ *equal in magnitude and opposite in direction to the weight of the food* $\mathbf{w}$*. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force* $\mathbf{N}$ *equal in magnitude and opposite in direction to the weight of the load.*

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a **normal force** and here is given the symbol **N**. (This is not the unit for force N.) The word normal means perpendicular to a surface. The

normal force can be less than the object's weight (for example, if the object is on an incline as you will see in the next example) or it can be larger than the object's weight (for example, the normal force on a roller-coaster car at the bottom of a loop).

## COMMON MISCONCEPTIONS: NORMAL FORCE (N) VS. NEWTON (N)

In this section we have introduced the quantity normal force, which is represented by the variable **N**. This should not be confused with the symbol for the newton, which is also represented by the letter **N**. These symbols are particularly important to distinguish because the units of a normal force (**N**) happen to be newtons (**N**). For example, the normal force **N** that the floor exerts on a chair might be **N = 100 N**. One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations! You will encounter more similarities among variables and units as you proceed in physics. Another example of this is the quantity work (**W**) and the unit watts (**W**).

## Example 1: Weight on an Incline, a Two-Dimensional Problem

Consider the skier on a slope shown in Figure 2. Her mass including equipment is 60.0 kg. (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is known to be 45.0 N? The details of the friction force are discussed in more detail in Chapter 4.6. For this question, we are only dealing the magnitude of the friction force (and our not concerned with why this force exists, or why the magnitude is the given value).

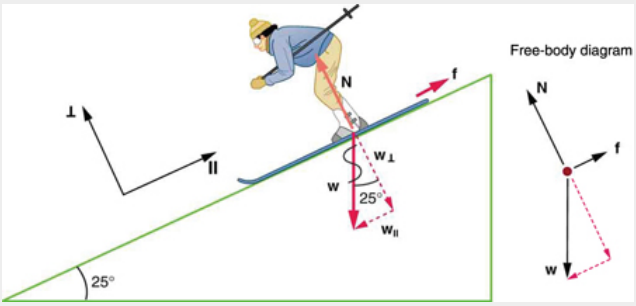



***Figure 2.*** *Since motion and friction are parallel to the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier).* **N** *is perpendicular to the slope and* ***f*** *is parallel to the slope, but* ***w*** *has components along both axes, namely* $\boldsymbol{w}_{\perp}$ *and* $\boldsymbol{w}_{\parallel}$. **N** *is equal in magnitude to the perpendicular component or* $\boldsymbol{w}_{\perp}$, *so that there is no motion perpendicular to the slope, but* ***f*** *is less than* $\boldsymbol{w}_{\parallel}$, *so that there is a downslope acceleration (along the parallel axis).*

### Strategy

This is a two-dimensional problem, since the forces on the skier (the system of interest) are not parallel. The approach we have used in two-dimensional kinematics also works very well here. Choose a convenient coordinate system and project the vectors onto its axes, creating *two* connected *one*-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Remember that motions along mutually perpendicular

axes are independent.) We use the symbols $\perp$ and $\parallel$ to represent perpendicular and parallel, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and because friction is always parallel to the surface between two objects. The only external forces acting on the system are the skier's weight, friction, and the support of the slope, respectively labeled $\boldsymbol{w}$, and $\mathbf{N}$ in Figure 2. $\mathbf{N}$ is always perpendicular to the slope, and $\boldsymbol{f}$ is parallel to it. But $\boldsymbol{w}$ is not in the direction of either axis, and so the first step we take is to project it into components along the chosen axes, defining $\boldsymbol{w}_{\parallel}$ to be the component of weight parallel to the slope and $\boldsymbol{w}_{\perp}$ the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

**Solution**

The magnitude of the component of the weight parallel to the slope is

$$\boldsymbol{w}_{\parallel} = \boldsymbol{w} \sin\left(\mathbf{25}^{o}\right) = \boldsymbol{mg} \sin\left(\mathbf{25}^{\mathbf{0}}\right),$$

and the magnitude of the component of the weight perpendicular to the slope is

$$\boldsymbol{w}_{\perp} = \boldsymbol{w} \cos\left(\mathbf{25}^{\mathbf{0}}\right) = \boldsymbol{mg} \cos\left(\mathbf{25}^{\mathbf{0}}\right).$$

(a) Neglecting friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the

amount of the skier's weight parallel to the slope $\boldsymbol{w_{\parallel}}$ and friction $\boldsymbol{f}$. Using Newton's second law, with subscripts to denote quantities parallel to the slope,

$$\boldsymbol{a_{\parallel}} = \frac{\boldsymbol{F_{\text{net}\parallel}}}{\boldsymbol{m}}$$

where $\boldsymbol{F_{\text{net}\parallel} = w_{\parallel} = mg \sin(25^0)}$, assuming no friction for this part, so that

$$\boldsymbol{a_{\parallel}} = \frac{\boldsymbol{F_{\text{net}\parallel}}}{\boldsymbol{m}} = \frac{\boldsymbol{mg \sin(25^0)}}{\boldsymbol{m}}$$

$$\boldsymbol{= g \sin(25^0)}$$

$$\boldsymbol{(9.80\ \text{m/s}^2)(0.4226) = 4.14\ \text{m/s}^2}$$

is the acceleration.

(b) Including friction. We now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

$$\boldsymbol{F_{\text{net}\parallel} = w_{\parallel} - f},$$

and substituting this into Newton's second law, $\boldsymbol{a_{\parallel} = \frac{F_{\text{net}}}{m}}$, gives

$$\boldsymbol{a_{\parallel}} = \frac{\boldsymbol{F_{\text{net}\parallel}}}{\boldsymbol{m}} = \frac{\boldsymbol{w_{\parallel} - f}}{\boldsymbol{m}} = \frac{\boldsymbol{mg \sin(25^0) - f}}{\boldsymbol{m}}.$$

We substitute known values to obtain

$$\boldsymbol{a_{\parallel}} =$$

$$\frac{(60.0\ \text{kg})(9.80\ \text{m/s}^2)(0.4226) - 45.0\ \text{N}}{60.0\ \text{kg}},$$

which yields

$$a_{||} = 3.39\ \text{m/s}^2,$$

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

**Discussion**

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. In fact, it is a general result that if friction on an incline is negligible, then the acceleration down the incline is $a = g \sin \theta$, *regardless of mass*. This is related to the previously discussed fact that all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

## RESOLVING WEIGHT INTO COMPONENTS

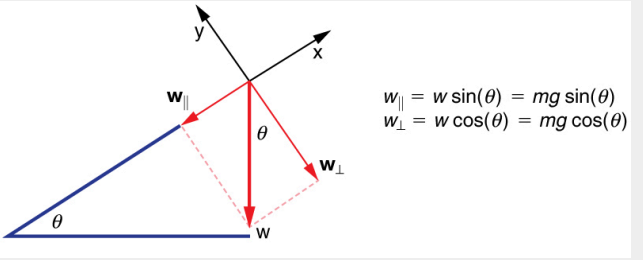



***Figure 3.*** *An object rests on an incline that makes an angle θ with the horizontal.*

When an object rests on an incline that makes an angle $\boldsymbol{\theta}$ with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane, $\mathbf{W}_{\perp}$, and a force acting parallel to the plane, $\mathbf{W}_{\parallel}$. The perpendicular force of weight, $\mathbf{W}_{\perp}$, is typically equal in magnitude and opposite in direction to the normal force, **N**. The force acting parallel to the plane, $\mathbf{W}_{\parallel}$, causes the object to accelerate down the incline. The force of friction, $\boldsymbol{f}$, opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline

is at an angle $\boldsymbol{\theta}$ to the horizontal, then the magnitudes of the weight components are

$$w_{\parallel} = w \sin(\theta) = mg \sin(\theta)$$

and

$$w_{\perp} = w \cos(\theta) = mg \cos(\theta).$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the $\boldsymbol{\theta}$ of the incline is the same as the angle formed between $\mathbf{w}$ and $\mathbf{w}_{\perp}$. Knowing this property, you can use trigonometry to determine the magnitude of the weight components:

$$\cos(\theta) \quad \frac{w_{\perp}}{w}$$

$$w_{\perp} \quad w \cos(\theta) = mg \cos(\theta)$$

$$\sin(\theta) \quad \frac{w_{\parallel}}{w}$$

$$w_{\parallel} \quad w \sin(\theta) = mg \sin(\theta)$$

## TAKE-HOME EXPERIMENT: FORCE

### PARALLEL

To investigate how a force parallel to an inclined plane changes, find a rubber band, some objects to hang from the end of the rubber band, and a board you can position at different angles. How much does the rubber band stretch when you hang the object from the end of the board? Now place the board at an angle so that the object slides off when placed on the board. How much does the rubber band extend if it is lined up parallel to the board and used to hold the object stationary on the board? Try two more angles. What does this show?

## Tension Forces

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word "tension" comes from a Latin word meaning "to stretch." Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: "You can't push a rope." The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in Figure 4.

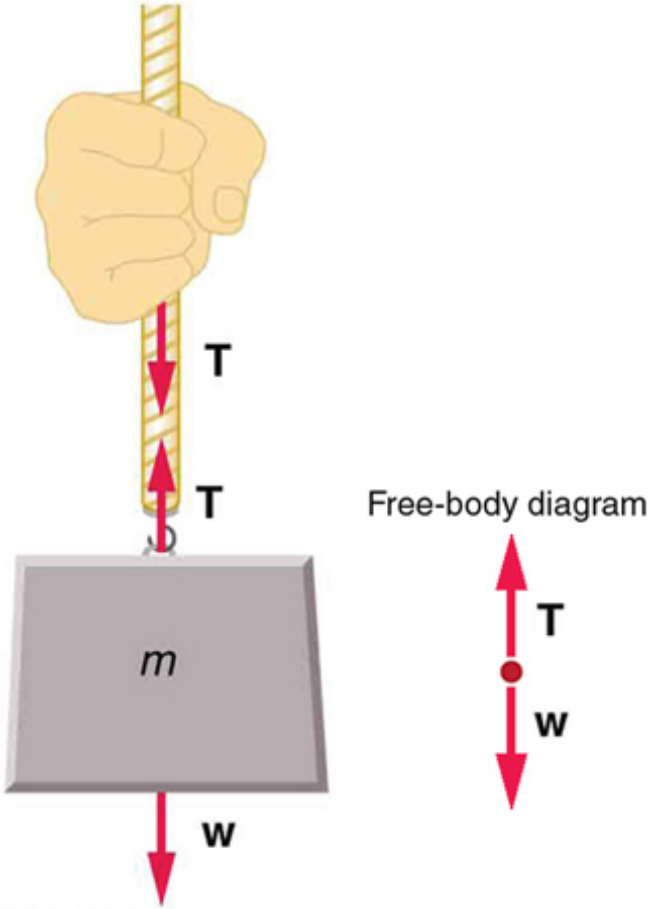



***Figure 4.*** *When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force* **T**, *that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.*

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton's second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus $\mathbf{F_{net} = 0}$. The only external forces acting on the mass are its weight $\boldsymbol{w}$ and the tension $\mathbf{T}$ supplied by the rope. Thus,

$$\boldsymbol{F_{\text{net}} = T - w = 0,}$$

where $\mathbf{T}$ and $\boldsymbol{w}$ are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

$$\boldsymbol{T = w = mg}$$

For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

$$\boldsymbol{T = mg = (5.00\ \text{kg})(9.80\ \text{m/s}^2) = 49.0\ \text{N}.}$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in Figure 5 (a) and (b).

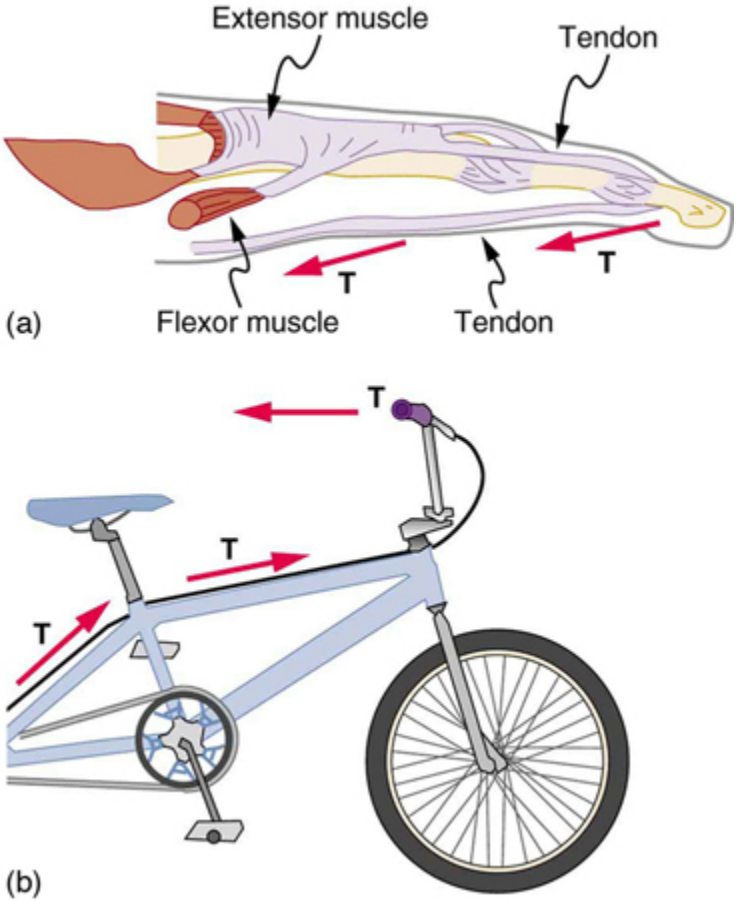



***Figure 5.*** *(a) Tendons in the finger carry force* **T** *from the muscles to other parts of the finger, usually changing the force's direction, but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension* **T** *from the handlebars to the brake mechanism. Again, the direction but not the magnitude of* **T** *is changed.*

## Example 2: What Is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in Figure 6.

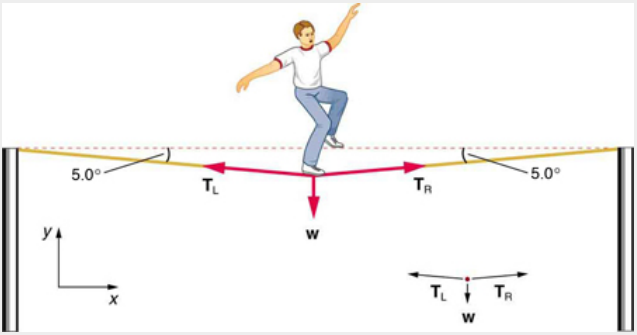



***Figure 6.*** *The weight of a tightrope walker causes a wire to sag by 5.0 degrees. The system of interest here is the point in the wire at which the tightrope walker is standing.*

### Strategy

As you can see in the figure, the wire is not perfectly horizontal (it cannot be!), but is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As usual, forces are vectors represented pictorially by arrows having the same directions as the forces and lengths proportional to their magnitudes. The system is

the tightrope walker, and the only external forces acting on him are his weight $\boldsymbol{w}$ and the two tensions $\mathbf{T_L}$ (left tension) and $\mathbf{T_R}$ (right tension), as illustrated. It is reasonable to neglect the weight of the wire itself. The net external force is zero since the system is stationary. A little trigonometry can now be used to find the tensions. One conclusion is possible at the outset—we can see from part (b) of the figure that the magnitudes of the tensions $\mathbf{T_L}$ and $\mathbf{T_R}$ must be equal. This is because there is no horizontal acceleration in the rope, and the only forces acting to the left and right are $\mathbf{T_L}$ and $\mathbf{T_R}$. Thus, the magnitude of those forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case the best coordinate system has one axis horizontal and the other vertical. We call the horizontal the $x$-axis and the vertical the $y$-axis.

**Solution**

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to draw a new free-body diagram showing all of the horizontal and vertical components of each force acting on the system.

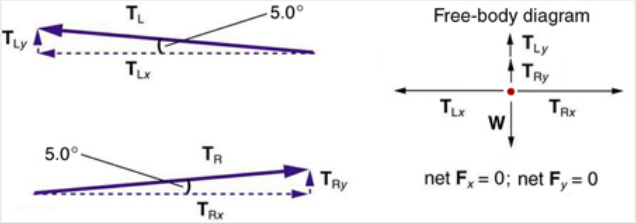



***Figure 7.*** *When the vectors are projected onto vertical and horizontal axes, their components along those axes must add to zero, since the tightrope walker is stationary. The small angle results in* **T** *being much greater than* $\boldsymbol{w}$.

Consider the horizontal components of the forces (denoted with a subscript $\boldsymbol{x}$):

$$\boldsymbol{F_{\text{net}x} = T_{\text{L}x} - T_{\text{R}x}}.$$

The net external horizontal force $\mathbf{F_{netx} = 0}$, since the person is stationary. Thus,

$$\boldsymbol{F_{\text{net}x} = 0 = \quad T_{\text{L}x} - T_{\text{R}x}}$$

$$\boldsymbol{T_{\text{L}x} = T_{\text{R}x}}$$

Now, observe Figure 7. You can use trigonometry to determine the magnitude of $\mathbf{T_L}$ and $\mathbf{T_R}$. Notice that:

$$\cos(5.0^\circ) = \frac{T_{\mathrm{L}x}}{T_{\mathrm{L}}}$$

$$T_{\mathrm{L}x} = T_{\mathrm{L}} \cos(5.0^\circ)$$

$$\cos(5.0^\circ) = \frac{T_{\mathrm{R}x}}{T_{\mathrm{R}}}$$

$$T_{\mathrm{R}x} = T_{\mathrm{R}} \cos(5.0^\circ)$$

Equating $T_{\mathrm{L}x}$ and $T_{\mathrm{R}x}$:

$$T_{\mathrm{L}}\cos(5.0^0) = T_{\mathrm{R}}\cos(5.0^0).$$

Thus,

$$T_{\mathrm{L}} = T_{\mathrm{R}} = T,$$

as predicted. Now, considering the vertical components (denoted by a subscript **y**), we can solve for **T**. Again, since the person is stationary, Newton's second law implies that net $F_y = 0$. Thus, as illustrated in the free-body diagram in Figure 7,

$$F_{\mathrm{net}y} = T_{\mathrm{L}y} + T_{\mathrm{R}y} - w = 0.$$

Observing Figure 7, we can use trigonometry to determine the relationship between $T_{\mathrm{L}y}$, $T_{\mathrm{R}y}$, and **T**. As we determined from the analysis in the horizontal direction, $T_{\mathrm{L}} = T_{\mathrm{R}} = T$:

$$\sin(5.0^\circ) = \frac{T_{Ly}}{T_L}$$

$$T_{Ly} = T_L \sin(5.0^\circ) = T \sin(5.0^\circ)$$

$$\sin(5.0^\circ) = \frac{T_{Ry}}{T_R}$$

$$T_{Ry} = T_R \sin(5.0^\circ) = T \sin(5.0^\circ)$$

Now, we can substitute the values for $T_{Ly}$ and $T_{Ry}$, into the net force equation in the vertical direction:

$$F_{net\,y} = T_{Ly} + T_{Ly} - w = 0$$

$$F_{net\,y} = T \sin(5.0^\circ) + T \sin(5.0^\circ) - w = 0$$

$$2T \sin(5.0^\circ) - w = 0$$

$$2T \sin(5.0^\circ) = w$$

and

$$T = \frac{w}{2 \sin(5.0^0)} = \frac{mg}{2 \sin(5.0^0)},$$

so that

$$T = \frac{(70.0 \text{ kg})(9.80 \text{ m/s}^2)}{2(0.0872)},$$

and the tension is

$$T = 3900 \text{ N}.$$

**Discussion**

Note that the vertical tension in the wire acts as a normal force that supports the weight of the tightrope

walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a small fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, and so most of the tension in the wire is not used to support the weight of the tightrope walker.

If we wish to *create* a very large tension, all we have to do is exert a force perpendicular to a flexible connector, as illustrated in Figure 8. As we saw in the last example, the weight of the tightrope walker acted as a force perpendicular to the rope. We saw that the tension in the roped related to the weight of the tightrope walker in the following way:

$$T = \frac{w}{2\sin(\theta)}.$$

We can extend this expression to describe the tension **T** created when a perpendicular force ($\mathbf{F_\perp}$) is exerted at the middle of a flexible connector:

$$T = \frac{F_\perp}{2\sin(\theta)}.$$

Note that $\boldsymbol{\theta}$ is the angle between the horizontal and the bent connector. In this case, **T** becomes very large as $\boldsymbol{\theta}$ approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e., $\boldsymbol{\theta} = 0$ and $\sin\boldsymbol{\theta} = 0$). (See Figure 8.)

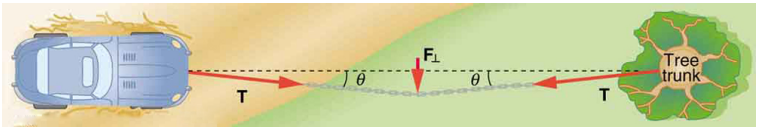



***Figure 8.*** *We can create a very large tension in the chain by pushing on it perpendicular to its length, as shown. Suppose we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as nearly straight as possible. The tension in the chain is given by* $T = F_{\perp} / 2 \sin\theta$*; since* $\theta$ *is small,* $T$ *is very large. This situation is analogous to the tightrope walker shown in Figure 6, except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where* $F_{\perp}$ *is applied.*



***Figure 9.*** *Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly distributed along the length. Suspension bridges—such as the Golden Gate Bridge shown in this image—are essentially very heavy flexible connectors. The weight of the bridge is evenly distributed along the length of flexible connectors, usually cables, which take on the characteristic shape. (credit: Leaflet, Wikimedia Commons)*

# Spring Forces and Hooke's Law

Forces can affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a **deformation**. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force—that is, for small deformations, **Hooke's law** is obeyed. In equation form, Hooke's law is given by

$$|\boldsymbol{F}| = \boldsymbol{k}\boldsymbol{\Delta L},$$

where **|F|** is the magnitude of the force that produced the amount of deformation **ΔL** (the change in length, for example), and **k** is a proportionality constant that depends on the shape and composition of the object and the direction of the force. Note that this force is a function of the deformation **ΔL** – it is not constant as a kinetic friction force is. Sometimes we use **Δx** instead of **ΔL.** The deformation can be along any axis. Rearranging this to

$$\boldsymbol{\Delta L} = \frac{\boldsymbol{F}}{\boldsymbol{k}}$$

makes it clear that the deformation is proportional to the applied force. Figure 10 shows the Hooke's law relationship between the extension **ΔL** of a spring or of a human bone. For metals or springs, the straight line region in which Hooke's law pertains is much larger. Bones are brittle and the elastic region is small and the fracture abrupt. Eventually a large enough stress to the material will cause it to break or fracture. **Tensile strength** is the breaking stress that will cause permanent deformation or fracture of a material.

## HOOKE'S LAW

$$F = k\Delta L,$$

where $\Delta L$ is the amount of deformation (the change in length, for example) produced by the force $F$, and $k$ is a proportionality constant that depends on the shape and composition of the object and the direction of the force.

$$\Delta L = \frac{F}{k}$$

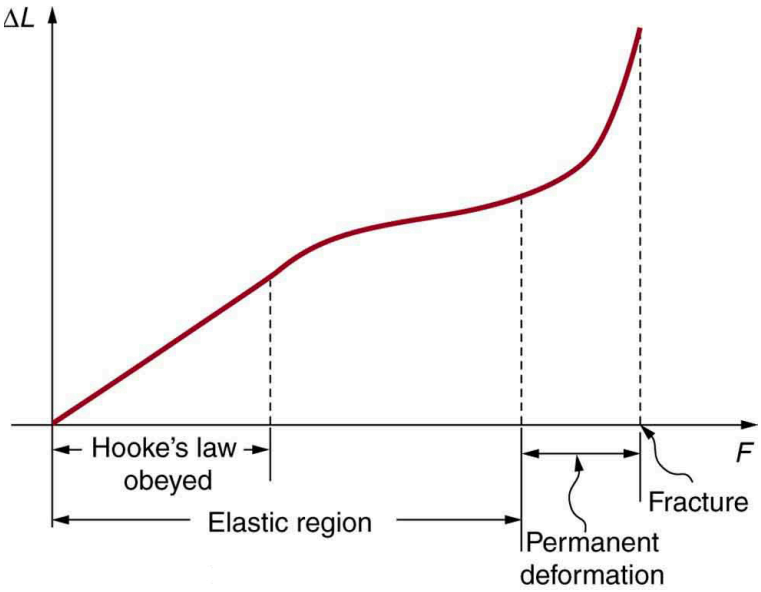



***Figure 10.*** *A graph of deformation* **ΔL** *versus applied force* **F**. *The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is* $\mathbf{1 / k}$. *For larger forces, the graph is curved but the deformation is still elastic—***ΔL** *will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force* **F** *is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in***F** *is producing a large increase in* **L** *near the fracture.*

The proportionality constant $\boldsymbol{k}$ depends upon a number of factors for the material. For example, a guitar string made of nylon stretches when it is tightened, and the elongation **ΔL** is proportional to the force applied (at least for small deformations). Thicker nylon strings and ones made of steel stretch less for the same applied force, implying they have a larger $\boldsymbol{k}$ (see Figure 11). Finally, all three strings return to their normal lengths when the force is removed, provided the deformation is small. Most materials will behave in this manner if the deformation is less than about 0.1% or about 1 part in $10^3$.

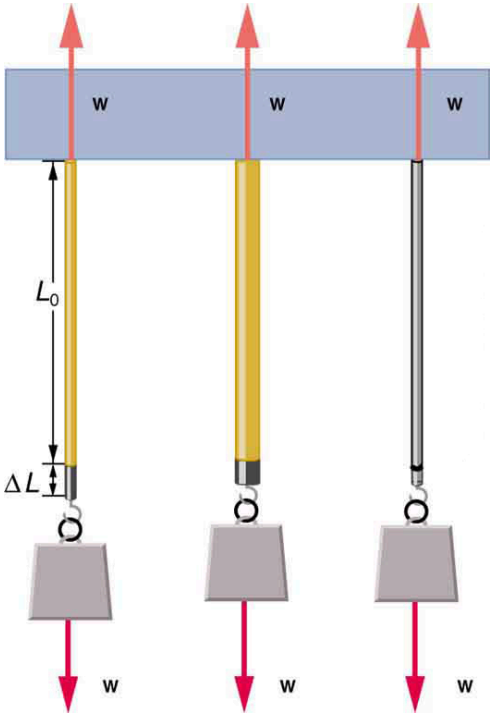



***Figure 11.*** *The same force, in this case a weight (**w**), applied to three different guitar strings of identical length produces the three different deformations shown as shaded segments. The string on the left is thin nylon, the one in the middle is thicker nylon, and the one on the right is steel.*

## STRETCH YOURSELF A LITTLE

How would you go about measuring the proportionality constant **k** of a rubber band? If a rubber band stretched 3 cm when a 100-g mass was attached to it, then how much would it stretch if two similar rubber bands were attached to the same mass—even if put together in parallel or alternatively if tied together in series?

As we have seen, the application of a force can cause the deformation of an object. However, Newton's Third Law says that the object will also exert a force (called a *restoring force*). For example, consider the mass and spring system shown in Figure 12. The mass experiences a restoring force (or spring force), which is given by Hooke's Law. The force is

$$\vec{F}_s = -k\Delta\vec{x},$$

where $\vec{F}_s$ is the spring force, **k** is the spring constant and $\Delta\vec{x}$ is the stretch or compression of the spring measured from its unstretched length. Note that the spring force is a *vector*, where the sign of the force indicates the direction. Likewise, the stretch or compression of the spring is also a vector, and indicates the direction of the stretch or compression. The negative sign in the equation means that the spring forces *acts in the opposite direction to the stretch or compression.* This relationship between the spring force and the direction of the stretch or compression is shown in Figure 3. In many cases, the stretch or compression will simply

be written as **x**, so the equation of the spring force is written as $\vec{F}_s = -kx$,. However, **x** still stands for a *change* in length.

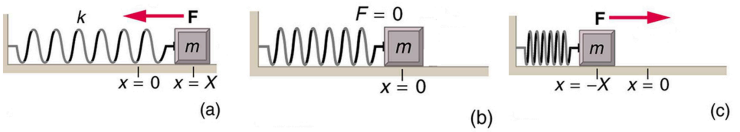



***Figure 12.****The spring exerts a force on the mass due to Hooke's Law. In (a), the spring is stretched in the positive direction (to the right), so the restoring forces acts in the negative direction (to the left). In (c), the spring is stretched in the negative direction (to the left), so the restoring force acts in the positive direction (to the right). In (b), the spring is not stretched so there is no restoring force.*

In some applications, it is better to work with the *magnitude* of the spring force

$$|F| = k|x|$$

where **|x|** is the magnitude of the stretch or compression. To determine the direction of this force, remember that the spring forces *acts in the opposite direction to the stretch or compression.* This can be useful when determining the direction of the force on a free body diagram that involves a spring force.

## Optional: Real Forces and Inertial Frames

There is another distinction among forces in addition to the types already mentioned. Some forces are real, whereas others are not. *Real forces* are those that have some physical origin, such as the gravitational pull. Contrastingly, *fictitious forces* are those that arise simply because an observer is in an accelerating frame of reference, such as one that rotates (like a merry-go-round) or undergoes linear acceleration (like a car slowing down). For example, if a satellite is

heading due north above Earth's northern hemisphere, then to an observer on Earth it will appear to experience a force to the west that has no physical origin. Of course, what is happening here is that Earth is rotating toward the east and moves east under the satellite. In Earth's frame this looks like a westward force on the satellite, or it can be interpreted as a violation of Newton's first law (the law of inertia). An **inertial frame of reference** is one in which all forces are real and, equivalently, one in which Newton's laws have the simple forms given in this chapter.

Earth's rotation is slow enough that Earth is nearly an inertial frame. You ordinarily must perform precise experiments to observe fictitious forces and the slight departures from Newton's laws, such as the effect just described. On the large scale, such as for the rotation of weather systems and ocean currents, the effects can be easily observed.

The crucial factor in determining whether a frame of reference is inertial is whether it accelerates or rotates relative to a known inertial frame. Unless stated otherwise, all phenomena discussed in this text are considered in inertial frames.

All the forces discussed in this section are real forces, but there are a number of other real forces, such as lift and thrust, that are not discussed in this section. They are more specialized, and it is not necessary to discuss every type of force. It is natural, however, to ask where the basic simplicity we seek to find in physics is in the long list of forces. Are some more basic than others? Are some different manifestations of the same underlying force? The answer to both questions is yes, as will be seen in the next (extended) section and in the treatment of modern physics later in the text.

### PHET EXPLORATIONS: FORCES IN 1 DIMENSION

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).



**PhET Interactive Simulation**

*Figure 10.* *Forces in 1 Dimension*

## Section Summary

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force, **N**.
- When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the

object:

$$N = mg.$$

- When objects rest on an inclined plane that makes an angle $\theta$ with the horizontal surface, the weight of the object can be resolved into components that act perpendicular ($\mathbf{W}_{\perp}$) and parallel ($\mathbf{W}_{||}$) to the surface of the plane. These components can be calculated using:

$$w_{||} = w \sin(\theta) = mg \sin(\theta)$$
$$w_{\perp} = w \cos(\theta) = mg \cos(\theta).$$

- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, **T**.When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object:

$$T = mg.$$

- Hooke's law is given by

$$F = k\Delta L, \text{ or } F = k\Delta x,$$

where **ΔL** is the amount of deformation (the change in length), **F** is the applied force, and **k** is a proportionality constant that depends on the shape and composition of the object and the direction of the force.

- The spring force is the restoring force provided by a spring. It has a magnitude of $|F| = k|x|$ and acts in the opposite direction to the stretch or compression.
- In any inertial frame of reference (one that is not accelerated or rotated), Newton's laws have the simple forms given in this chapter and all forces are real forces having a physical origin.

## *Conceptual Questions*

**1:** If a leg is suspended by a traction setup as shown in Figure 13, what is the tension in the rope?

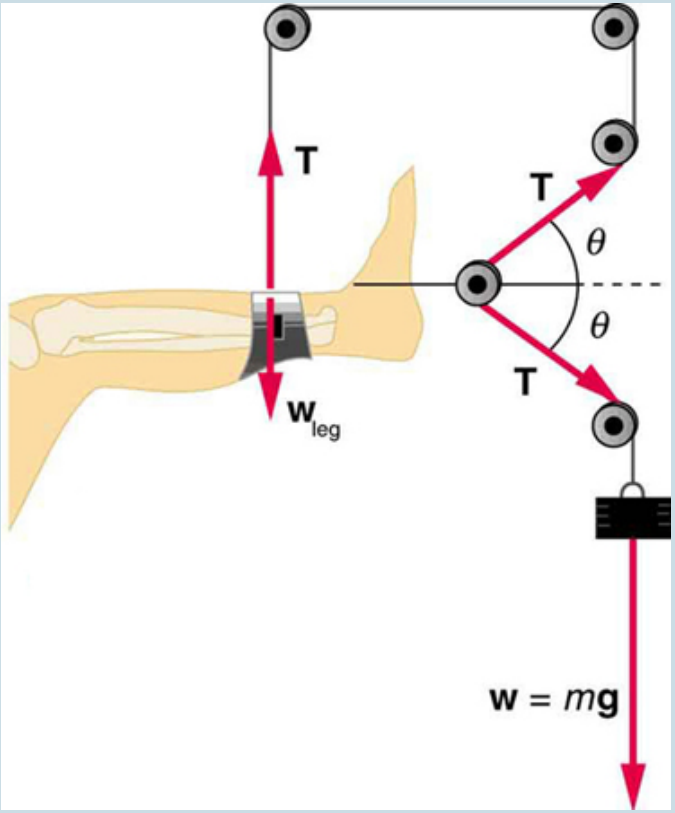



***Figure 13***. *A leg is suspended by a traction system in which wires are used to transmit forces. Frictionless pulleys change the direction of the force T without changing its magnitude.*

**2:** In a traction setup for a broken bone, with pulleys and rope available, how might we be able to increase the force along the tibia using the same weight? (See Figure 13.) (Note that the tibia is the shin bone shown in this image.)

**3:** How is a bow and arrow like a spring?

## *Problems & Exercises*

**1:** Two teams of nine members each engage in a tug of war. Each of the first team's members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team's members has an average mass of 73 kg and exerts an average force of 1365 N horizontally. (a) What is magnitude of the acceleration of the two teams? (b) What is the tension in the section of rope between the teams?

**2:** What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at 7.50 $m/s^2$? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.

**3:** (a) Calculate the tension in a vertical strand of spider web if a spider of mass $8.00 \times 10^{-5}$ kg hangs motionless on it. (b) Calculate the tension in a horizontal strand of spider web if the same spider sits motionless in the middle of it much like the tightrope walker in Figure 6. The strand sags

at an angle of 12° below the horizontal. Compare this with the tension in the vertical strand (find their ratio).

**4:** Suppose a 60.0-kg gymnast climbs a rope. (a) What is the tension in the rope if he climbs at a constant speed? (b) What is the tension in the rope if he accelerates upward at a rate of 1.50 $m/s^2$?

**5:** Show that, as stated in the text, a force $F_\perp$ exerted on a flexible medium at its center and perpendicular to its length (such as on the tightrope wire in Figure 6) gives rise to a tension of magnitude $T = \dfrac{F_\perp}{2\,\sin(\theta)}$.

**6:** A mass is suspended from a vertical spring such that it exerts a downwards force of magnitude 5.02 newtons on the spring. The spring stretches 0.0456 metres when the mass is attached to it. What is the spring constant in newton/metres? Hint: Suspended means hanging in the air not moving. What is the net force on a mass that is not moving?

**7:** The shock absorbers on my car have a spring constant of 97,722 newton/metres. When a certain person sits in the car they exert a downwards force of 987 newtons on the car. How far "down" does the car move after they sit in it? (Watch the movie "The Italian Job" and see how they used this type of physics to figure out which truck was actually carrying the gold.)

**8:** A spring balance has a spring constant of 34.5 newton/metres and it stretches 3.21 centimetres when an unknown

mass is attached to it. What is the force in newtons exerted on the spring by the unknown mass?

**9:** If an applied force of 12 newtons stretches a certain spring 4.0 centimetres, how much stretch will occur for an applied force of 18 newtons?

**10:** Consider the baby being weighed in Figure 14. (a) What is the mass of the child and basket if a scale reading of 55 N is observed? (b) What is the tension $T_1$ in the cord attaching the baby to the scale? (c) What is the tension $T_2$ in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Draw a sketch of the situation indicating the system of interest used to solve each part. The masses of the cords are negligible.

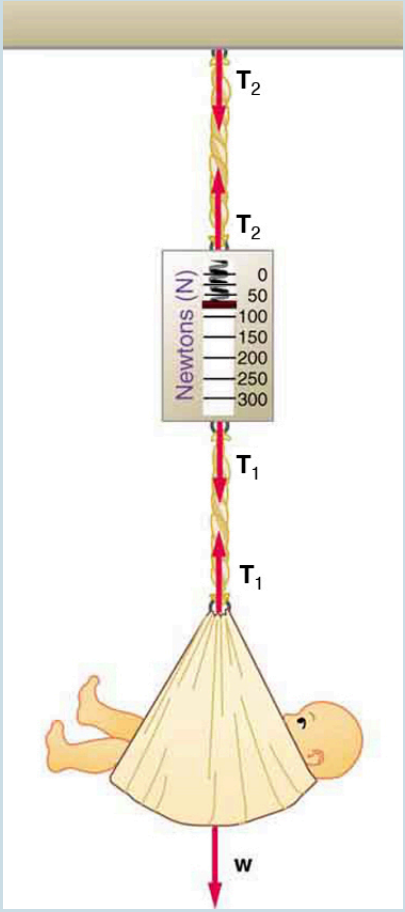



***Figure 14.*** *A baby is weighed using a spring scale.*

## Glossary

**inertial frame of reference**
: a coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious forces that are observed due to an accelerating frame of reference

**normal force**
: the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

**tension**
: the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

## *Solutions*

**Problems & Exercises**

**1:** (a) $\mathbf{0.11\ m/s^2}$ (b) $\mathbf{1.2 \times 10^4\ N}$

**3:** (a) $\mathbf{7.84 \times 10^{-4}\ N}$ (b) $\mathbf{1.89 \times 10^{-3}\ N.}$
This is 2.41 times the tension in the vertical strand.

**5:** Newton's second law applied in vertical direction gives

$$\boldsymbol{F_y = F - 2T \sin\theta = 0}$$
$$\boldsymbol{F = 2T \sin\theta}$$
$$\boldsymbol{T = \frac{F}{2 \sin\theta}.}$$

**7:** 0.0101 m = 1.01 cm

**9:** 6.0 cm

# 24. 4.6 Friction

*Summary*

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitude of static and kinetic friction.

**Friction** is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behaviour of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

## FRICTION

Friction is a force that opposes relative motion between systems in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the objects.

### KINETIC FRICTION

If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

Figure 1 is a crude pictorial representation of how friction occurs

at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.

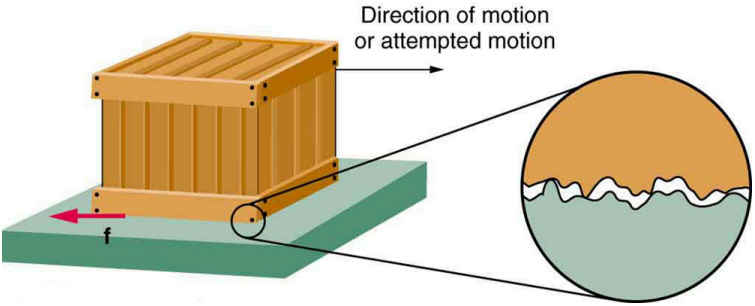



***Figure 1.*** *Frictional forces, such as* ***f****, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.*

The magnitude of the frictional force has two forms: one for static

situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction $f_s$** is

$$f_s \leq \mu_s N,$$

where $\mu_s$ is the coefficient of static friction and **N** is the magnitude of the normal force (the force perpendicular to the surface).

### MAGNITUDE OF STATIC FRICTION

Magnitude of static friction $f_s$ is

$$f_s \leq \mu_s N,$$

where $\mu_s$ is the coefficient of static friction and **N** is the magnitude of the normal force.

The symbol ≤ means *less than or equal to*, implying that static friction can have a minimum value of 0 (in other words, there can be no static friction) and a maximum value of $\mu_s N$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $f_{s(max)}$, the object will move. Thus

$$f_{s(max)} = \mu_s N.$$

Once an object is moving, the **magnitude of kinetic friction $f_k$** is given by

$$f_k = \mu_k N,$$

where $\mu_k$ is the coefficient of kinetic friction. A system in which $f_k = \mu_k N$ is described as a system in which *friction behaves simply*.

### MAGNITUDE OF KINETIC FRICTION

The magnitude of kinetic friction $f_k$ is given by

$$f_k = \mu_k N,$$

where $\mu_k$ is the coefficient of kinetic friction.

As seen in Table 1, the coefficients of kinetic friction are less than their static counterparts. That values of $\mu$ in Table 1 are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

| System | Static friction, $\mu_s$ | Kinetic friction, $\mu_k$ |
|---|---|---|
| Rubber on dry concrete | 1.0 | 0.7 |
| Rubber on wet concrete | 0.7 | 0.5 |
| Wood on wood | 0.5 | 0.3 |
| Waxed wood on wet snow | 0.14 | 0.1 |
| Metal on wood | 0.5 | 0.3 |
| Steel on steel (dry) | 0.6 | 0.3 |
| Steel on steel (oiled) | 0.05 | 0.03 |
| Teflon on steel | 0.04 | 0.04 |
| Bone lubricated by synovial fluid | 0.016 | 0.015 |
| Shoes on wood | 0.9 | 0.7 |
| Shoes on ice | 0.1 | 0.05 |
| Ice on ice | 0.1 | 0.03 |
| Steel on ice | 0.4 | 0.02 |

**Table 1.** Coefficients of Static and Kinetic Friction.

The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $\mathbf{W = mg = (100\ kg)(9.80\ m/s^2) = 980\ N}$, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than $\mathbf{f_{s(max)} = \mu N = (0.45)(980\ N) = 440\ N}$ to move

the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ($\boldsymbol{f_k = \mu_k N = (0.30)(980\ N) = 290\ N}$) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

## TAKE-HOME EXPERIMENT

Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table, simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth,

almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (Figure 2). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.

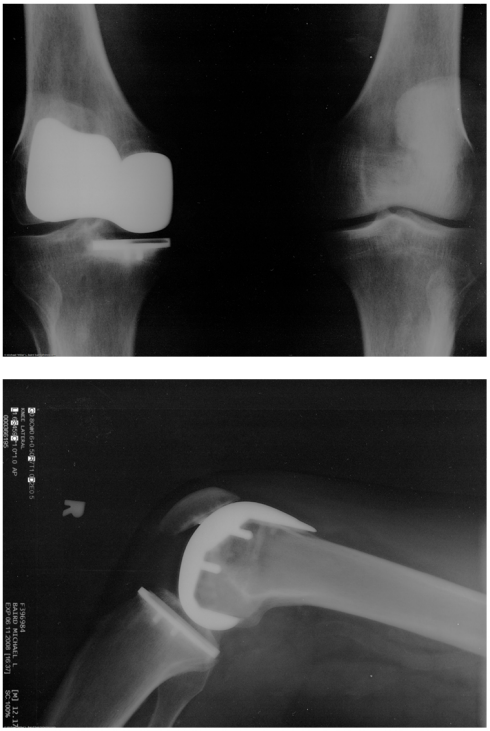


***Figure 2.*** *Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op x rays of the right knee joint replacement. (credit: Mike Baird, Flickr)*

Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person

moves. Artificial lubricants are also common in hospitals and doctor's clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to mover freely over the skin.

## Example 1: Skiing Exercise

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

**Strategy**

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force **N** as $\boldsymbol{f_k = \mu_k N}$; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See the skier and free-body diagram in Figure 3.)

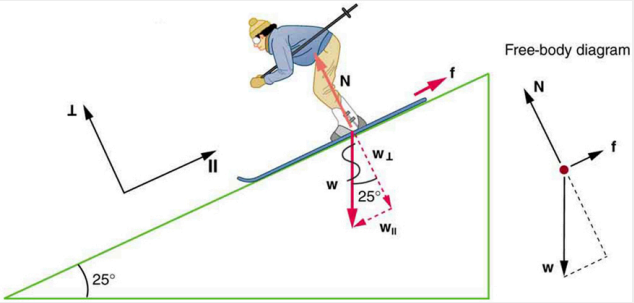



***Figure 3.*** *The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier).* **N** *(the normal force) is perpendicular to the slope, and* ***f*** *(the friction) is parallel to the slope, but* ***w*** *(the skier's weight) has components along both axes, namely* $\boldsymbol{w_\perp}$ *and* $\boldsymbol{w_{/\!/}}$. **N** *is equal in magnitude to* $\boldsymbol{w_\perp}$, *so there is no motion perpendicular to the slope. However,* ***f*** *is less than* $\boldsymbol{w_{/\!/}}$ *in magnitude, so there is acceleration down the slope (along the x-axis).*

That is,

$$\boldsymbol{N = w_\perp = w\cos 25^0 = mg\cos 25^0.}$$

Substituting this into our expression for kinetic friction, we get

$$\boldsymbol{f_k = \mu_k mg\cos 25^0,}$$

which can now be solved for the coefficient of kinetic friction $\boldsymbol{\mu_k}$.

**Solution**

Solving for $\boldsymbol{\mu_k}$ gives

$$\mu_k = \frac{f_k}{N} = \frac{f_k}{w\cos 25^0} = \frac{f_k}{mg\cos 25^0}.$$

Substituting known values on the right-hand side of the equation,

$$\mu_k = \frac{45.0\ \text{N}}{(62\ \text{kg})(9.80\ \text{m/s}^2)(0.906)} = 0.082.$$

**Discussion**

This result is a little smaller than the coefficient listed in Table 1 for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass $m$ slides down a slope that makes an angle $\theta$ with the horizontal, friction is given by $f_k = \mu_k mg \cos\theta$. All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter's Problems and Exercises.

## TAKE-HOME EXPERIMENT

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of

kinetic friction between two objects. As shown in Example 1, the kinetic friction on a slope $f_k = \mu_k mg \cos\theta$. The component of the weight down the slope is equal to $mg \sin\theta$ (see the free-body diagram in Figure 3). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out:

$$f_k = Fg_x$$
$$\mu_k mg \cos\theta = mg \sin\theta.$$

Solving for $\mu_k$, we find that

$$\mu_k = \frac{mg \sin\theta}{mg \cos\theta} = \tan\theta.$$

Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find $\mu_k$. Note that the coin will not start to slide at all until an angle greater than $\theta$ is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Discuss how this may affect the value for $\mu_k$ and its uncertainty.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

## MAKING CONNECTIONS: SUBMICROSCOPIC EXPLANATIONS OF FRICTION

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

Figure 4 illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.

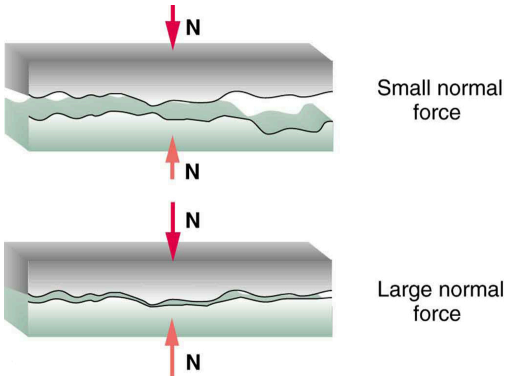



***Figure 4.*** *Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.*

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. Figure 5 shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of $10^{12}$) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.

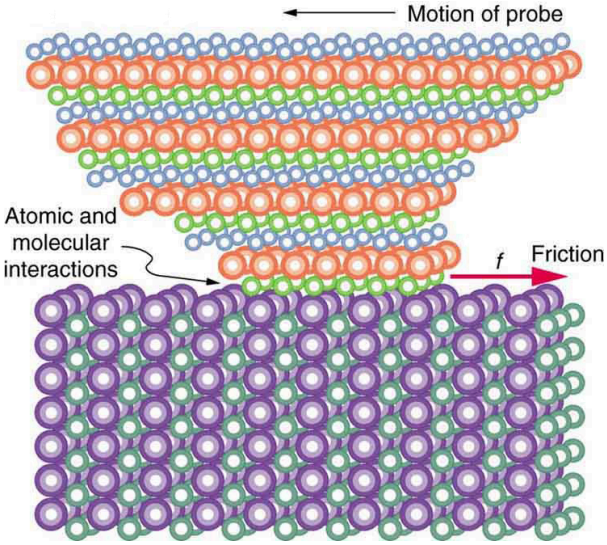



***Figure 5.*** *The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.*

## PHET EXPLORATIONS: FORCES AND MOTION

Explore the forces at work when you try to push things. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and

acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).



PhET Interactive Simulation

*Figure 6.* Forces and Motion

## Section Summary

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force **N** pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction $f_s$ between systems stationary relative to one another is given by

$$f_s \leq \mu N,$$

  where $\mu_s$ is the coefficient of static friction, which depends on both of the materials.
- The kinetic friction force $f_k$ between systems moving relative to one another is given by

$$f_k = \mu_k N,$$

  where $\mu_k$is the coefficient of kinetic friction, which also depends on both materials.

## *Conceptual Questions*

**1:** Define normal force. What is its relationship to friction when friction behaves simply?

**2:** The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.

**3:** When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.

**4:** When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

## *Problems & Exercises*

**1:** A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only 0.200 N. Knowing the coefficient

of kinetic friction between the two materials, he quickly calculates the normal force. What is it?

**2:** (a) When rebuilding her car's engine, a physics major must exert 300 N of force to insert a dry steel piston into a steel cylinder. What is the magnitude of the normal force between the piston and cylinder? (b) What is the magnitude of the force would she have to exert if the steel parts were oiled?

**3:** (a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

**4:** Suppose you have a 120-kg wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be?

**5:** (a) If half of the weight of a small $1.00 \times 10^3$ kg utility truck is supported by its two drive wheels, what is the magnitude of the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.

**6:** A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.

**7:** Consider the 65.0-kg ice skater being pushed by two others shown in Figure 7. (a) Find the direction and magnitude of $F_{tot}$, the total force exerted on her by the others, given that the magnitudes $F_1$ and $F_2$ are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of $F_{tot}$? (c) What is her acceleration assuming she is already moving in the direction of $F_{tot}$? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)

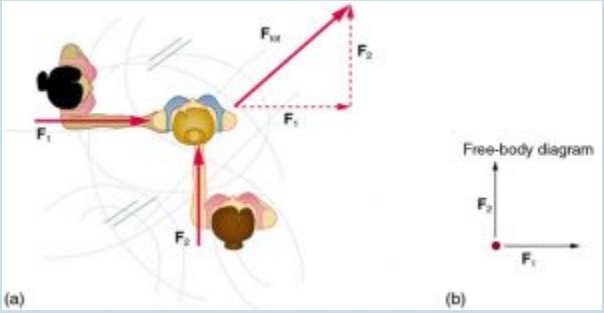



***Figure 7.***

**8:** Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is $a = g \sin \theta$. (Note that this acceleration is independent of mass.)

**9:** Show that the acceleration of any object down an incline where friction behaves simply (that is, where $f_k = \mu_k N$) is $a = g (\sin \theta - \mu_k \cos \theta)$. Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small ($\mu_k = 0$).

**10:** Calculate the deceleration of a snow boarder going up a 5.0°, slope assuming the coefficient of friction for waxed wood on wet snow. The result of Exercise 9 may be useful, but be careful to consider the fact that the snow boarder is going uphill. Explicitly show how you follow the steps in Chapter 4.6 Problem-Solving Strategies.

**11:** (a) Calculate the acceleration of a skier heading down a 10.0° slope, assuming the coefficient of friction for waxed wood on wet snow. (b) Find the angle of the slope down which this skier could coast at a constant velocity. You can neglect air resistance in both parts, and you will find the result of Exercise 9 to be useful. Explicitly show how you follow the steps in the Chapter 4.6 Problem-Solving Strategies.

**12:** If an object is to rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes. Show that the maximum angle of an incline above the horizontal for which an object will not slide down is $\theta = \tan^{-1} \mu_s$. You may use the result of the previous problem. Assume that $a$=0 and that static friction has reached its maximum value.

**13:** Calculate the maximum deceleration of a car that is heading down a 6° slope (one that makes an angle of 6° with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s$ = 0.100, the same as for shoes on ice.

**14:** Calculate the maximum acceleration of a car that is heading up a 4° slope (one that makes an angle of 4° with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static

friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s$ = 0.100, the same as for shoes on ice.

**15:** Repeat Exercise 14 for a car with four-wheel drive.

**16:** A freight train consists of two $8.00 \times 10^5$-kg engines and 45 cars with average masses of $5.50 \times 10^5$ kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00 \times 10^{-2}$ $m/s^2$ if the force of friction is $7.50 \times 10^5$ N, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the magnitude of the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

**17:** Consider the 52.0-kg mountain climber in Figure 8. (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?

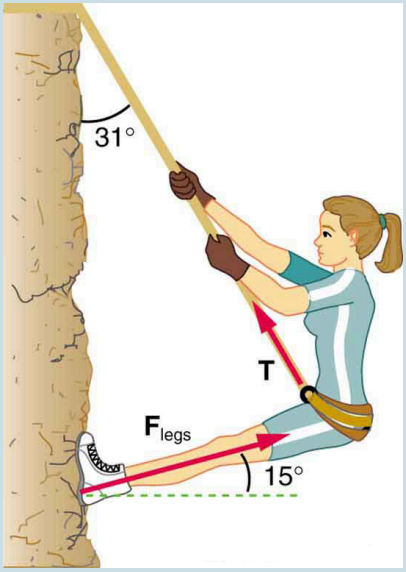



***Figure 8.*** *Part of the climber's weight is supported by her rope and part by friction between her feet and the rock face.*

**18:** A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown in Figure 9(a). (a) Calculate the minimum force *F* he must exert to get the block moving. (b) What is the magnitude of its acceleration once it starts to move, if that force is maintained?

**19:** Repeat Exercise 18 with the contestant pulling the block of ice with a rope over his shoulder at the same angle above the horizontal as shown in Figure 9(b).

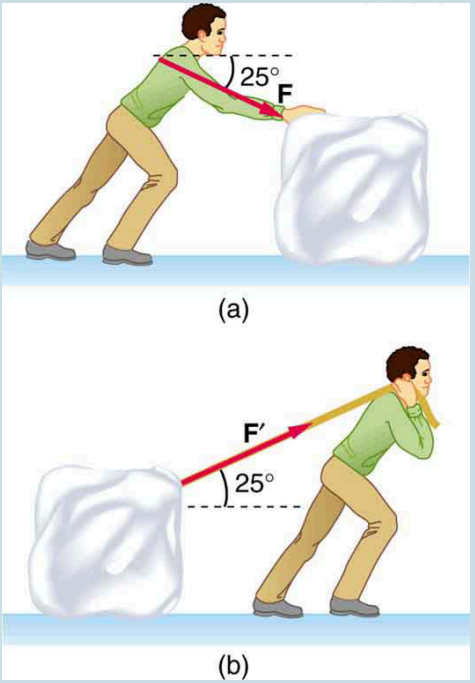



***Figure 9.*** *Which method of sliding a block of ice requires less force—(a) pushing or (b) pulling at the same angle above the horizontal?*

## Glossary

**friction**

a force that opposes relative motion or attempts at motion between systems in contact

**kinetic friction**
: a force that opposes the motion of two systems that are in contact and moving relative to one another

**static friction**
: a force that opposes the motion of two systems that are in contact and are not moving relative to one another

**magnitude of static friction**
: $f_s \leq \mu_s N$, where $\mu_s$ is the coefficient of static friction and $N$ is the magnitude of the normal force

**magnitude of kinetic friction**
: $f_k = \mu_k N$, where $\mu_k$ is the coefficient of kinetic friction

## *Solutions*

**Problems & Exercises**

**1:** 5.00 N

**4:** (a) $588\ \text{N}$ (b) $1.96\ \text{m/s}^2$

**6:** (a) $3.29\ \text{m/s}^2$ (b) $3.52\ \text{m/s}^2$ (c) $980\ \text{N};\ 945\ \text{N}$

**10:** $1.83\ \text{m/s}^2$

**14:** (a) $4.20\ \text{m/s}^2$ (b) $2.74\ \text{m/s}^2$ (c) $-0.195\ \text{m/s}^2$

**16:** (a) $1.03 \times 10^6\ \text{N}$ (b) $3.48 \times 10^5\ \text{N}$

**18:** (a) $51.0\ \text{N}$ (b) $0.720\ \text{m/s}^2$

# 25. 4.7 Problem-Solving Strategies

*Summary*

- Understand and apply a problem-solving procedure to solve problems using Newton's laws of motion.

Success in problem solving is obviously necessary to understand and apply physical principles, not to mention the more immediate need of passing exams. The basics of problem solving, presented earlier in this text, are followed here, but specific strategies useful in applying Newton's laws of motion are emphasized. These techniques also reinforce concepts that are useful in many other areas of physics. Many problem-solving strategies are stated outright in the worked examples, and so the following techniques should reinforce skills you have already begun to develop.

## Problem-Solving Strategy for Newton's Laws of Motion

Step 1. As usual, it is first necessary to identify the physical principles involved. *Once it is determined that Newton's laws of motion are involved (if the problem involves forces), it is particularly important to draw a careful sketch of the situation.* Such a sketch is

shown in Figure 1(a). Then, as in Figure 1(b), use arrows to represent all forces, label them carefully, and make their lengths and directions correspond to the forces they represent (whenever sufficient information exists).

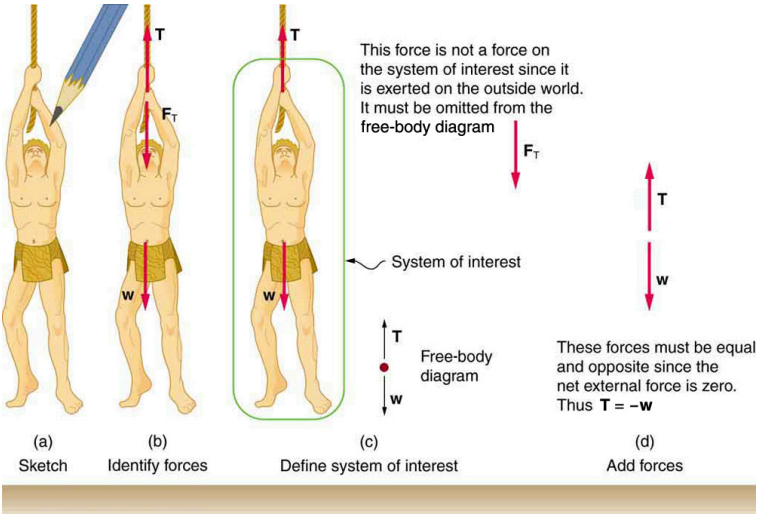



***Figure 1.*** *(a) A sketch of Tarzan hanging from a vine. (b) Arrows are used to represent all forces.* $\mathbf{T}$ *is the tension in the vine above Tarzan,* $\mathbf{F_T}$ *is the force he exerts on the vine, and* $\boldsymbol{w}$ *is his weight. All other forces, such as the nudge of a breeze, are assumed negligible. (c) Suppose we are given the ape man's mass and asked to find the tension in the vine. We then define the system of interest as shown and draw a free-body diagram.* $\mathbf{F_T}$ *is no longer shown, because it is not a force acting on the system of interest; rather,* $\mathbf{F_T}$ *acts on the outside world. (d) Showing only the arrows, the head-to-tail method of addition is used. It is apparent that* $\mathbf{T = -w}$*, if Tarzan is stationary.*

Step 2. Identify what needs to be determined and what is known or can be inferred from the problem as stated. That is, make a list of knowns and unknowns. *Then carefully determine the system of interest.* This decision is a crucial step, since Newton's second law involves only external forces. Once the system of interest has been identified, it becomes possible to determine which forces are external and which are internal, a necessary step to employ

Newton's second law. (See Figure 1(c).) Newton's third law may be used to identify whether forces are exerted between components of a system (internal) or between the system and something outside (external). As illustrated earlier in this chapter, the system of interest depends on what question we need to answer. This choice becomes easier with practice, eventually developing into an almost unconscious process. Skill in clearly defining systems will be beneficial in later chapters as well.

A diagram showing the system of interest and all of the external forces is called a **free-body diagram**. Only forces are shown on free-body diagrams, not acceleration or velocity. We have drawn several of these in worked examples. Figure 1(c) shows a free-body diagram for the system of interest. Note that no internal forces are shown in a free-body diagram.

Step 3. Once a free-body diagram is drawn, *Newton's second law can be applied to solve the problem.* This is done in Figure 1(d) for a particular situation. In general, once external forces are clearly identified in free-body diagrams, it should be a straightforward task to put them into equation form and solve for the unknown, as done in all previous examples. If the problem is one-dimensional—that is, if all forces are parallel—then they add like scalars. If the problem is two-dimensional, then it must be broken down into a pair of one-dimensional problems. This is done by projecting the force vectors onto a set of axes chosen for convenience. As seen in previous examples, the choice of axes can simplify the problem. For example, when an incline is involved, a set of axes with one axis parallel to the incline and one perpendicular to it is most convenient. It is almost always convenient to make one axis parallel to the direction of motion, if this is known.

## Applying Newton's Second Law

Before you write net force equations, it is critical to determine whether the system is accelerating in a particular direction. If the acceleration is zero in a particular direction, then the net force is zero in that direction. Similarly, if the acceleration is nonzero in a particular direction, then the net force is described by the equation: $\mathbf{F}_{\text{net}} = m\mathbf{a}$. For example, if the system is accelerating in the horizontal direction, but it is not accelerating in the vertical direction, then you will have the following conclusions:

$$\boldsymbol{F}_{\text{net } x} = m\boldsymbol{a},$$
$$\boldsymbol{F}_{\text{net } y} = \mathbf{0}.$$

You will need this information in order to determine unknown forces acting in a system.

Step 4. As always, *check the solution to see whether it is reasonable.* In some cases, this is obvious. For example, it is reasonable to find that friction causes an object to slide down an incline more slowly than when no friction exists. In practice, intuition develops gradually through problem solving, and with experience it becomes progressively easier to judge whether an answer is reasonable. Another way to check your solution is to check the units. If you are solving for force and end up with units of m/s, then you have made a mistake.

## Section Summary

- To solve problems involving Newton's laws of motion, follow the procedure described:

    1. Draw a sketch of the problem.
    2. Identify known and unknown quantities, and identify the system of interest. Draw a free-body diagram, which is a sketch showing all of the forces acting on an object. The object is represented by a dot, and the forces are represented by vectors extending in different directions from the dot. If vectors act in directions that are not horizontal or vertical, resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.
    3. Write Newton's second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the $x$-direction) then $\mathbf{F_{netx} = 0}$. If the object does accelerate in that direction, $\mathbf{F_{netx} = ma}$.
    4. Check your answer. Is the answer reasonable? Are the units correct?

### *Problems & Exercises*

**1:** A $5.00 \times 10^5$-kg rocket is accelerating straight up. Its engines produce $1.250 \times 10^7$ N of thrust, and air resistance is $4.50 \times 10^6$ N. What is the rocket's acceleration? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

**2:** The wheels of a midsize car exert a force of 2100 N backward on the road to accelerate the car in the forward direction. If the force of friction including air resistance is 250 N and the acceleration of the car is 1.80 $m/s^2$, what is the mass of the car plus its occupants? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. For this situation, draw a free-body diagram and write the net force equation.

**3:** Calculate the force a 70.0-kg high jumper must exert on the ground to produce an upward acceleration 4.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

**4:** When landing after a spectacular somersault, a 40.0-kg gymnast decelerates by pushing straight down on the mat. Calculate the force she must exert if her deceleration is 7.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

**5:** A freight train consists of two $8.00 \times 10^4$-kg engines and 45 cars with average masses of $5.50 \times 10^4$ kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00 \times 10^{-2}$ $m/s^2$ if the force of friction is $7.50 \times 10^5$ N, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same

mass and that friction is evenly distributed among all of the cars and engines?

**6:** Commercial airplanes are sometimes pushed out of the passenger loading area by a tractor. (a) An 1800-kg tractor exerts a force of $1.75 \times 10^4$ N backward on the pavement, and the system experiences forces resisting motion that total 2400 N. If the acceleration is 0.150 $m/s^2$, what is the mass of the airplane? (b) Calculate the force exerted by the tractor on the airplane, assuming 2200 N of the friction is experienced by the airplane. (c) Draw two sketches showing the systems of interest used to solve each part, including the free-body diagrams for each.

**7:** A 1100-kg car pulls a boat on a trailer. (a) What total force resists the motion of the car, boat, and trailer, if the car exerts a 1900-N force on the road and produces an acceleration of 0.550 $m/s^2$? The mass of the boat plus trailer is 700 kg. (b) What is the force in the hitch between the car and the trailer if 80% of the resisting forces are experienced by the boat and trailer?

**8:** (a) Find the magnitudes of the forces $F_1$ and $F_2$ that add to give the total force $F_{tot}$ shown in Figure 2. This may be done either graphically or by using trigonometry. (b) Show graphically that the same total force is obtained independent of the order of addition of $F_1$ and $F_2$. (c) Find the direction and magnitude of some other pair of vectors that add to give $F_{tot}$. Draw these to scale on the same drawing used in part (b) or a similar picture.

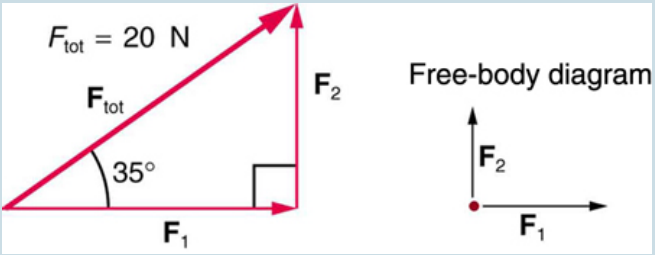



***Figure 2.***

**9:** Two children pull a third child on a snow saucer sled exerting forces $F_1$ and $F_2$ as shown from above in Figure 3. Find the acceleration of the 49.00-kg sled and child system. Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of $F_1$ and $F_2$.

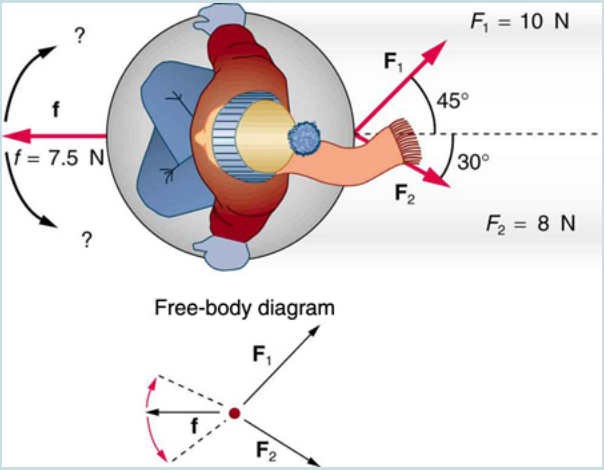



***Figure 3.*** *An overhead view of the horizontal forces acting on a child's snow saucer sled.*

**10:** Suppose your car was mired deeply in the mud and you wanted to use the method illustrated in Figure 4 to pull it out. (a) What force would you have to exert perpendicular to the center of the rope to produce a force of 12,000 N on the car if the angle is 2.00°? In this part, explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. (b) Real ropes stretch under such forces. What force would be exerted on the car if the angle increases to 7.00° and you still apply the force found in part (a) to its center?

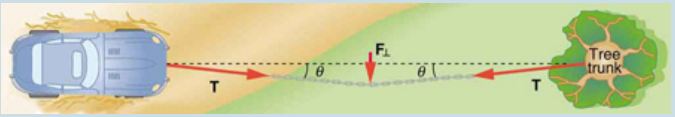



***Figure 4.***

**11:** What force is exerted on the tooth in Figure 5 if the tension in the wire is 25.0 N? Note that the force applied to the tooth is smaller than the tension in the wire, but this is necessitated by practical considerations of how force can be applied in the mouth. Explicitly show how you follow steps in the Problem-Solving Strategy for Newton's laws of motion.

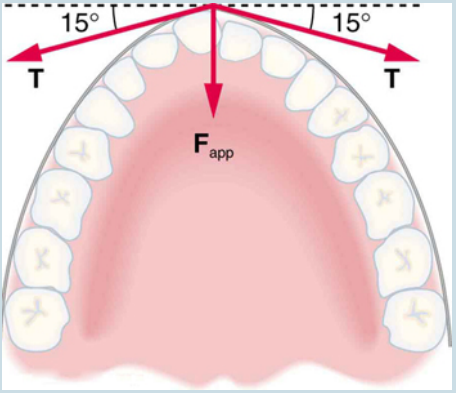



***Figure 5.*** *Braces are used to apply forces to teeth to realign them. Shown in this figure are the tensions applied by the wire to the protruding tooth. The total force applied to the tooth by the wire,* $\mathbf{F_{app}}$*, points straight toward the back of the mouth.*

**12:** Figure 6 shows Superhero and Trusty Sidekick hanging motionless from a rope. Superhero's mass is 90.0 kg, while Trusty Sidekick's is 55.0 kg, and the mass of the rope is negligible. (a) Draw a free-body diagram of the situation showing all forces acting on Superhero, Trusty Sidekick, and the rope. (b) Find the tension in the rope above Superhero. (c) Find the tension in the rope between Superhero and Trusty Sidekick. Indicate on your free-body diagram the system of interest used to solve each part.



***Figure 6.*** *Superhero and Trusty Sidekick hang motionless on a rope as they try to figure out what to do next. Will the tension be the same everywhere in the rope?*

**13:** A nurse pushes a cart by exerting a force on the

handle at a downward angle 35.0° below the horizontal. The loaded cart has a mass of 28.0 kg, and the force of friction is 60.0 N. (a) Draw a free-body diagram for the system of interest. (b) What force must the nurse exert to move at a constant velocity?

**14: Construct Your Own Problem** Consider the tension in an elevator cable during the time the elevator starts from rest and accelerates its load upward to some cruising velocity. Taking the elevator and its load to be the system of interest, draw a free-body diagram. Then calculate the tension in the cable. Among the things to consider are the mass of the elevator and its load, the final velocity, and the time taken to reach that velocity.

**15: Construct Your Own Problem** Consider two people pushing a toboggan with four children on it up a snow-covered slope. Construct a problem in which you calculate the acceleration of the toboggan and its load. Include a free-body diagram of the appropriate system of interest as the basis for your analysis. Show vector forces and their components and explain the choice of coordinates. Among the things to be considered are the forces exerted by those pushing, the angle of the slope, and the masses of the toboggan and children.

**16: Unreasonable Results** (a) Repeat Exercise 7, but assume an acceleration of 1.20 $m/s^2$ is produced. (b) What is unreasonable about the result? (c) Which premise is unreasonable, and why is it unreasonable?

**17: Unreasonable Results** (a) What is the initial acceleration of a rocket that has a mass of $1.50 \times 10^6$ kg at

takeoff, the engines of which produce a thrust of $2.00 \times 10^6$ N? Do not neglect gravity. (b) What is unreasonable about the result? (This result has been unintentionally achieved by several real rockets.) (c) Which premise is unreasonable, or which premises are inconsistent? (You may find it useful to compare this problem to the rocket problem earlier in this section.)

## *Solutions*

**Problems & Exercises**

**1:**

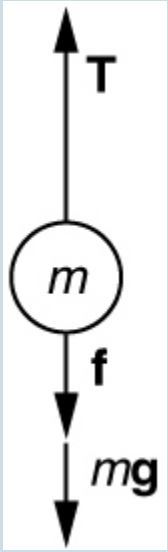



***Figure 7.***

Using the free-body diagram:

$\boldsymbol{F_{\text{net}} = T - f - mg = ma}$, so that

$$a = \frac{T - f - mg}{m} = \frac{1.250 \times 10^7\ \text{N} - 4.50 \times 10^6\ N - (5.00 \times 10^5\ \text{kg})(9.80\ \text{m/s}^2)}{5.00 \times 10^5\ \text{kg}} = 6.20\ \text{m/s}^2$$

**3:**

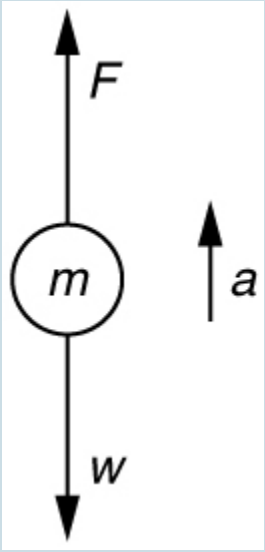



***Figure 8.***

1. Use Newton's laws of motion
2. Given :
   $a = 4.00g = (4.00)(9.80\ \text{m/s}^2) = 39.2\ \text{m/s}^2$ ; $m = 70.0\ \text{kg}$,
   Find: $\boldsymbol{F}$.
3. $\sum \boldsymbol{F} = +\boldsymbol{F} - \boldsymbol{w} = \boldsymbol{ma}$, so that
   $\boldsymbol{F} = \boldsymbol{ma} + \boldsymbol{w} = \boldsymbol{ma} + \boldsymbol{mg} = \boldsymbol{m(a+g)}.$
   $F = (70.0\ \text{kg})[(39.2\ \text{m/s}^2) + (9.80\ \text{m/s}^2)] = 3.43 \times 10^3\ \text{N}.$
   The force exerted by the high-jumper is actually down on the ground, but $\boldsymbol{F}$ is up from the ground and makes him jump.
4. This result is reasonable, since it is quite possible for a person to exert a force of the magnitude of

$10^3$ N.

**5:** (a) $4.41 \times 10^5$ N (b) $1.50 \times 10^5$ N

**7:** (a) 910 N (b) $1.11 \times 10^3$ N

**9:** $a = 0.139\ \text{m/s}, \theta = 12.4^o$ north of east

**11:**

1. Use Newton's laws since we are looking for forces.
2. Draw a free-body diagram:

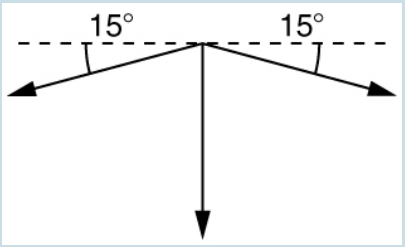



***Figure 9.***

3. The tension is given as $T = 25.0$ N. Find $F_{\text{app}}$. Using Newton's laws gives: $\sum F_y = 0$, so that applied force is due to the $y$-components of the two tensions:
   $F_{\text{app}} = 2T\sin\theta = 2(25.0\ \text{N})\sin(15^0) = 12.9\ \text{N}$
   The $x$-components of the tension cancel.
   $\sum F_x = 0.$
4. This seems reasonable, since the applied tensions should be greater than the force applied to the tooth.

# 26. 4.8 Further Applications of Newton's Laws of Motion

*Summary*

- Apply problem-solving techniques to solve for quantities in more complex systems of forces.
- Integrate concepts from kinematics to solve problems using Newton's laws of motion.

There are many interesting applications of Newton's laws of motion, a few more of which are presented in this section. These serve also to illustrate some further subtleties of physics and to help build problem-solving skills.

## Example 1: Drag Force on a Barge

Suppose two tugboats push on a barge at different angles, as shown in Figure 1. The first tugboat exerts a force of $\mathbf{2.7 \times 10^5\ N}$ in the $x$-direction, and the second tugboat exerts a force of $\mathbf{3.6 \times 10^5\ N}$ in the $y$-direction.

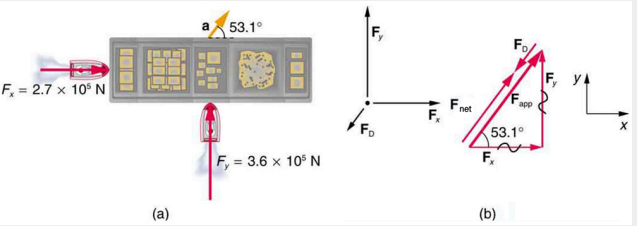



***Figure 1.*** *(a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the x- and y-axes are in the same direction as* $\mathbf{F_x}$ *and* $\mathbf{F_y}$*. The problem quickly becomes a one-dimensional problem along the direction of* $\mathbf{F_{app}}$*, since friction is in the direction opposite to* $\mathbf{F_{app}}$*.*

If the mass of the barge is $\mathbf{5.0 \times 10^6\ kg}$ and its acceleration is observed to be $\mathbf{7.5 \times 10^{-2}\ m/s^2}$ in the direction shown, what is the drag force of the water on the barge resisting the motion? (Note: drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object.)

**Strategy**

The directions and magnitudes of acceleration and the applied forces are given in Figure 1(a). We will define the total force of the tugboats on the barge as $\mathbf{F_{app}}$ so that:

$$\boldsymbol{F_{\text{app}} = F_x + F_y}$$

Since the barge is flat bottomed, the drag of the water $\mathbf{F_D}$ will be in the direction opposite to $\mathbf{F_{app}}$, as shown in

the free-body diagram in Figure 1(b). The system of interest here is the barge, since the forces on *it* are given as well as its acceleration. Our strategy is to find the magnitude and direction of the net applied force $\mathbf{F_{app}}$, and then apply Newton's second law to solve for the drag force $\mathbf{F_D}$.

**Solution**

Since $\mathbf{F_x}$ and $\mathbf{F_y}$ are perpendicular, the magnitude and direction of $\mathbf{F_{app}}$ are easily found. First, the resultant magnitude is given by the Pythagorean theorem:

$$F_{\text{app}} = \sqrt{F_x^2 + F_y^2}$$

$F_{\text{app}} = \sqrt{(2.7 \times 10^5 \text{ N})^2 + (3.6 \times 10^5 \text{ N})^2} = 4.5 \times 10^5 \text{ N}.$

The angle is given by

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{3.6 \times 10^5 \text{ N}}{2.7 \times 10^5 \text{ N}}\right) = 53^o,$$

which we know, because of Newton's first law, is the same direction as the acceleration. $\mathbf{F_D}$ is in the opposite direction of $\mathbf{F_{app}}$, since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as $\mathbf{F_{app}}$, but its magnitude is slightly less than $\mathbf{F_{app}}$. The problem is now one-dimensional. From Figure 1(b), we can see that

$$F_{\text{net}} = F_{\text{app}} - F_{\text{D}}.$$

But Newton's second law states that

$$F_{\text{net}} = ma.$$

Thus,

$$\boldsymbol{F_{\text{app}} - F_{\text{D}} = ma.}$$

This can be solved for the magnitude of the drag force of the water $\mathbf{F_D}$ in terms of known quantities:

$$\boldsymbol{F_{\text{D}} = F_{\text{app}} - ma.}$$

Substituting known values gives

$$F_{\text{D}} = (4.5 \times 10^5 \text{ N}) - (5.0 \times 10^6 \text{ kg})(7.5 \times 10^{-2} \text{ m/s}^2) = 7.5 \times 10^4 \text{ N}.$$

The direction of $\mathbf{F_D}$ has already been determined to be in the direction opposite to $\mathbf{F_{app}}$, or at an angle of **53°** south of west.

**Discussion**

The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where $\mathbf{F_D}$ is less than 1/600th of the weight of the ship.

In the earlier example of a tightrope walker we noted that the tensions in wires supporting a mass were equal only because the angles on either side were equal. Consider the following example, where the angles are not equal; slightly more trigonometry is involved.

## Example 2: Different Tensions at Different Angles

Consider the traffic light (mass 15.0 kg) suspended from two wires as shown in Figure 2. Find the tension in each wire, neglecting the masses of the wires.

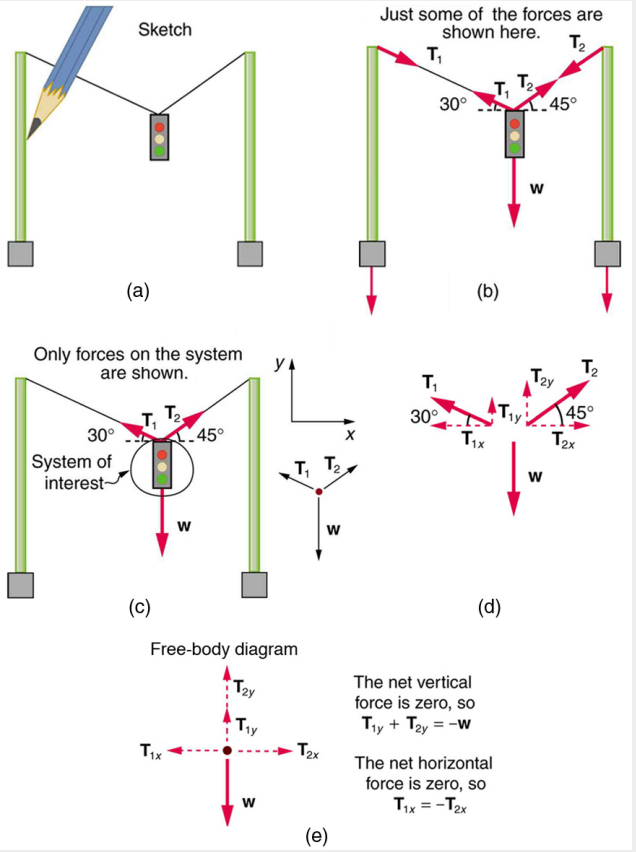



***Figure 2.*** *A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical (y) and horizontal (x) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The free-body diagram shows the vertical and horizontal forces acting on the traffic light.*

**Strategy**

The system of interest is the traffic light, and its free-body diagram is shown in Figure 2(c). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in part (d) of the figure. There are two unknowns in this problem ($\mathbf{T_1}$ and $\mathbf{T_2}$), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

**Solution**

First consider the horizontal or $x$-axis:

$$\boldsymbol{F_{\text{net}x} = T_{2x} - T_{1x} = 0.}$$

Thus, as you might expect,

$$\boldsymbol{T_{1x} = T_{2x}.}$$

This gives us the following relationship between $\mathbf{T_1}$ and $\mathbf{T_2}$:

$$\boldsymbol{T_1 \cos(30^0) = T_2 \cos(45^0).}$$

Thus,

$$\boldsymbol{T_2 = (1.225) T_1.}$$

Note that $\mathbf{T_1}$ and $\mathbf{T_2}$ are not equal in this case, because the angles on either side are not equal. It is reasonable that $\mathbf{T_2}$ ends up being greater than $\mathbf{T_1}$, because it is exerted more vertically than $\mathbf{T_1}$

Now consider the force components along the vertical or $y$-axis:

$$F_{\text{net}y} = T_{1y} + T_{2y} - w = 0.$$

This implies

$$T_{1y} + T_{2y} = w.$$

Substituting the expressions for the vertical components gives

$$T_1 \sin(30^0) + T_2 \sin(45^0) = w.$$

There are two unknowns in this equation, but substituting the expression for $T_2$ in terms of $T_1$ reduces this to one equation with one unknown:

$$T_1(0.500) + (1.225T_1)(0.707) = w = mg,$$

which yields

$$(1.366)T_1 = (15.0 \text{ kg})(9.80 \text{ m/s}^2).$$

Solving this last equation gives the magnitude of $T_1$ to be

$$T_1 = 108 \text{ N}.$$

Finally, the magnitude of $T_2$ is determined using the relationship between them, $T_2 = 1.225\ T_1$ found above. Thus we obtain

$$T_2 = 132 \text{ N}.$$

**Discussion**

Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker).

The bathroom scale is an excellent example of a normal force acting

on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the dial of a bathroom scale if you stood on it during an elevator ride? Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed: will the scale still read more than your weight at rest? Consider the following example.

### Example 3: What Does the Bathroom Scale Read in an Elevator?

Figure 3 shows a 75.0-kg man (weight of about 165 lb) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of $\mathbf{1.20\ m/s^2}$, and (b) if the elevator moves upward at a constant speed of 1 m/s.

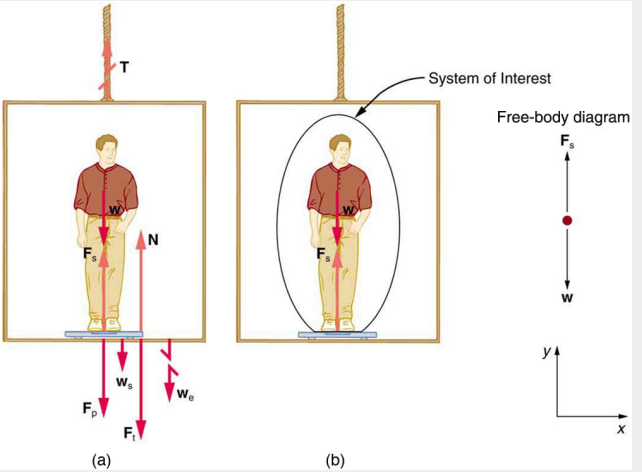



***Figure 3.*** *(a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale.* **T** *is the tension in the supporting cable,* $\boldsymbol{w}$ *is the weight of the person,* $\boldsymbol{w_s}$ *is the weight of the scale,* $\boldsymbol{w_e}$*is the weight of the elevator,* $\mathbf{F_s}$ *is the force of the scale on the person,* $\mathbf{F_p}$ *is the force of the person on the scale,* $\mathbf{F_t}$ *is the force of the scale on the floor of the elevator, and* **N** *is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting on the designated system of interest—the person.*

### Strategy

If the scale is accurate, its reading will equal $\mathbf{F_p}$, the magnitude of the force the person exerts downward on it. Figure 3(a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a

free-body diagram is drawn as in Figure 3(b). Analysis of the free-body diagram using Newton's laws can produce answers to both parts (a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight $\boldsymbol{w}$ and the upward force of the scale $\mathbf{F_s}$. According to Newton's third law $\mathbf{F_p}$ and $\mathbf{F_s}$ are equal in magnitude and opposite in direction, so that we need to find $\mathbf{F_s}$ in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,

$$\boldsymbol{F_{\text{net}} = ma.}$$

From the free-body diagram we see that $\mathbf{F_{net} = F_s - w}$, so that

$$\boldsymbol{F_s - w = ma.}$$

Solving for $\mathbf{F_s}$ gives an equation with only one unknown:

$$\boldsymbol{F_s = ma + w,}$$

or, because ***weight = gravity force downwards = mg***, simply

$$\boldsymbol{F_s = ma + mg.}$$

No assumptions were made about the acceleration, and so this solution should be valid for a variety of accelerations in addition to the ones in this exercise.

**Solution for (a)**

In this part of the problem, $\boldsymbol{a = 1.20\ \text{m/s}^2}$, so that

$$\boldsymbol{F_s = (75.0\ \text{kg})(1.20\ \text{m/s}^2) + (75.0\ \text{kg})(9.80\ \text{m/s}^2) = 825N,}$$

**Discussion for (a)**

This is about 185 lb. What would the scale have read if

he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

$$F_{\text{net}} = ma = 0 = F_{\text{s}} - w$$
$$F_{\text{s}} = w = mg$$
$$F_{\text{s}} = (75.0\ \text{kg})(9.80\ \text{m/s}^2) = 735N$$

So, the scale reading in the elevator is greater than his 735-N (165 lb) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators.

**Solution for (b)**

Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because $a = \frac{\Delta v}{\Delta t}$, and $\Delta v = 0$.

Thus,

$$F_{\text{s}} = ma + mg = 0 + mg$$

Now

$$F_{\text{s}} = (75.0\ \text{kg})(9.80\ \text{m/s}^2) = 735N$$

**Discussion for (b)**

The scale reading is 735 N, which equals the person's weight. This will be the case whenever the elevator has a

constant velocity—moving up, moving down, or stationary.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward, $\boldsymbol{a}$ is negative, and the scale reading is *less* than the weight of the person, until a constant downward velocity is reached, at which time the scale reading again becomes equal to the person's weight. If the elevator is in free-fall and accelerating downward at $\boldsymbol{g}$, then the scale reading will be zero and the person will *appear* to be weightless.

## Integrating Concepts: Newton's Laws of Motion and Kinematics

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton's laws of motion can also be integrated with other concepts that have been discussed previously in this text to solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters. When approaching problems that involve various types of forces, acceleration, velocity, and/or position, use the following steps to approach the problem:

**Problem-Solving Strategy**

Step 1. *Identify which physical principles are involved.* Listing the givens and the quantities to be calculated will allow you to identify the principles involved.

Step 2. *Solve the problem using strategies outlined in the text.* If these are available for the specific topic, you should refer to them. You should also refer to the sections of the text that deal with a particular topic. The following worked example illustrates how these strategies are applied to an integrated concept problem.

## Example 4: What Force Must a Soccer Player Exert to Reach Top Speed?

A soccer player starts from rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player's mass is 70.0 kg, and air resistance is negligible.

**Strategy**

1. *To solve an integrated concept problem*, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example considers *acceleration* along a straight line. This is a topic of *kinematics*. Part (b) deals with force, a topic of *dynamics* found in this chapter.
2. The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so forth.

**Solution for (a)**

We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is $\Delta v = 8.00\ \text{m/s}$. We are given the elapsed time, and so $\Delta t = 2.50\ \text{s}$. The unknown is acceleration, which can be found from its definition:

$$a = \frac{\Delta v}{\Delta t}.$$

Substituting the known values yields

$$a = \frac{8.00\ \text{m/s}}{2.50\ \text{s}} = 3.20\ \text{m/s}^2.$$

**Discussion for (a)**

This is an attainable acceleration for an athlete in good condition.

**Solution for (b)**

Here we are asked to find the average force the player exerts backward to achieve this forward acceleration. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes his acceleration. Since we now know the player's acceleration and are given his mass, we can use Newton's second law to find the force exerted. That is,

$$F_{\text{net}} = ma.$$

Substituting the known values of $m$ and $a$ gives

$$F_{\text{net}} = (70.0\ \text{kg})(3.20\ \text{m/s}^2) = 224N$$

**Discussion for (b)**

This is about 50 pounds, a reasonable average force.

This worked example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These strategies are found throughout the text, and many worked examples show how to use them for single topics. You will find these techniques for integrated concept problems useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

## Summary

- Newton's laws of motion can be applied in numerous situations to solve problems of motion.
- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether $\mathbf{F_{net}} = m\boldsymbol{a}$ or $\mathbf{F_{net}} = \mathbf{0}$.
- The normal force on an object is not always equal in magnitude

to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.

- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

## *Conceptual Questions*

**1:** To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at $g$. Why will they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?

**2:** A cartoon shows the toupee coming off the head of an elevator passenger when the elevator rapidly stops during an upward ride. Can this really happen without the person being tied to the floor of the elevator? Explain your answer.

## *Problems & Exercises*

**1:** A flea jumps by exerting a force of $1.20 \times 10^{-5}$ N straight

down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of $0.500 \times 10^{-6}$ N on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is $6.00 \times 10^{-7}$ kg. Do not neglect the gravitational force.

**2:** Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in Figure 4. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?

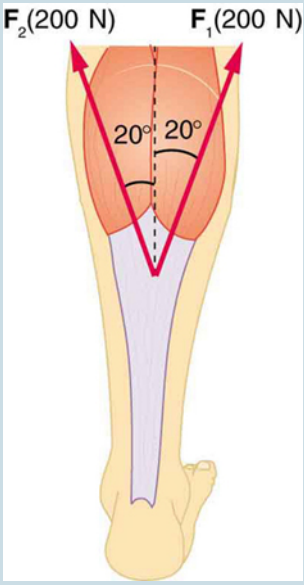



***Figure 4.*** *Achilles tendon*

**3:** A 76.0-kg person is being pulled away from a burning building as shown in Figure 5. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.

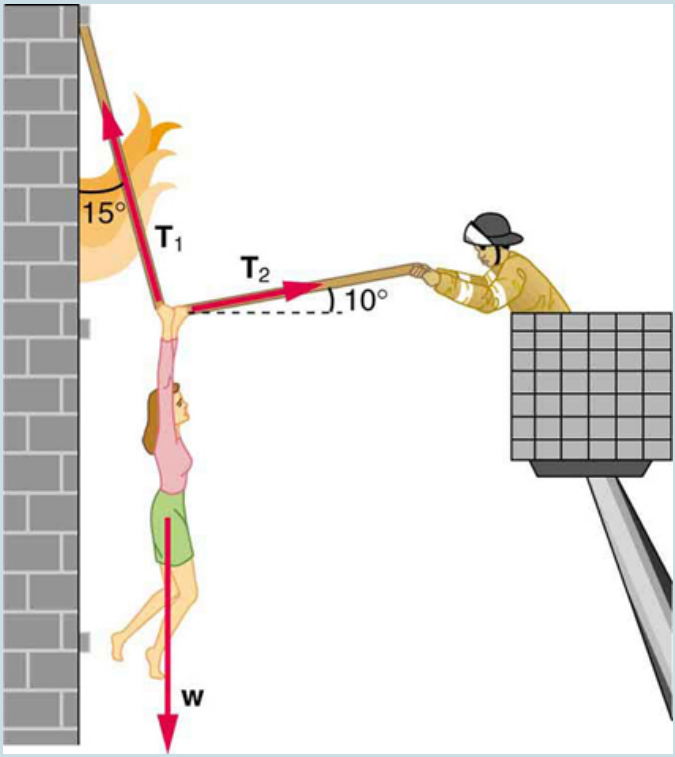



***Figure 5.*** *The force* $\mathbf{T_2}$ *needed to hold steady the person being rescued from the fire is less than her weight and less than the force* $\mathbf{T_1}$ *in the other rope, since the more vertical rope supports a greater part of her weight (a vertical force)*

**4: Integrated Concepts** A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow him if he was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)

**5: Integrated Concepts** When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel? Hint: Find his acceleration first then use kinematics.

**6: Integrated Concepts** A large rocket has a mass of $2.00 \times 10^6$ kg at takeoff, and its engines produce a thrust of $3.50 \times 10^7$ N. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust? (c) In reality, the mass of a rocket decreases significantly as its fuel is consumed. Describe qualitatively how this affects the acceleration and time for this motion.

**7: Integrated Concepts** A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force he exerts on the floor to do this, given that his mass is 110 kg.

**8: Integrated Concepts** A 2.50-kg fireworks shell is fired

straight up from a mortar and reaches a height of 110 m. (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell's velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.

**9: Integrated Concepts** Repeat Exercise 8 for a shell fired at an angle 10.0° from the vertical.

**10: Integrated Concepts** An elevator filled with passengers has a mass of 1700 kg. (a) The elevator accelerates upward from rest at a rate of 1.20 $m/s^2$ for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of 0.600 $m/s^2$ for 3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?

**11: Unreasonable Results** (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of 0.400 $m/s^2$ for 50.0 s? (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

**12: Unreasonable Results** A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in

newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

## *Solutions*

**Problems & Exercises**

**1: $10.2\ \mathrm{m/s^2}, 4.67^0$ from vertical**

**3:** $T_1 = 736$ N $T_2 = 194$ N as net force is 0 N so using magnitudes only

T1 cos $15^o$ + T2 sin $10^o$ = Weight = mg and T1 sin $15^o$ = T2 cos$10^o$

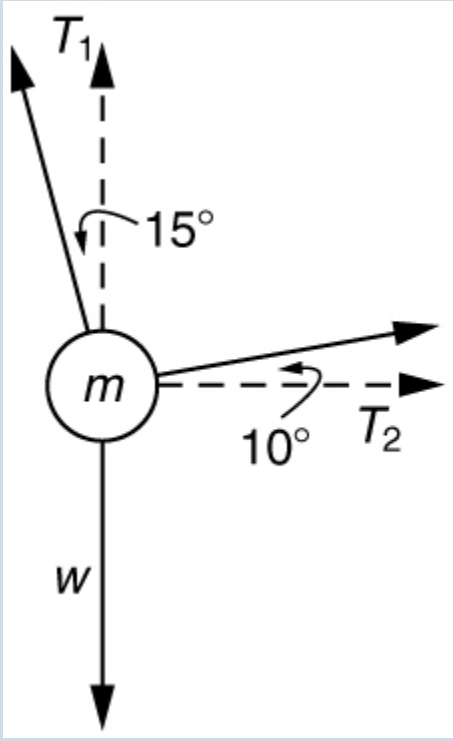



***Figure 6.***

**5:** (a) 7.43 m/s (b) 2.97 m as the acceleration is 650 N / 70.0 kg = 9.29 $m/s^2$

**7:** (a) 4.20 m/s 7 (b) 29.4 $m/s^2$ 7 (c) 4.31 x $10^3$ N

**9:** (a) 47.1 m/s 9 (b) $\mathbf{2.47 \times 10^3\ m/s^2}$ (c ) $\mathbf{6.18 \times 10^3\ N}$. The average force is 252 times the shell's weight.

PART VI

# CHAPTER 5 UNIFORM CIRCULAR MOTION AND GRAVITATION

Many motions, such as the arc of a bird's flight or Earth's path around the Sun, are curved. Recall that Newton's first law tells us that motion is along a straight line at constant speed unless there is a net external force. We will therefore study not only motion along curves, but also the forces that cause it, including gravitational forces. In some ways, this chapter is a continuation of Chapter 4 Dynamics: Newton's Laws of Motion as we study more applications of Newton's laws of motion.

This chapter deals with the simplest form of curved motion, **uniform circular motion**, motion in a circular path at constant speed. Studying this topic illustrates most concepts associated with rotational motion and leads to the study of many new topics we group under the name *rotation*. Pure *rotational motion* occurs when points in an object move in circular paths centered on one point. Pure *translational motion* is motion with no rotation. Some motion combines both types, such as a rotating hockey puck moving along ice.

## Glossary

**uniform circular motion**
the motion of an object in a circular path at constant speed

# 27. 5.1 Centripetal Acceleration

*Summary*

- Establish the expression for centripetal acceleration.
- Explain the centrifuge.

We know from kinematics that acceleration is a change in velocity, either in its magnitude or in its direction, or both. In uniform circular motion, the direction of the velocity changes constantly, so there is always an associated acceleration, even though the magnitude of the velocity might be constant. You experience this acceleration yourself when you turn a corner in your car. (If you hold the wheel steady during a turn and move at constant speed, you are in uniform circular motion.) What you notice is a sideways acceleration because you and the car are changing direction. The sharper the curve and the greater your speed, the more noticeable this acceleration will become. In this section we examine the direction and magnitude of that acceleration.

Figure 1 shows an object moving in a circular path at constant speed. The direction of the instantaneous velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity, which points directly toward the center of rotation (the center of the circular path). This pointing is shown with the vector diagram in the figure. We call the acceleration of an object moving in uniform circular motion (resulting from a net external force) the

**centripetal acceleration** ($a_c$); centripetal means "toward the center" or "center seeking."

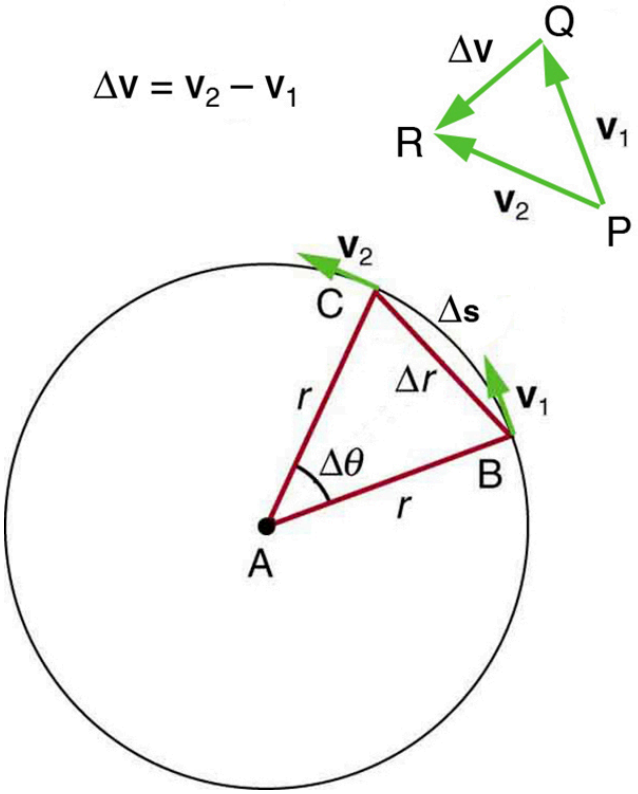



***Figure 1.*** *The directions of the velocity of an object at two different points are shown, and the change in velocity* $\Delta \mathbf{v}$ *is seen to point directly toward the center of curvature. (See small inset.) Because* $\mathbf{a_c} = \Delta \mathbf{v}/\Delta \mathbf{t}$*, the acceleration is also toward the center; ac is called centripetal acceleration. (Because* $\Delta \boldsymbol{\theta}$ *is very small, the arc length* $\Delta \mathbf{s}$ *is equal to the chord length* $\Delta \mathbf{r}$ *for small time differences.)*

The direction of centripetal acceleration is toward the center of curvature, but what is its magnitude? Note that the triangle formed by the velocity vectors and the one formed by the radii $\boldsymbol{r}$ and $\Delta \mathbf{s}$ are similar. Both the triangles ABC and PQR are isosceles triangles (two

equal sides). The two equal sides of the velocity vector triangle are the speeds $v_1 = v_2 = v$. Using the properties of two similar triangles, we obtain

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}.$$

Acceleration is $\Delta v/\Delta t$, and so we first solve this expression for $\Delta v$:

$$\Delta v = \frac{v}{r}\Delta s.$$

Then we divide this by $\Delta t$, yielding

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \times \frac{\Delta s}{\Delta t}.$$

Finally, noting that $\Delta v/\Delta t = a_c$ and that $\Delta s/\Delta t = v$, the linear or tangential speed, we see that the magnitude of the centripetal acceleration is

$$a_c = \frac{v^2}{r},$$

which is the acceleration of an object in a circle of radius $r$ at a speed $v$. So, centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you have noticed when driving a car. But it is a bit surprising that $a_c$ is proportional to speed squared, implying, for example, that it is four times as hard to take a curve at 100 km/h than at 50 km/h. A sharp corner has a small radius, so that $a_c$ is greater for tighter turns, as you have probably noticed.

A centrifuge (see Figure 2b) is a rotating device used to separate specimens of different densities. High centripetal acceleration significantly decreases the time it takes for separation to occur, and makes separation possible with small samples. Centrifuges are used in a variety of applications in science and medicine, including the separation of single cell suspensions such as bacteria, viruses, and blood cells from a liquid medium and the separation of macromolecules, such as DNA and protein, from a solution.

Centrifuges are often rated in terms of their centripetal acceleration relative to acceleration due to gravity ($g$); maximum centripetal acceleration of several hundred thousand $g$ is possible in a vacuum. Human centrifuges, extremely large centrifuges, have been used to test the tolerance of astronauts to the effects of accelerations larger than that of Earth's gravity.

## Example 1: How Does the Centripetal Acceleration of a Car Around a Curve Compare with That Due to Gravity?

What is the magnitude of the centripetal acceleration of a car following a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h)? Compare the acceleration with that due to gravity for this fairly gentle curve taken at highway speed. See Figure 2(a).

**Strategy**

Because $v$ and $r$ are given, we use the expression $a_c = \frac{v^2}{r}$.

**Solution**

Entering the given values of $v = 25.0\ \text{m/s}$ and $r = 500\ \text{m}$ into the first expression for $a_c$ gives

$$a_c = \frac{v^2}{r} = \frac{25.0^2}{500} = 1.25\ \text{m/s}^2$$

**Discussion**

To compare this with the acceleration due to gravity ($g = 9.80\ \text{m/s}^2$), we take the ratio of $a_c / g = (1.25\ \text{m/s}^2)/(9.80\ \text{m/s}^2) = 0.128$. Thus, $a_c = 0.128\ g$ and is noticeable especially if you were not wearing a seat belt.

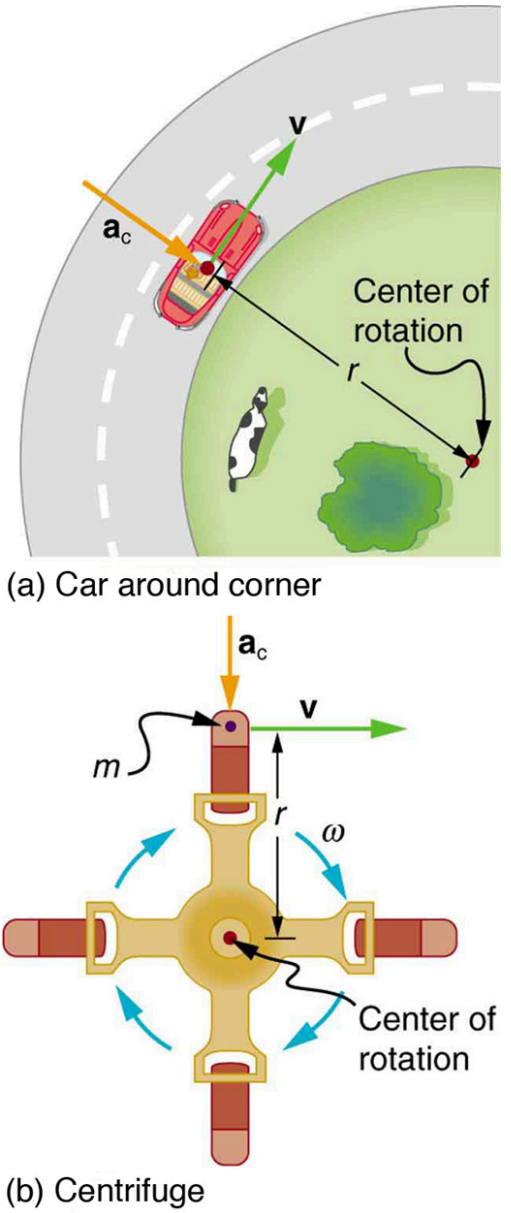



***Figure 2.*** *(a) The car following a circular path at constant speed is accelerated perpendicular to its velocity, as shown. The magnitude of this*

*centripetal acceleration is found in Example 1. (b) A particle of mass in a centrifuge is rotating at constant angular velocity . It must be accelerated perpendicular to its velocity or it would continue in a straight line. The magnitude of the necessary acceleration is found in Example 2.*

## Example 2: What is the speed of an Ultracentrifuge?

Calculate the speed of a point 7.50 cm from the axis of a spinning **ultracentrifuge** if the centripetal acceleration at that point is $\mathbf{4.63 \times 10^6\ m/s^2}$

**Strategy**

Both $r$ and $a_c$ are given, so we can use the equation $a_c = \frac{v^2}{r}$ to determine the speed of the centrifuge.

**Solution**

First we solve for the speed of the centrifugre

$$a_c = \frac{v^2}{r} \rightarrow v = \sqrt{a_c r}$$

Converting 7.50 cm to meters and substituting known values gives

$$v = \sqrt{a_c r} = \sqrt{(4.63 \times 10^6)(0.0750)} = 590\ \text{m/s}.$$

**Discussion**

The given centripetal acceleration is 472,000 times as strong as ***g***. It is no wonder that such high velocity centrifuges are called ultracentrifuges. The extremely large accelerations involved greatly decrease the time needed to cause the sedimentation of blood cells or other materials.

Of course, a net external force is needed to cause any acceleration, just as Newton proposed in his second law of motion. So a net external force is needed to cause a centripetal acceleration. In the next section on centripetal force, we will consider the forces involved in circular motion.

## PHET EXPLORATIONS: LADYBUG MOTION 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

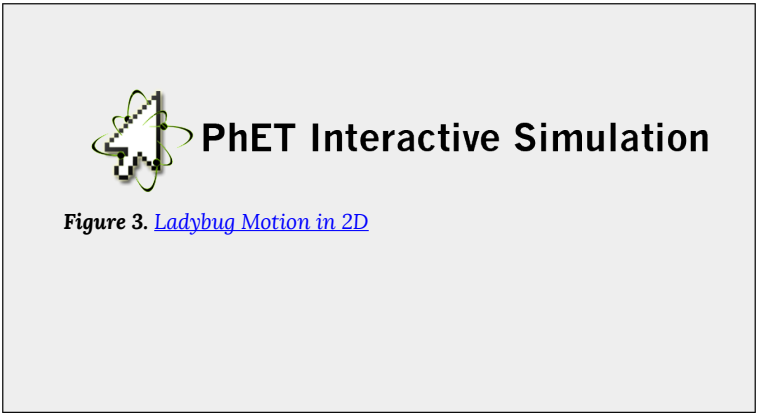



***Figure 3.*** *Ladybug Motion in 2D*

## Section Summary

- Centripetal acceleration $a_c$ is the acceleration experienced while in uniform circular motion. It always points toward the center of rotation. It is perpendicular to the linear velocity $v$ and has the magnitude

$$a_c = \frac{v^2}{r}$$

- The unit of centripetal acceleration is $\mathbf{m/s^2}$.

### *Conceptual Questions*

**1:** Can centripetal acceleration change the speed of circular motion? Explain.

## *Problems & Exercises*

**1:** A fairground ride spins its occupants inside a flying saucer-shaped container. If the horizontal circular path the riders follow has an 8.00 m radius, at how many revolutions per minute will the riders be subjected to a centripetal acceleration whose magnitude is 1.50 times that due to gravity?

**2:** A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of 30 m. If he completes the 200 m dash in 23.2 s and runs at constant speed throughout the race, what is the magnitude of his centripetal acceleration as he runs the curved portion of the track?

**3:** Taking the age of Earth to be about $4 \times 10^9$ years and assuming its orbital radius of $1.5 \times 10^{11}$ has not changed and is circular, calculate the approximate total distance Earth has traveled since its birth (in a frame of reference stationary with respect to the Sun). *Recall that the circumference of a circle is* $2\pi r$.

**4:** The propeller of a World War II fighter plane is 2.30 m in diameter and the tip of the blade spins at 145 m/s. What is the centripetal acceleration of the propeller tip under these conditions? Calculate it in meters per second squared and convert to multiples of *g*.

**5: Integrated Concepts**

Riders in an amusement park ride shaped like a Viking ship hung from a large pivot are rotated back and forth like

a rigid pendulum. Sometime near the middle of the ride, the ship is momentarily motionless at the top of its circular arc. The ship then swings down under the influence of gravity.

(a) Assuming negligible friction, find the speed of the riders at the bottom of its arc, given the system's center of mass travels in an arc having a radius of 14.0 m and the riders are near the center of mass.

(b) What is the centripetal acceleration at the bottom of the arc?

(c) Draw a free body diagram of the forces acting on a rider at the bottom of the arc.

(d) Find the force exerted by the ride on a 60.0 kg rider and compare it to her weight.

(e) Discuss whether the answer seems reasonable.

**6: Unreasonable Results**

A mother pushes her child on a swing so that his speed is 9.00 m/s at the lowest point of his path. The swing is suspended 2.00 m above the child's center of mass.

(a) What is the magnitude of the centripetal acceleration of the child at the low point?

(b) What is the magnitude of the force the child exerts on the seat if his mass is 18.0 kg?

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent?

## Glossary

**centripetal acceleration**
the acceleration of an object moving in a circle, directed toward the center

**ultracentrifuge**
a centrifuge optimized for spinning a rotor at very high speeds

## *Solutions*

**Problems & Exercises**

1: $10.8\ \mathrm{m/s}$

3: $4 \times 10^{21}\ \mathrm{m}$

5: (a) $23.4\ \mathrm{m/s}$ (b) $39.1\ \mathrm{m/s^2}$ (c)

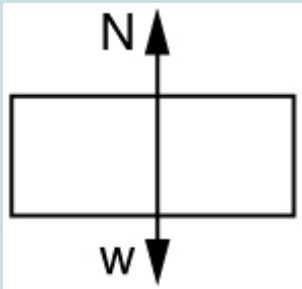



***Figure 4.***

(d) $1.76 \times 10^3\ \mathrm{N}$ or $3.00\ \omega$, that is, the normal force (upward) is three times her weight. (e) This

answer seems reasonable, since she feels like she's being forced into the chair MUCH stronger than just by gravity.

**6:** (a) $\mathbf{40.5\ m/s^2}$ (b) 905 N (c) The force in part (b) is very large. The acceleration in part (a) is too much, about 4 g. (d) The speed of the swing is too large. At the given velocity at the bottom of the swing, there is enough kinetic energy to send the child all the way over the top, ignoring friction.

# 28. 5.2 Centripetal Force

*Summary*

- Calculate coefficient of friction on a car tire.
- Calculate ideal speed and angle of a car on a turn.

Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration: net $\boldsymbol{F = ma}$. For uniform circular motion, the acceleration is the centripetal acceleration– $\boldsymbol{a = a_c}$. Thus, the magnitude of centripetal force $\mathbf{F_c}$ is

$$\boldsymbol{F_c = ma_c}.$$

By using the expressions for centripetal acceleration $\boldsymbol{a_c}$ from $\boldsymbol{a_c = \frac{v^2}{r}}$ we get an expression for the centripetal force $\mathbf{F_c}$ in terms of mass, velocity, and radius of curvature:

$$\boldsymbol{F_c = \frac{mv^2}{r}}$$

The centripetal force $\mathbf{F_c}$ is always perpendicular to the path and

pointing to the center of curvature, because $\boldsymbol{a_c}$ is perpendicular to the velocity and pointing to the center of curvature.

Note that if you solve the first expression for $\boldsymbol{r}$, you get

$$\boldsymbol{r} = \frac{\boldsymbol{mv^2}}{\boldsymbol{F_c}}.$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.

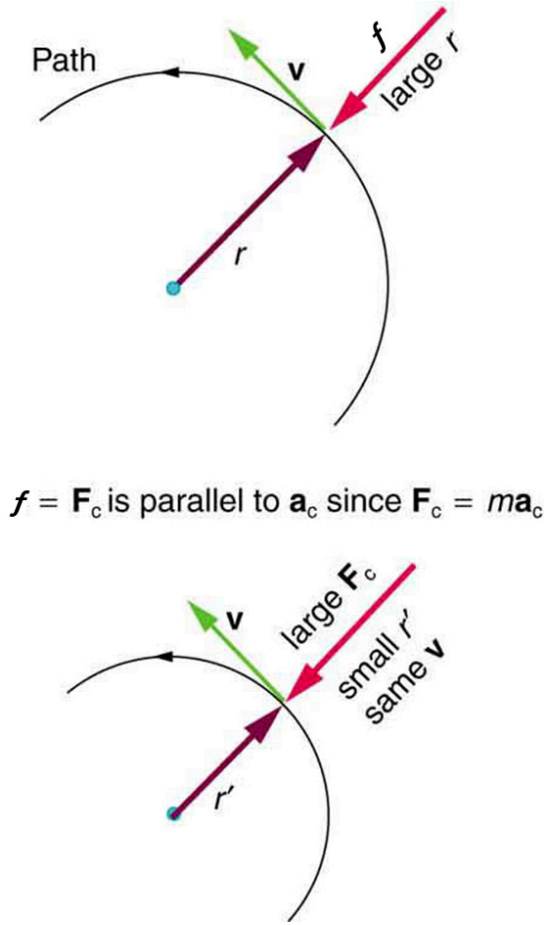



***Figure 1.*** *The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the* $\mathbf{F_c}$*, the smaller the radius of curvature* $\mathbf{r}$ *and the sharper the curve. The second curve has the same* $\mathbf{v}$*, but a larger* $\mathbf{F_c}$ *produces a smaller* $\mathbf{r'}$*.*

## Example 1: What Coefficient of Friction Do Care Tires Need on a Flat Curve?

(a) Calculate the centripetal force exerted on a 900 kg car that negotiates a 500 m radius curve at 25.0 m/s.

(b) Assuming an unbanked curve, find the minimum static coefficient of friction, between the tires and the road, static friction being the reason that keeps the car from slipping (see Figure 2).

**Strategy and Solution for (a)**

We know that $F_c = \frac{mv^2}{r}$. Thus,

$$F_c = \frac{mv^2}{r} = \frac{(900\ \text{kg})(25.0\ \text{m/s})^2}{(500\ \text{m})} = 1125\ \text{N}.$$

**Strategy for (b)**

Figure 2 shows the forces acting on the car on an unbanked (level ground) curve. Friction is to the left, keeping the car from slipping, and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is $\mu_s N$, where $\mu_s$ is the static coefficient of friction and $N$ is the normal force. The normal force

equals the car's weight on level ground, so that $\mathbf{N} = \boldsymbol{mg}$. Thus the centripetal force in this situation is

$$\boldsymbol{F_c = f = \mu_s N = \mu_s mg.}$$

Now we have a relationship between centripetal force and the coefficient of friction. Using the first expression for $\mathbf{F_c}$ from the equation

$$\left.\begin{aligned} \boldsymbol{F_c} &= \boldsymbol{m\frac{v^2}{r}} \\ \boldsymbol{F_c} &= \boldsymbol{mr\omega^2} \end{aligned}\right\},$$

$$\boldsymbol{m\frac{v^2}{r} = \mu_s mg.}$$

We solve this for $\boldsymbol{\mu_s}$, noting that mass cancels, and obtain

$$\boldsymbol{\mu_s = \frac{v^2}{rg}.}$$

**Solution for (b)**

Substituting the knowns,

$$\boldsymbol{\mu_s = \frac{(25.0\ \mathrm{m/s})^2}{(500\ \mathrm{m})(9.80\ \mathrm{m/s^2})} = 0.13.}$$

(Because coefficients of friction are approximate, the answer is given to only two digits.)

**Discussion**

The coefficient of friction found in part (b) is much smaller than is typically found between tires and roads. The car will still negotiate the curve if the coefficient is greater than 0.13, because static friction is a responsive force, being able to assume a value less than but no

more than $\mu_s N$. A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be less as will be discussed below.

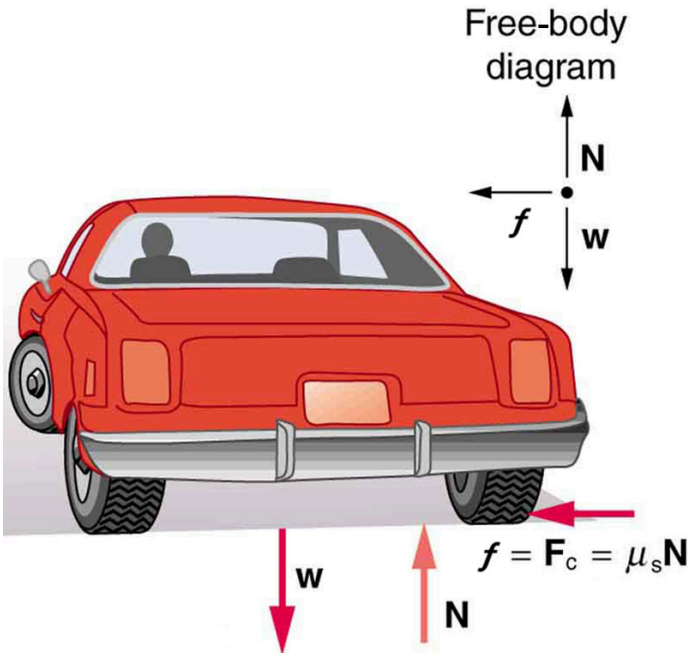



***Figure 2.*** *This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.*

## Optional: Banked Curves

Let us now consider **banked curves**, where the slope of the road helps you negotiate the curve. See Figure 3. The greater the angle $\boldsymbol{\theta}$, the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an "ideally banked curve," the angle $\boldsymbol{\theta}$ is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for $\boldsymbol{\theta}$ for an ideally banked curve and consider an example related to it.

For **ideal banking**, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force **N** in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.

Figure 3 shows a free body diagram for a car on a frictionless banked curve. If the angle $\boldsymbol{\theta}$ is ideal for the speed and radius, then the net external force will equal the necessary centripetal force. The only two external forces acting on the car are its weight $\boldsymbol{w}$ and the normal force of the road **N**. (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude $\boldsymbol{mv^2/r}$. Because this is the crucial force and it is horizontal, we use a coordinate system with vertical and horizontal axes. Only the normal force has a horizontal component, and so this must equal the centripetal force—that is,

$$\boldsymbol{N\sin\theta = \frac{mv^2}{r}.}$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From the figure, we see that the vertical component of the normal force is $\boldsymbol{N\cos\theta}$, and the only other vertical force is the car's weight. These must be equal in magnitude; thus,

$$\boldsymbol{N\cos\theta = mg.}$$

Now we can combine the last two equations to eliminate **N** and get an expression for $\boldsymbol{\theta}$, as desired. Solving the second equation for $\boldsymbol{N = mg/(\cos\theta)}$, and substituting this into the first yields

$$\boldsymbol{mg\frac{\sin\theta}{\cos\theta} = \frac{mv^2}{r}}$$

$$mg\tan(\theta) = \frac{mv^2}{r}$$

$$\tan\theta = \frac{v^2}{rg}.$$

Taking the inverse tangent gives

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

**(ideally banked curve, no friction).**

This expression can be understood by considering how $\theta$ depends on $v$ and $r$. A large $\theta$ will be obtained for a large $v$ and a small $r$. That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve is frictionless. Note that $\theta$ does not depend on the mass of the vehicle.

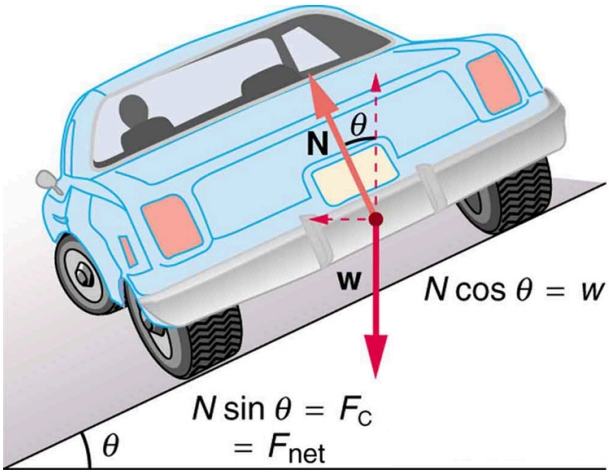



***Figure 3.*** *The car on this banked curve is moving away and turning to the left.*

## Example 2: What Is the Ideal Speed to Take a Steeply Banked Tight Curve?

Curves on some test tracks and race courses, such as the Daytona International Speedway in Florida, are very steeply banked. This banking, with the aid of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 100 m radius curve banked at 65.0° should be driven if the road is frictionless.

**Strategy**

We first note that all terms in the expression for the ideal angle of a banked curve except for speed are known; thus, we need only rearrange it so that speed appears on the left-hand side and then substitute known quantities.

**Solution**

Starting with

$$\tan\theta = \frac{v^2}{rg}$$

we get

$$v = (rg \tan \theta)^{1/2}.$$

Noting that tan 65.0° = 2.14, we obtain

$$\begin{aligned} v &= [(100\ \text{m})(9.80\ \text{m/s}^2)(2.14)]^{1/2} \\ &= 45.8\ \text{m/s}. \end{aligned}$$

**Discussion**

This is just about 165 km/h, consistent with a very steeply banked and rather sharp curve. Tire friction enables a vehicle to take the curve at significantly higher speeds.

Calculations similar to those in the preceding examples can be performed for a host of interesting situations in which centripetal force is involved—a number of these are presented in this chapter's Problems and Exercises.

## TAKE-HOME EXPERIMENT

Ask a friend or relative to swing a golf club or a tennis racquet. Take appropriate measurements to estimate the centripetal acceleration of the end of the club or racquet. You may choose to do this in slow motion.

PHET EXPLORATIONS: GRAVITY AND ORBITS

Move the sun, earth, moon and space station to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies, and turn off gravity to see what would happen without it!




***Figure 4.*** *Gravity and Orbits*

## Section Summary

- Centripetal force $\mathbf{F_c}$ is any force causing uniform circular motion. It is a "center-seeking" force that always points toward the center of rotation. It is perpendicular to linear velocity $\boldsymbol{v}$ and has magnitude

$$\boldsymbol{F_c = ma_c},$$

which can also be expressed as

$$F_c = m\frac{v^2}{r}$$

## Conceptual Questions

### *Conceptual Questions*

**1:** If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.

**2:** Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?

**3:** If centripetal force is directed toward the center, why do you feel that you are 'thrown' away from the center as a car goes around a curve? Explain.

**4:** Race car drivers routinely cut corners as shown in Figure 5. Explain how this allows the curve to be taken at the greatest speed.

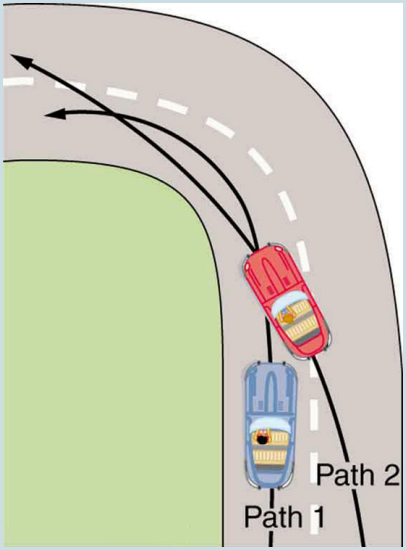



***Figure 5.*** *Two paths around a race track curve are shown. Race car drivers will take the inside path (called cutting the corner) whenever possible because it allows them to take the curve at the highest speed.*

**5:** A number of amusement parks have rides that make vertical loops like the one shown in Figure 6. For safety, the cars are attached to the rails in such a way that they cannot fall off. If the car goes over the top at just the right speed, gravity alone will supply the centripetal force. What other force acts and what is its direction if:

(a) The car goes over the top at faster than this speed?

(b)The car goes over the top at slower than this speed?

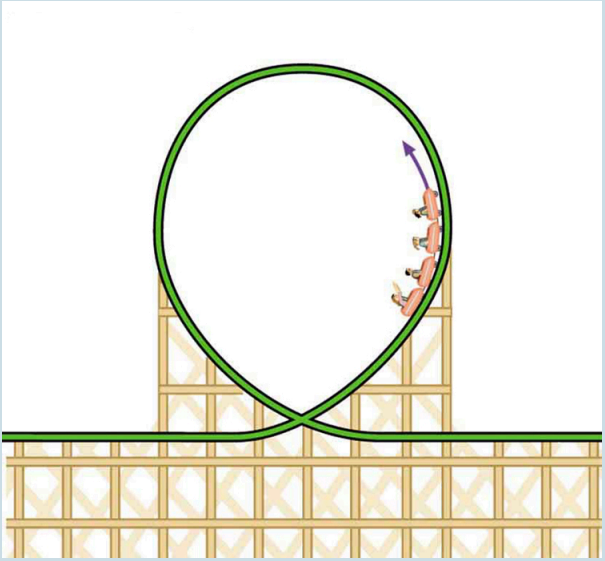


***Figure 6.*** *Amusement rides with a vertical loop are an example of a form of curved motion.*

**7:** What is the direction of the force exerted by the car on the passenger as the car goes over the top of the amusement ride pictured in Figure 6 under the following circumstances:

(a) The car goes over the top at such a speed that the gravitational force is the only force acting?

(b) The car goes over the top faster than this speed?

(c) The car goes over the top slower than this speed?

**8:** As a skater forms a circle, what force is responsible for making her turn? Use a free body diagram in your answer.

**9:** Suppose a child is riding on a merry-go-round at a distance about halfway between its center and edge. She has a lunch box resting on wax paper, so that there is very little friction between it and the merry-go-round. Which path shown in Figure 7 will the lunch box take when she lets go? The lunch box leaves a trail in the dust on the merry-go-round. Is that trail straight, curved to the left, or curved to the right? Explain your answer.

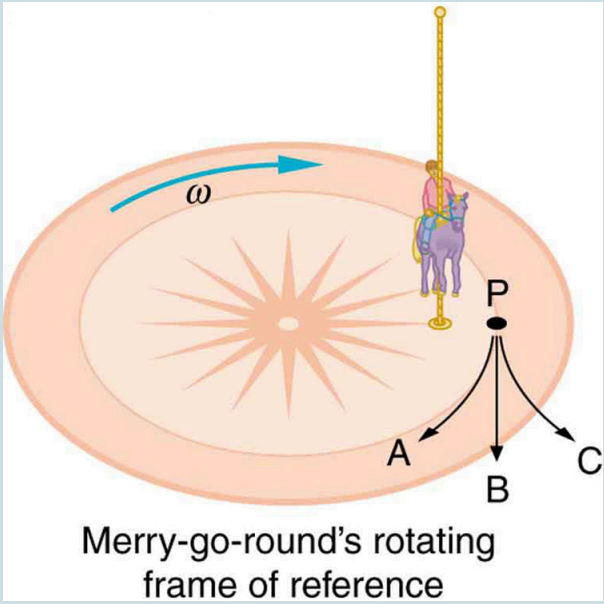



***Figure 7.*** *A child riding on a merry-go-round releases her lunch box at point P. This is a view from above the clockwise rotation. Assuming it slides with negligible friction, will it follow path A, B, or C, as viewed from Earth's frame of reference? What will be the shape of the path it leaves in the dust on the merry-go-round?*

**10:** Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?

**11:** Suppose a mass is moving in a circular path on a frictionless table as shown in figure. In the Earth's frame of reference, there is no centrifugal force pulling the mass away from the centre of rotation, yet there is a very real force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.

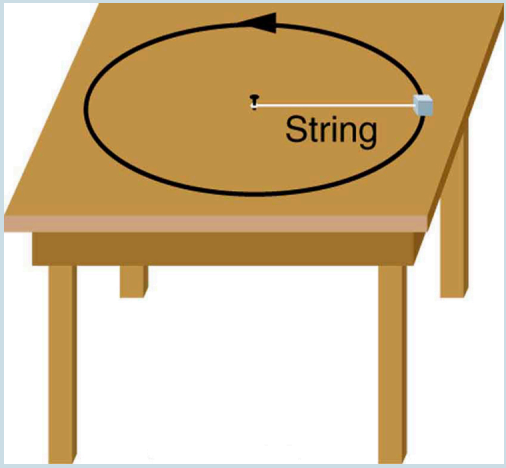



***Figure 8.*** *A mass attached to a nail on a frictionless table moves in a circular path. The force stretching the string is real and not fictional. What is the physical origin of the force on the string?*

## *Problems & Exercises*

**1:** A 22.0 kg child is riding a playground merry-go-round is moving at a speed of 0.0698 m/s when she is 1.25 m from the center. What is the centripetal force exerted on her? Compare this force to her weight.

**2:** What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?

**3:** What is the ideal speed to take a 100 m radius curve banked at a 20.0° angle?

**4:** (a) What is the radius of a bobsled turn banked at 75.0° and taken at 30.0 m/s, assuming it is ideally banked?

(b) Calculate the centripetal acceleration.

(c) Does this acceleration seem large to you?

**5:** Part of riding a bicycle involves leaning at the correct angle when making a turn, as seen in Figure 9. To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components—friction parallel to the road (this must supply the centripetal force), and the vertical normal force (which must equal the system's weight).

(a) Show that $\theta$ (as defined in the figure) is related to the speed $v$ and radius of curvature $r$ of the turn in the same way as for an ideally banked roadway—that is, $\theta = \tan^{-1} v^2/rg$.

(b) Calculate θ for a 12.0 m/s turn of radius 30.0 m (as in a race).

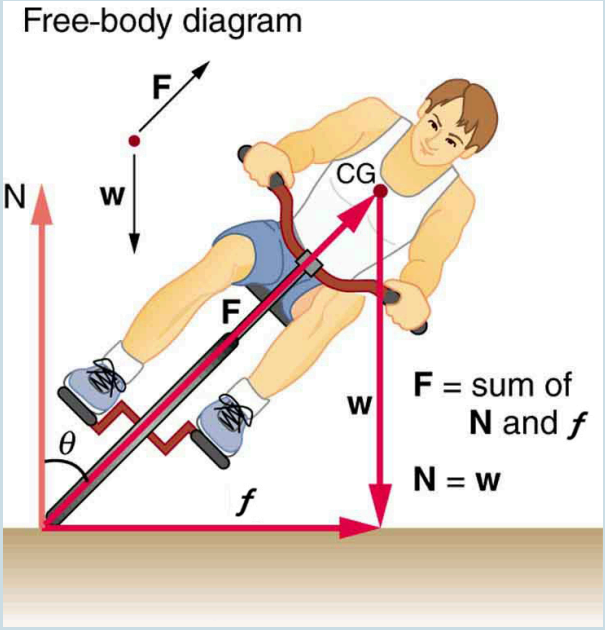



***Figure 9.*** *A bicyclist negotiating a turn on level ground must lean at the correct angle—the ability to do this becomes instinctive. The force of the ground on the wheel needs to be on a line through the center of gravity. The net external force on the system is the centripetal force. The vertical component of the force on the wheel cancels the weight of the system while its horizontal component must supply the centripetal force. This process produces a relationship among the angle* ***θ****, the speed* ***v****, and the radius of curvature* ***r*** *of the turn similar to that for the ideal banking of roadways.*

**6: Integrated Concepts**

If a car takes a banked curve at less than the ideal speed, friction is needed to keep it from sliding toward the inside

of the curve (a real problem on icy mountain roads). (a) Calculate the ideal speed to take a 100 m radius curve banked at 15.0°. (b) What is the minimum coefficient of friction needed for a frightened driver to take the same curve at 20.0 km/h?

**7:** Modern roller coasters have vertical loops like the one shown in Figure 11. The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats. What is the speed of the roller coaster at the top of the loop if the radius of curvature there is 15.0 m and the downward acceleration of the car is 1.50 g?

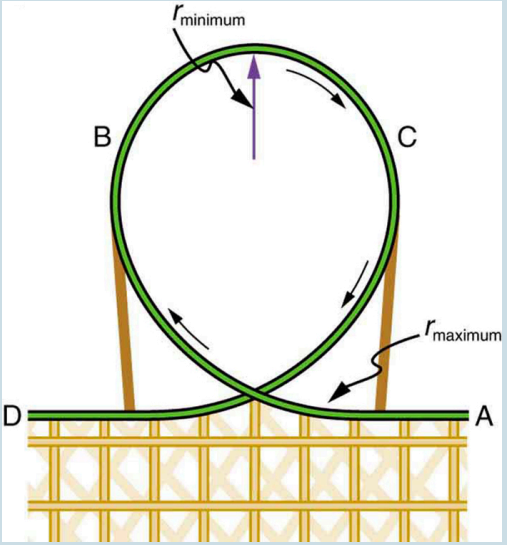



***Figure 11.*** *Teardrop-shaped loops are used in the latest roller coasters so that the radius of curvature gradually decreases to a minimum at the top. This means that the centripetal acceleration builds from zero to a maximum at the top and gradually decreases again. A circular loop would cause a jolting change in acceleration at entry, a disadvantage discovered long ago in railroad curve design. With a small radius of curvature at the top, the centripetal acceleration can more easily be kept greater than* ***g*** *so that the passengers do not lose contact with their seats nor do they need seat belts to keep them in place.*

**8: Unreasonable Results**

(a) Calculate the minimum coefficient of friction needed for a car to negotiate an unbanked 50.0 m radius curve at 30.0 m/s.

(b) What is unreasonable about the result?

(c) Which premises are unreasonable or inconsistent?

## Glossary

**centripetal force**
any net force causing uniform circular motion

**ideal banking**
the sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction

**ideal speed**
the maximum safe speed at which a vehicle can turn on a curve without the aid of friction between the tire and the road

**ideal angle**
the angle at which a car can turn safely on a steep curve, which is in proportion to the ideal speed

**banked curve**
the curve in a road that is sloping in a manner that helps a vehicle negotiate the curve

## *Solutions*

**Problems & Exercises**

**1:** (a) 483 N (b) 2.24 times her weight

**2:** $\mathbf{4.14^0}$

**4:** 24.6 m (b) $\mathbf{36.6\ m/s^2}$ (c) $\boldsymbol{a_c = 3.73\ g.}$ This does not seem too large, but it is clear that bobsledders feel a lot of force on them going through sharply banked turns.

**6:** (a) 16.2 m/s (b) 0.234

**8:** (a)1.84 (b) A coefficient of friction this much greater than 1 is unreasonable . (c) The assumed speed is too great for the tight curve.

# 29. 5.3 Newton's Universal Law of Gravitation

*Summary*

- Explain Earth's gravitational force.
- Describe the gravitational effect of the Moon on Earth.
- Discuss weightlessness in space.
- Examine the Cavendish experiment

What do aching feet, a falling apple, and the orbit of the Moon have in common? Each is caused by the gravitational force. Our feet are strained by supporting our weight—the force of Earth's gravity on us. An apple falls from a tree because of the same force acting a few meters above Earth's surface. And the Moon orbits Earth because gravity is able to supply the necessary centripetal force at a distance of hundreds of millions of meters. In fact, the same force causes planets to orbit the Sun, stars to orbit the center of the galaxy, and galaxies to cluster together. Gravity is another example of underlying simplicity in nature. It is the weakest of the four basic forces found in nature, and in some ways the least understood. It is a force that acts at a distance, without physical contact, and is expressed by a formula that is valid everywhere in the universe, for masses and distances that vary from the tiny to the immense.

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See Figure 1. But Newton was not

the first to suspect that the same force caused both our weight and the motion of planets. His forerunner Galileo Galilei had contended that falling bodies and planetary motions had the same cause. Some of Newton's contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph—it had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose a mechanism that caused them to follow these paths and not others.



***Figure 1.*** *According to early accounts, Newton was inspired to make the connection between falling bodies and astronomical motions when he saw an apple fall from a tree and realized that if the gravitational force could extend above the ground to a tree, it might also reach the Sun. The inspiration of Newton's apple is a part of worldwide folklore and may even be based in fact. Great importance is attached to it because Newton's universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Scientists still expect underlying simplicity to emerge from their ongoing inquiries into nature.*

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Stated in modern language, **Newton's universal law of gravitation** states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

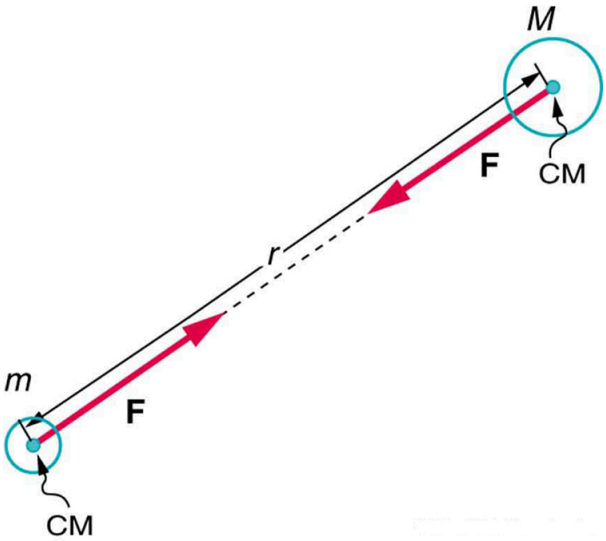



***Figure 2.*** *Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton's third law.*

## MISCONCEPTION ALERT

The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton's third law.

The bodies we are dealing with tend to be large. To simplify the situation we assume that the body acts as if its entire mass is concentrated at one specific point called the **center of mass** (CM). For two bodies having masses $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ with a distance $\boldsymbol{r}$ between their centers of mass (for this course, we will assume that objects have a uniform density, which means the center of mass is the geometric center of the object), the equation for Newton's universal law of gravitation is

$$F_{\text{g}} = \frac{Gm_1 m_2}{r^2},$$

where $\mathbf{F_g}$ is the magnitude of the gravitational force and **G** is a proportionality factor called the **gravitational constant**. **G** is a universal gravitational constant—that is, it is thought to be the same everywhere in the universe. It has been measured experimentally to be

$$G = 6.674 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

in SI units. Note that the units of **G** are such that a force in newtons is obtained from Newton's universal law of gravitation when considering masses in kilograms and distance in meters. For

example, two 1.000 kg masses separated by 1.000 m will experience a gravitational attraction of $\mathbf{6.674 \times 10^{-11}}$ **N**. This is an extraordinarily small force. The small magnitude of the gravitational force is consistent with everyday experience. We are unaware that even large objects like mountains exert gravitational forces on us. In fact, our body weight is the force of attraction of the *entire Earth* on us with a mass of $\mathbf{6 \times 10^{24}}$ **kg**.

Recall that the acceleration due to gravity $\boldsymbol{g}$ is about $\mathbf{9.80\ m/s^2}$ on Earth. We can now determine why this is so. The weight of an object $mg$ is the gravitational force between it and Earth. Substituting $\boldsymbol{m_1 g}$ for **F** in Newton's universal law of gravitation gives

$$m_1 g = \frac{G m_1 m_2}{r^2},$$

where $\boldsymbol{m_1}$ is the mass of the object, $\boldsymbol{m_2}$ is the mass of Earth, and $\boldsymbol{r}$ is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See Figure 3. The mass $\boldsymbol{m_1}$ of the object cancels, leaving an equation for $\boldsymbol{g}$:

$$g = \frac{G m_2}{r^2}.$$

Substituting known values for Earth's mass and radius (to three significant figures),

$$g = (6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}) \times \frac{5.98 \times 10^{24}\ \text{kg}}{(6.38 \times 10^6\ \text{m})^2},$$

and we obtain a value for the acceleration of a falling body:

$$g = 9.80\ \text{m/s}^2.$$

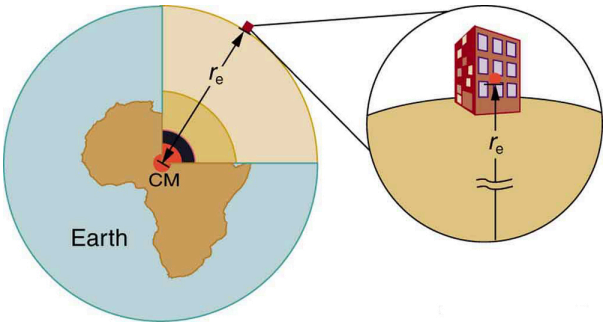



***Figure 3.*** *The distance between the centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object.*

This is the expected value *and is independent of the body's mass*. Newton's law of gravitation takes Galileo's observation that all masses fall with the same acceleration a step further, explaining the observation in terms of a force that causes objects to fall—in fact, in terms of a universally existing force of attraction between masses.

## TAKE-HOME EXPERIMENT

Take a marble, a ball, and a spoon and drop them from the same height. Do they hit the floor at the same time? If you drop a piece of paper as well, does it behave like the other objects? Explain your observations.

## MAKING CONNECTIONS

Attempts are still being made to understand the gravitational force. Modern physics is exploring the connections of gravity to other forces, space, and time. General relativity alters our view of gravitation, leading us to think of gravitation as bending space and time.

In the following example, we make a comparison similar to one made by Newton himself. He noted that if the gravitational force caused the Moon to orbit Earth, then the acceleration due to gravity should equal the centripetal acceleration of the Moon in its orbit. Newton found that the two accelerations agreed "pretty nearly."

### Example 1: Earth's Gravitational Force Is the Centripetal Force Making the Moon Move in a Curved Path

(a) Find the acceleration due to Earth's gravity at the distance of the Moon.

(b) Calculate the centripetal acceleration needed to keep the Moon in its orbit (assuming a circular orbit

about a fixed Earth), and compare it with the value of the acceleration due to Earth's gravity that you have just found. The orbital period of the Moon is 27.3 days.

**Strategy for (a)**

This calculation is the same as the one finding the acceleration due to gravity at Earth's surface, except that $r$ is the distance from the center of Earth to the center of the Moon. The radius of the Moon's nearly circular orbit is $3.84 \times 10^8$ m.

**Solution for (a)**

Substituting known values into the expression for $g$ found above, remembering that $m_2$ is the mass of Earth not the Moon, yields

$$g = G\frac{m_2}{r^2} = (6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{kg^2}) \times \frac{5.98 \times 10^{24} \text{ kg}}{(3.84 \times 10^8 \text{ m})^2}$$
$$= 2.70 \times 10^{-3} \text{ m/s}^2.$$

**Strategy for (b)**

Centripetal acceleration can be calculated using

$$a_{\text{c}} = \frac{v^2}{r}$$

where $r$ the distance to the Moon and $v$ is the orbital speed of the Moon.

**Solution for (b)**

Given that the period (the time it takes to make one complete rotation) of the Moon's orbit is 27.3 days, and using

$$\mathbf{27.3\ d \times 24\frac{hr}{d} \times 60\frac{min}{hr} \times 60\frac{s}{min}}$$
$$\mathbf{= 2.34 \times 10^6\ s}$$

we see that

$$v = \frac{2\pi r}{T} = \frac{2\pi 3.84 \times 10^8}{2.34 \times 10^6} = 1030\ \text{m/s}.$$

The centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(1030^2}{3.84 \times 10^8} = 2.76 \times 10^{-3}\ \text{m/s}^2$$

The direction of the acceleration is toward the center of the Earth.

**Discussion**

The centripetal acceleration of the Moon found in (b) differs by about 2% from the acceleration due to Earth's gravity found in (a). This agreement is approximate because the Moon's orbit is slightly elliptical, and Earth is not stationary (rather the Earth-Moon system rotates about its center of mass, which is located some 1700 km below Earth's surface). The clear implication is that Earth's gravitational force causes the Moon to orbit Earth.

Why does Earth not remain stationary as the Moon orbits it? This is because, as expected from Newton's third law, if Earth exerts a force on the Moon, then the Moon should exert an equal and opposite force on Earth (see Figure 4). We do not sense the Moon's effect on Earth's motion, because the Moon's gravity moves our bodies right along with Earth but there are other signs on Earth that clearly show the effect of the Moon's gravitational force, such as tides.

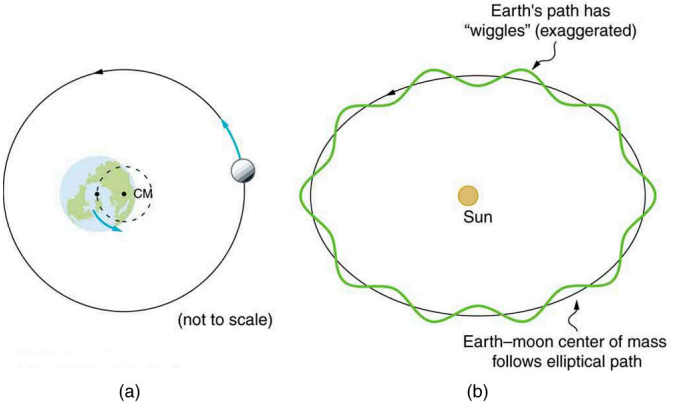



***Figure 4.*** *(a) Earth and the Moon rotate approximately once a month around their common center of mass. (b) Their center of mass orbits the Sun in an elliptical orbit, but Earth's path around the Sun has "wiggles" in it. Similar wiggles in the paths of stars have been observed and are considered direct evidence of planets orbiting those stars. This is important because the planets' reflected light is often too dim to be observed.*

## Optional: Tides

Ocean tides are one very observable result of the Moon's gravity acting on Earth. Figure 5 is a simplified drawing of the Moon's position relative to the tides. Because water easily flows on Earth's surface, a high tide is created on the side of Earth nearest to the Moon, where the Moon's gravitational pull is strongest. Why is there also a high tide on the opposite side of Earth? The answer is that Earth is pulled toward the Moon more than the water on the far side, because Earth is closer to the Moon. So the water on the side of Earth closest to the Moon is pulled away from Earth, and Earth is pulled away from water on the far side. As Earth rotates, the tidal bulge (an effect of the tidal forces between an orbiting natural satellite and the primary planet that it orbits) keeps its orientation with the Moon. Thus there are two tides per day (the actual tidal

period is about 12 hours and 25.2 minutes), because the Moon moves in its orbit each day as well).




***Figure 5.*** *The Moon causes ocean tides by attracting the water on the near side more than Earth, and by attracting Earth more than the water on the far side. The distances and sizes are not to scale. For this simplified representation of the Earth-Moon system, there are two high and two low tides per day at any location, because Earth rotates under the tidal bulge.*

The Sun also affects tides, although it has about half the effect of the Moon. However, the largest tides, called spring tides, occur when Earth, the Moon, and the Sun are aligned. The smallest tides, called neap tides, occur when the Sun is at a **90°** angle to the Earth-Moon alignment.

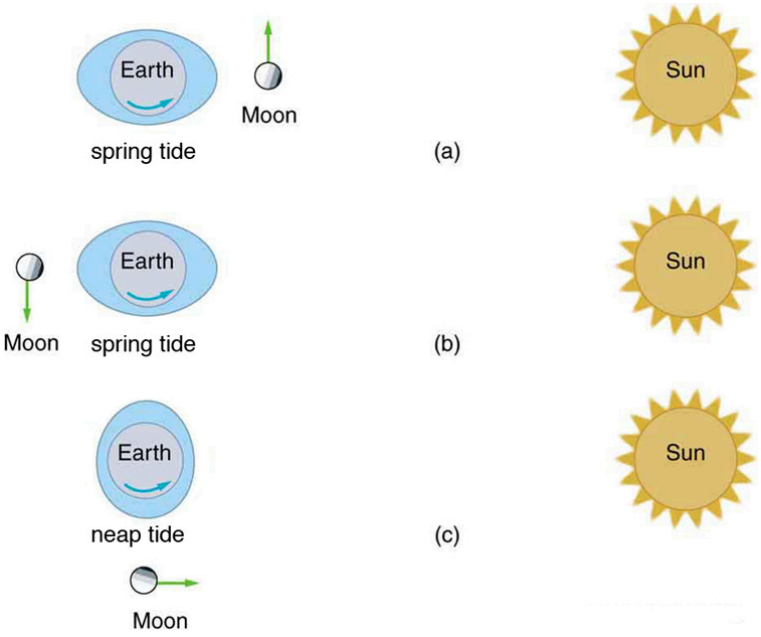



***Figure 6.*** *(a, b) Spring tides: The highest tides occur when Earth, the Moon, and the Sun are aligned. (c) Neap tide: The lowest tides occur when the Sun lies at* $\mathbf{90^0}$ *to the Earth-Moon alignment. Note that this figure is not drawn to scale.*

Tides are not unique to Earth but occur in many astronomical systems. The most extreme tides occur where the gravitational force is the strongest and varies most rapidly, such as near black holes (see Figure 7). A few likely candidates for black holes have been observed in our galaxy. These have masses greater than the Sun but have diameters only a few kilometers across. The tidal forces near them are so great that they can actually tear matter from a companion star.

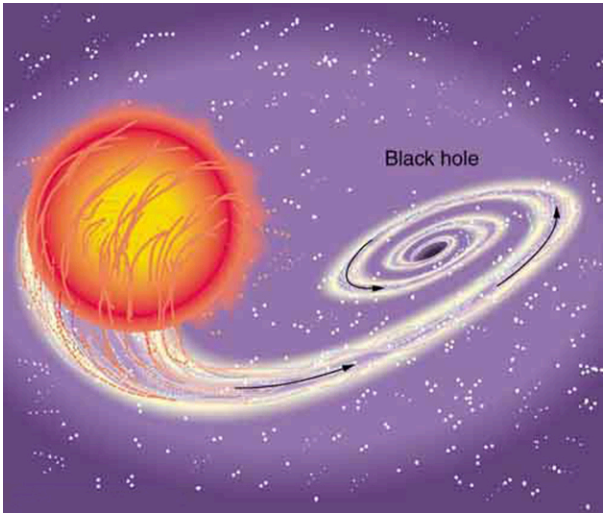



***Figure 7.*** *A black hole is an object with such strong gravity that not even light can escape it. This black hole was created by the supernova of one star in a two-star system. The tidal forces created by the black hole are so great that it tears matter from the companion star. This matter is compressed and heated as it is sucked into the black hole, creating light and X-rays observable from Earth.*

## "Weightlessness" and Microgravity

In contrast to the tremendous gravitational force near black holes is the apparent gravitational field experienced by astronauts orbiting Earth. What is the effect of "weightlessness" upon an astronaut who is in orbit for months? Or what about the effect of weightlessness upon plant growth? Weightlessness doesn't mean that an astronaut is not being acted upon by the gravitational force. There is no "zero gravity" in an astronaut's orbit. The term just means that the astronaut is in free-fall, accelerating with the acceleration due to gravity. If an elevator cable breaks, the passengers inside will be

in free fall and will experience weightlessness. You can experience short periods of weightlessness in some rides in amusement parks.

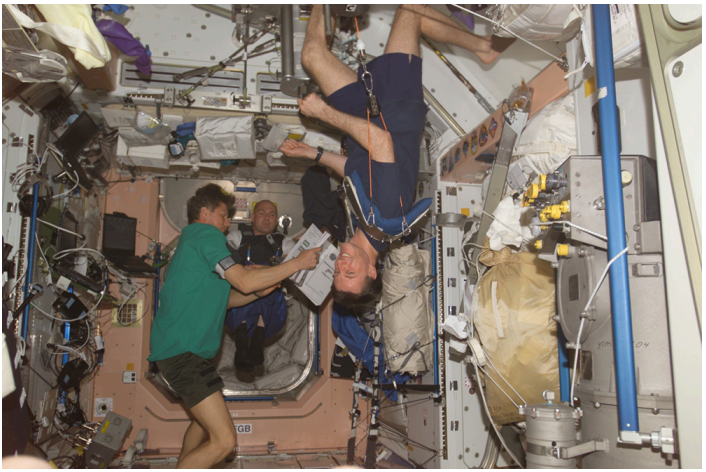


***Figure 8.*** *Astronauts experiencing weightlessness on board the International Space Station. (credit: NASA)*

**Microgravity** refers to an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface. Many interesting biology and physics topics have been studied over the past three decades in the presence of microgravity. Of immediate concern is the effect on astronauts of extended times in outer space, such as at the International Space Station. Researchers have observed that muscles will atrophy (waste away) in this environment. There is also a corresponding loss of bone mass. Study continues on cardiovascular adaptation to space flight. On Earth, blood pressure is usually higher in the feet than in the head, because the higher column of blood exerts a downward force on it, due to gravity. When standing, 70% of your blood is below the level of the heart, while in a horizontal position, just the opposite occurs. What difference does the absence of this pressure differential have upon the heart?

Some findings in human physiology in space can be clinically

important to the management of diseases back on Earth. On a somewhat negative note, spaceflight is known to affect the human immune system, possibly making the crew members more vulnerable to infectious diseases. Experiments flown in space also have shown that some bacteria grow faster in microgravity than they do on Earth. However, on a positive note, studies indicate that microbial antibiotic production can increase by a factor of two in space-grown cultures. One hopes to be able to understand these mechanisms so that similar successes can be achieved on the ground. In another area of physics space research, inorganic crystals and protein crystals have been grown in outer space that have much higher quality than any grown on Earth, so crystallography studies on their structure can yield much better results.

Plants have evolved with the stimulus of gravity and with gravity sensors. Roots grow downward and shoots grow upward. Plants might be able to provide a life support system for long duration space missions by regenerating the atmosphere, purifying water, and producing food. Some studies have indicated that plant growth and development are not affected by gravity, but there is still uncertainty about structural changes in plants grown in a microgravity environment.

## The Cavendish Experiment: Then and Now

As previously noted, the universal gravitational constant **G** is determined experimentally. This definition was first done accurately by Henry Cavendish (1731–1810), an English scientist, in 1798, more than 100 years after Newton published his universal law of gravitation. The measurement of **G** is very basic and important because it determines the strength of one of the four forces in nature. Cavendish's experiment was very difficult because he measured the tiny gravitational attraction between two ordinary-

sized masses (tens of kilograms at most), using apparatus like that in Figure 9. Remarkably, his value for **G** differs by less than 1% from the best modern value.

One important consequence of knowing **G** was that an accurate value for Earth's mass could finally be obtained. This was done by measuring the acceleration due to gravity as accurately as possible and then calculating the mass of Earth $m_2$ from the relationship Newton's universal law of gravitation gives

$$m_1 g = G\frac{m_1 m_2}{r^2},$$

where $m_1$ is the mass of the object, $m_2$ is the mass of Earth, and $r$ is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See Figure 2. The mass $m_1$ of the object cancels, leaving an equation for $g$:

$$g = G\frac{m_2}{r^2}.$$

Rearranging to solve for $m_2$ yields

$$m_2 = \frac{gr^2}{G}.$$

So $m_2$ can be calculated because all quantities on the right, including the radius of Earth $r$, are known from direct measurements. Interestingly, of all the fundamental constants in physics, **G** is by far the least well determined.

The Cavendish experiment is also used to explore other aspects of gravity. One of the most interesting questions is whether the gravitational force depends on substance as well as mass—for example, whether one kilogram of lead exerts the same gravitational pull as one kilogram of water. A Hungarian scientist named Roland von Eötvös pioneered this inquiry early in the 20th century. He found, with an accuracy of five parts per billion, that the gravitational force does not depend on the substance. Such experiments continue today, and have improved upon Eötvös'

measurements. Cavendish-type experiments such as those of Eric Adelberger and others at the University of Washington, have also put severe limits on the possibility of a fifth force and have verified a major prediction of general relativity–that gravitational energy contributes to rest mass. Ongoing measurements there use a torsion balance and a parallel plate (not spheres, as Cavendish used) to examine how Newton's law of gravitation works over sub-millimeter distances. On this small-scale, do gravitational effects depart from the inverse square law? So far, no deviation has been observed.

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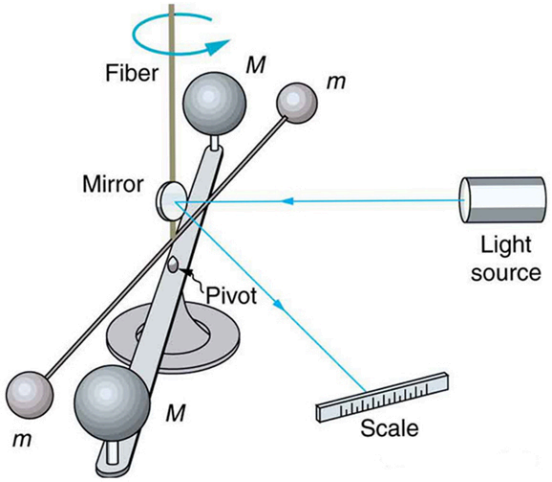



***Figure 9.*** *Cavendish used an apparatus like this to measure the gravitational attraction between the two suspended spheres* **(m)** *and the two on the stand* **(M)** *by observing the amount of torsion (twisting) created in the fiber. Distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity.*

## Section Summary

- Newton's universal law of gravitation: Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In equation form, this is

$$F_{\text{g}} = \frac{Gm_1m_2}{r^2},$$

where **F** is the magnitude of the gravitational force. **G** is the gravitational constant, given by

$$\boldsymbol{G = 6.674 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2}$$

- Newton's law of gravitation applies universally.

### *Conceptual Questions*

**1:** Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

**2:** Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not 9.80 $\text{m/s}^2$. Who do you agree with and why?

**3:** Draw a free body diagram for a satellite in an elliptical

orbit showing why its speed increases as it approaches its parent body and decreases as it moves away.

**4:** Newton's laws of motion and gravity were among the first to convincingly demonstrate the underlying simplicity and unity in nature. Many other examples have since been discovered, and we now expect to find such underlying order in complex situations. Is there proof that such order will always be found in new explorations?

## *Problems & Exercises*

**1:** (a) Calculate Earth's mass given the acceleration due to gravity at the North Pole is 9.830 $m/s^2$ and the radius of the Earth is 6371 km from center to pole.

(b) Compare this with the accepted value of $5.979 \times 10^{24}$ kg.

**2:** (a) Calculate the magnitude of the acceleration due to gravity on the surface of Earth due to the Moon.

(b) Calculate the magnitude of the acceleration due to gravity at Earth due to the Sun.

(c) Take the ratio of the Moon's acceleration to the Sun's and comment on why the tides are predominantly due to the Moon in spite of this number.

**3:** (a) What is the acceleration due to gravity on the surface of the Moon?

(b) On the surface of Mars? The mass of Mars is $6.418 \times 10^{23}$ kg and its radius is $3.38 \times 10^{6}$ m.

**4:** (a) Calculate the acceleration due to gravity on the surface of the Sun.

(b) By what factor would your weight increase if you could stand on the Sun? (Never mind that you cannot.)

**5:** The Moon and Earth rotate about their common center of mass, which is located about 4700 km from the center of Earth. (This is 1690 km below the surface.)

(a) Calculate the magnitude of the acceleration due to the Moon's gravity at that point.

(b) Calculate the magnitude of the centripetal acceleration of the center of Earth as it rotates about that point once each lunar month (about 27.3 d) and compare it with the acceleration found in part (a). Comment on whether or not they are equal and why they should or should not be.

**6:** Solve part (b) of Example 1 using $a_c = v^2/r$.

**7:** Astrology, that unlikely and vague pseudoscience, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational.

(a) Calculate the magnitude of the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child).

(b) Calculate the magnitude of the force on the baby due to Jupiter if it is at its closest distance to Earth, some $6.29 \times 10^{11}$ m away. How does the force of Jupiter on the baby

compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)

**8:** The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune's orbit. Pluto was subsequently discovered near its predicted position. But it now appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune's orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared with the closest planet to Neptune:

(a) Calculate the acceleration due to gravity at Neptune due to Pluto when they are $4.50 \times 10^{12}$ m apart, as they are at present. The mass of Pluto is $1.4 \times 10^{22}$ kg.

(b) Calculate the acceleration due to gravity at Neptune due to Uranus, presently about $2.50 \times 10^{12}$ m apart, and compare it with that due to Pluto. The mass of Uranus is $8.62 \times 10^{25}$ kg.

**9:** (a) The Sun orbits the Milky Way galaxy once each $2.60 \times 10^{8}$ y, with a roughly circular orbit averaging $3.00 \times 10^{4}$ light years in radius. (A light year is the distance traveled by light in 1 y.) Calculate the centripetal acceleration of the Sun in its galactic orbit. Does your result support the contention that a nearly inertial frame of reference can be located at the Sun?

(b) Calculate the average speed of the Sun in its galactic orbit. Does the answer surprise you?

**10: Unreasonable Result**

A mountain 10.0 km from a person exerts a gravitational force on him equal to 2.00% of his weight.

(a) Calculate the mass of the mountain.

(b) Compare the mountain's mass with that of Earth.

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent? (Note that accurate gravitational measurements can easily detect the effect of nearby mountains and variations in local geology.)

## Glossary

**gravitational constant, G**
: a proportionality factor used in the equation for Newton's universal law of gravitation; it is a universal constant—that is, it is thought to be the same everywhere in the universe

**center of mass**
: the point where the entire mass of an object can be thought to be concentrated

**microgravity**
: an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface

**Newton's universal law of gravitation**
: every particle in the universe attracts every other particle with a force along a line joining them; the force is directly proportional to the product of their masses and inversely

proportional to the square of the distance between them

## *Solutions*

**Problems & Exercises**

**1:** (a) $\mathbf{5.979 \times 10^{24}\ kg}$ (b) This is identical to the best value to three significant figures.

**3:** (a) $\mathbf{1.62\ m/s^2}$ (b) $\mathbf{3.75\ m/s^2}$

**5:** (a) $\mathbf{3.42 \times 10^{-5}\ m/s^2}$ (b) $\mathbf{3.34 \times 10^{-5}\ m/s^2}$

The values are nearly identical. One would expect the gravitational force to be the same as the centripetal force at the core of the system.

**7:** (a) $\mathbf{7.01 \times 10^{-7}\ N}$ (b) $\mathbf{1.35 \times 10^{-6}\ N,\ 0.521}$

**9:** (a) $\mathbf{1.66 \times 10^{-10}\ m/s^2}$ (b) $\mathbf{2.17 \times 10^{5}\ m/s}$

**10:** (a) $\mathbf{2.937 \times 10^{17}\ kg}$ (b) $\mathbf{4.91 \times 10^{-8}}$ of the Earth's mass. (c) The mass of the mountain and its fraction of the Earth's mass are too great. (d) The gravitational force assumed to be exerted by the mountain is too great.

PART VII

# CHAPTER 6 WORK AND ENERGY



***Figure 1.*** *How many forms of energy can you identify in this photograph of a wind farm in Iowa? (credit: Jürgen from Sandesneben, Germany, Wikimedia Commons)*

*Energy* plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is *conserved*.

**Conservation of energy** (as physicists like to call the principle that energy can neither be created nor destroyed) is based on experiment. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the

most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy in his famous equation $\mathbf{E = mc^2}$.

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define **energy** as the ability to do work, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.

# 30. 6.1 Work: The Scientific Definition

*Summary*

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

## What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be displacement in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force acts.*

For one-way motion in one dimension, this is expressed in equation form as

$$W = |\vec{\mathbf{F}}|(\cos\theta)|\mathbf{d}|,$$

where **W** is work, ***d*** is the displacement of the system, and $\boldsymbol{\theta}$ is the angle between the force vector $\vec{\mathrm{F}}$ and the displacement vector ***d***, as in Figure 1. We can also write this as

$$W = Fd\cos\theta.$$

To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

### WHAT IS WORK?

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts.* For one-way motion in one dimension, this is expressed in equation form as

$$W = Fd\cos\theta,$$

where **W** is work, **F** is the magnitude of the force on the system, ***d*** is the magnitude of the displacement of the system, and $\boldsymbol{\theta}$ is the angle between the force vector $\vec{\mathrm{F}}$ and the displacement vector ***d***.

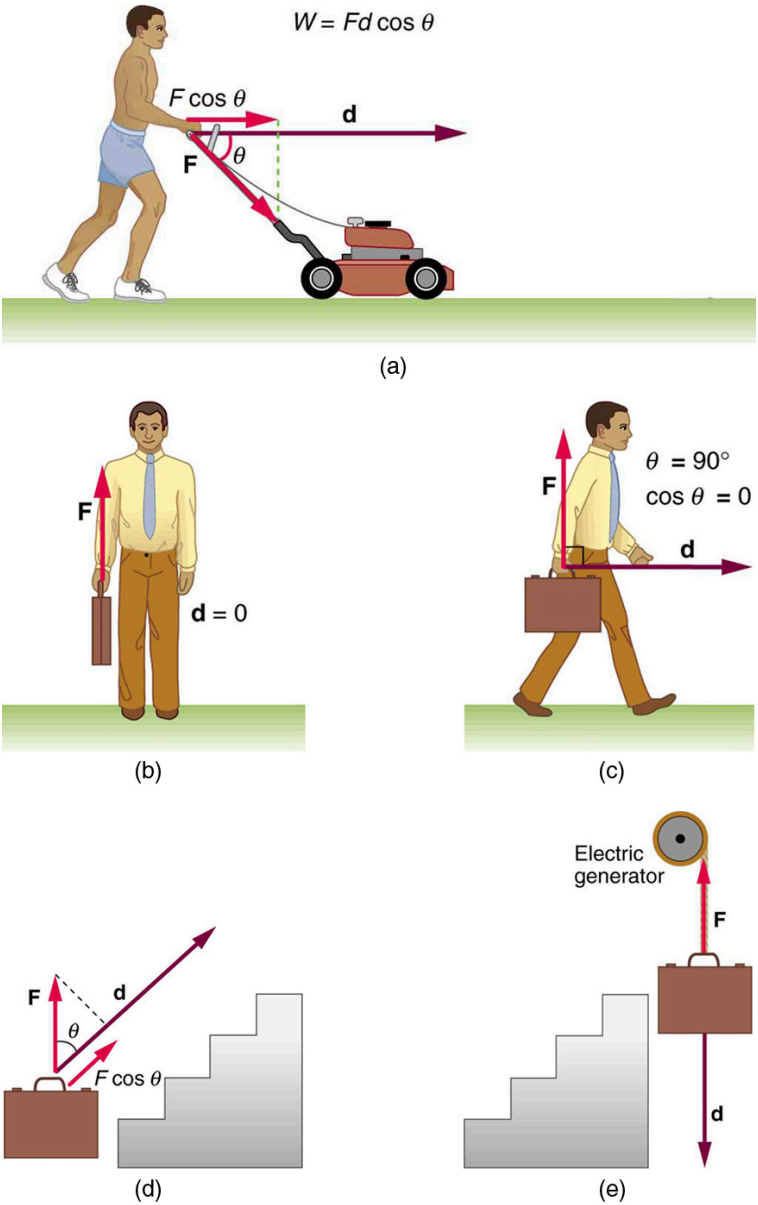



***Figure 1.*** *Examples of work. (a) The work done by the force* **F** *on this lawn mower is* ***Fd cosθ****. Note that* ***F cosθ*** *is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no displacement. No energy is transferred to or from the*

*briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work is done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force* **F** *in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because* **F** *and* $\boldsymbol{d}$ *are in opposite directions.*

To examine what the definition of work means, let us consider the other situations shown in Figure 1. The person holding the briefcase in Figure 1(b) does no work, for example. Here $\boldsymbol{d} = \mathbf{0}$, so $\mathbf{W} = \mathbf{0}$. Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the "briefcase-Earth system"—see Chapter 6.3 Gravitational Potential Energy for more details). There must be displacement for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in Figure 1(c) does no work on it, because the force is perpendicular to the motion. That is, $\cos 90^\circ = 0$, and so $\mathbf{W} = \mathbf{0}$.

In contrast, when a force exerted on the system has a component in the direction of motion, such as in Figure 1(d), work *is* done—energy is transferred to the briefcase. Finally, in Figure 1(e), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase's weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward on the briefcase, and the displacement downward. This makes $\boldsymbol{\theta} = 180^\circ$, and $\cos 180^\circ = -1$; therefore, **W** is negative.

## Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and **1 J = 1 Nm = 1 kgm$^2$/s$^2$**. One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

### Example 1: Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in Figure 1(a) if he exerts a constant force of **75.0 N** at an angle 35° below the horizontal and pushes the mower **25.0 m** on level ground? Convert the amount of work from joules to kilocalories and compare it with this person's average daily intake of **10,000 kJ** (about **2400 kcal**) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by 1 °C, and is equivalent to **4.184 J**, while one *food calorie* (1 kcal) is equivalent to **4184 J**.

**Strategy**

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation $W = Fd \cos \theta$. The force, angle,

and displacement are given, so that only the work **W** is unknown.

**Solution**

The equation for the work is

$$W = Fd\cos\theta.$$

Substituting the known values gives

$$\begin{aligned} W &= (75.0\ \text{N})(25.0\ \text{m})\cos(35.0^0) \\ &= 1536\ \text{J} = 1.54 \times 10^3\ \text{J} \end{aligned}.$$

Converting the work in joules to kilocalories yields **W = (1536 J)(1 kcal/4184 J) = 0.367 kcal**. The ratio of the work done to the daily consumption is

$$\frac{W}{2400\ \text{kcal}} = 1.53 \times 10^{-4}.$$

**Discussion**

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we "work" all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

## Section Summary

- Work is the transfer of energy by a force acting on an object as it is displaced.

- The work **W** that a force $\vec{F}$ does on an object is the product of the magnitude **F** of the force, times the magnitude ***d*** of the displacement, times the cosine of the angle $\boldsymbol{\theta}$ between them. In symbols,

$$W = Fd\cos\theta.$$

- The SI unit for work and energy is the joule (J), where **1 J = 1 Nm = 1 kgm$^2$/s$^2$**.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

## *Conceptual Questions*

**1:** Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.

**2:** Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

**3:** Describe a situation in which a force is exerted for a long time but does no work. Explain.

## *Problems & Exercises*

**1:** How much work does a supermarket checkout attendant do on a can of soup that is pushed 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.

**2:** A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

**3:** (a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. Hint: You need to draw a force diagram and calculate the force exerted by the cable first. What is the net force ? (b) What is the work done on the lift by the gravitational force in this process? (c) what is the work done by the friction force? (d) What is the total work done on the lift?

**4:** Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See Chapter 6.6 Table 1 for the energy content of gasoline.) (a) What is the magnitude of the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?

**5:** Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0° with the horizontal. (See Figure 2.) He exerts a

force of 500 N on the crate parallel to the ramp and moves at a constant speed.

a) What is the work done on the crate, b) what is the work done on himself to lift him up and c) what is the total work = a + b? This is the work he does on the crate *and* on his body to get up the ramp.

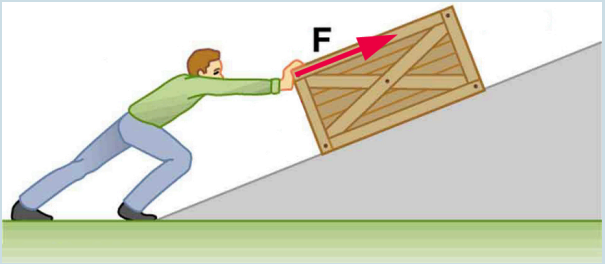



***Figure 2.*** *A man pushes a crate up a ramp.*

**6:** How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in Figure 3? Assume no friction acts on the wagon.

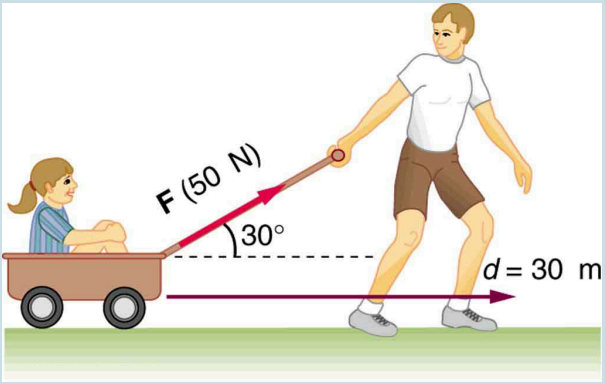



***Figure 3.*** *The boy does work on the system of the wagon and the child when he pulls them as shown.*

**7:** A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?

**8:** Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed, as shown in Figure 4. The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the

gravitational force on the sled? (d) What is the total work done?

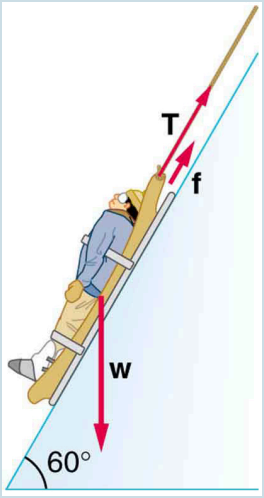



***Figure 4.*** *A rescue sled and victim are lowered down a steep slope.*

## Glossary

**energy**
: the ability to do work

**work**
: the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the

displacement

**joule**

SI unit of work and energy, equal to one newton-meter

## *Solutions*

**Problems & Exercises**

**1:** 3.00 J = 7.17 x $10^{-4}$ kcal

**3:** (a) 5.92 x $10^{5}$ J b) -5.88 x $10^{5}$ J c) – 4000 J = – 4 x $10^{3}$ J d) The net force is zero so the net work = 0 J. The force due to gravity = 1.47 x $10^{4}$ N down, force due to friction is 100 N down so the force due to the cable = 1.48 x $10^{4}$ N up. The net force = 0 N

4: 1 gallon of gasoline = 1.2 x $10^{8}$ J so 2 gallons = 2.4 x $10^{8}$ J 30% = 7.2 x $10^{7}$ J Energy = force x distance so force = energy / distance = (7.2 x $10^{7}$ J ) / ( 108 x $10^{3}$ m) = 667 N b) (28/30) x 2 gallons = 1.87 gallons

**5: a)** work = force x distance = 2000 J as angle between force and 4 m is 0 degrees b) m g h where h = 4m sin 20.0 = 1140 c) 3140 J in total

**7:** (a) -700 J (b) 0 J (c) 700 J (d) 38.6 N (e) 0 J

# 31. 6.2 Kinetic Energy and the Work-Energy Theorem

*Summary*

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

## Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if the lawn mower in Chapter 6.1 Figure 1(a) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in Chapter 6.1 Figure 1(d) is stored in the briefcase-Earth system and can be recovered at any time, as shown in Chapter 6.1 Figure 1(e). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

## Net Work and the Work-Energy Theorem

We know from the study of Newton's laws in Chapter 4 Dynamics: Force and Newton's Laws of Motion that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is, **net work** is the work done by the net external force $\mathbf{F_{net}}$. In equation form, this is $\mathbf{W_{net} = F_{net} d \cos \theta}$ where $\boldsymbol{\theta}$ is the angle between the force vector and the displacement vector.

Figure 1(a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an $\mathbf{F \cos \theta}$ vs. $\boldsymbol{d}$ graph. In this case, $\mathbf{F \cos \theta}$ is constant. You can see that the area under the graph is $\mathbf{Fd \cos \theta}$, or the work done. Figure 1(b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force $\mathbf{(F \cos \theta)_{i(ave)}}$. The work done is $\mathbf{(F \cos \theta)_{i(ave)} d_i}$ for each strip, and the total work done is the sum of the $\mathbf{W_i}$. Thus the total work done is the total area under the curve, a useful property to which we shall refer later.

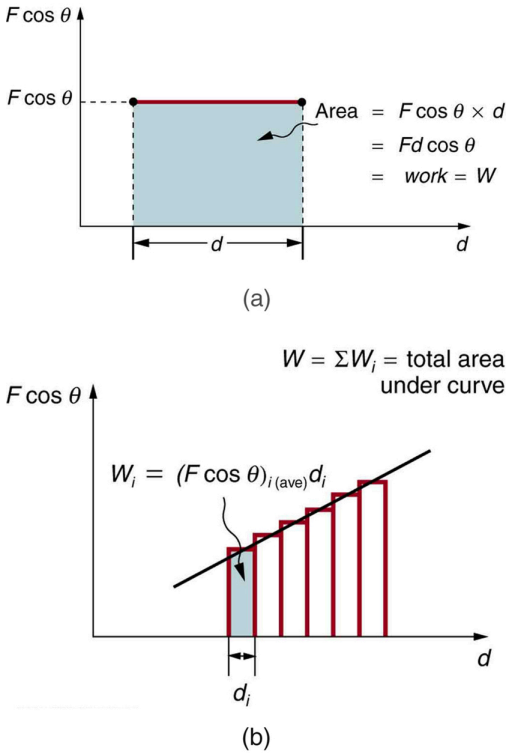



***Figure 1.*** *(a) A graph of* $\mathbf{F}\cos\boldsymbol{\theta}$ *vs.* $\mathbf{d}$*, when* $\mathbf{F}\cos\boldsymbol{\theta}$ *is constant. The area under the curve represents the work done by the force. (b) A graph of* $\mathbf{F}\cos\boldsymbol{\theta}$ *vs.* $\mathbf{d}$ *in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.*

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in Figure 2.

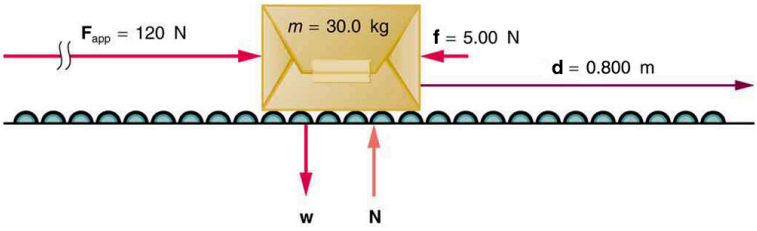



***Figure 2.*** *A package on a roller belt is pushed horizontally through a distance* ***d****.*

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force Fapp and the horizontal friction force $\boldsymbol{f}$. Thus, as expected, the net force is parallel to the displacement, so that $\boldsymbol{\theta = 0°}$ and $\boldsymbol{\cos\theta = 1}$, and the net work is given by

$$\boldsymbol{W_{\text{net}} = F_{\text{net}}d.}$$

The effect of the net force $\mathbf{F_{net}}$ is to accelerate the package from $\boldsymbol{v_0}$ to $\boldsymbol{v}$. The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See Example 1.) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $\boldsymbol{F_{net} = ma}$ from Newton's second law gives

$$\boldsymbol{W_{\text{net}} = mad.}$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $\boldsymbol{d = x - x_0}$ and use the equation studied in Chapter 2.6 Motion Equations for Constant Acceleration in One Dimension for the change in speed over a distance $\boldsymbol{d}$ if the acceleration has the constant value $\boldsymbol{a}$; namely, $\boldsymbol{v^2 = v_0^2 + 2ad}$ (note that $\boldsymbol{a}$ appears in the expression for the net

work). Solving for acceleration gives $a = \frac{v^2 - v_0^2}{2d}$. When $a$ is substituted into the preceding expression for $W_{net}$, we obtain

$$W_{net} = m\left(\frac{v^2 - v_0^2}{2d}\right)d.$$

The $d$ cancels, and we rearrange this to obtain

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

### THE WORK-ENERGY THEOREM

The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to

be the translational kinetic energy (KE) of a mass $m$ moving at a speed $v$. (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

$$\mathbf{KE} = \frac{1}{2}mv^2,$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in Figure 2, up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50 km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

### Example 1: Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in Figure 2 is moving at 0.500 m/s. What is its kinetic energy?

**Strategy**

Because the mass $m$ and speed $v$ are given, the kinetic

energy can be calculated from its definition as given in the equation $\mathbf{KE} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{mv}^{\mathbf{2}}$.

**Solution**

The kinetic energy is given by

$$\mathbf{KE} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{mv}^{\mathbf{2}},$$

Entering known values gives

$$\mathbf{KE} = \mathbf{0.5(30.0\ kg)(0.500\ m/s)^2},$$

which yields

$$\mathbf{KE} = \mathbf{3.75\ k\cdot m^2/s^2} = \mathbf{3.75\ J}.$$

**Discussion**

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

## Example 2: Determining the Work to

## Accelerate a Package

Suppose that you push on the 30.0-kg package in Figure 2 with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

**Strategy and Concept for (a)**

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See Figure 2.) As expected, the net work is the net force times distance.

**Solution for (a)**

The net force is the push force minus friction, or $\mathbf{F_{net} = 120\ N - 5.00\ N = 115\ N}$. Thus the net work is

$$\begin{aligned} W_{\text{net}} &= F_{\text{net}} d = (115\ \text{N})(0.800\ \text{m}) \\ &= 92.0\ \text{N} \cdot \text{m} = 92.0\ \text{J}. \end{aligned}$$

**Discussion for (a)**

This value is the net work done on the package. The person actually does more work than this, because

friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

**Strategy and Concept for (b)**

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

**Solution for (b)**

The applied force does work.

$$\begin{aligned} W_{\text{app}} &= F_{\text{app}} d \cos(0^\circ) = F_{\text{app}} d \\ &= (120\ \text{N})(0.800\ \text{m}) \\ &= 96.0\ \text{J} \end{aligned}$$

The friction force and displacement are in opposite directions, so that $\theta = 180°$, and the work done by friction is

$$\begin{aligned} W_{\text{fr}} &= F_{\text{fr}} d \cos 180^\circ = -F_{\text{fr}} d \\ &= (-5.00\ \text{N})(0.800\ \text{m}) \\ &= -4.00\ \text{J}. \end{aligned}$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

$$\begin{aligned} W_{\text{gr}} &= 0, \\ W_{\text{N}} &= 0, \\ W_{\text{app}} &= 96.0\ \text{J}, \\ W_{\text{fr}} &= -4.00\ \text{J}. \end{aligned}$$

The total work done as the sum of the work done by each force is then seen to be

$$W_{\text{total}} = W_{\text{gr}} + W_{\text{N}} + W_{\text{app}} + W_{\text{fr}} = 92.0\ \text{J}.$$

**Discussion for (b)**

The calculated total work $W_{\text{total}}$ as the sum of the work by each force agrees, as expected, with the work $W_{\text{net}}$ done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

## Example 3: Determining Speed from Work and Energy

Find the speed of the package in Figure 2 at the end of the push, using work and energy concepts.

**Strategy**

Here the work-energy theorem can be used, because we have just calculated the net work, $W_{\text{net}}$, and the

initial kinetic energy, $\frac{1}{2}mv_0^2$. These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$, and thus the final speed $v$.

**Solution**

The work-energy theorem in equation form is

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Solving for $\frac{1}{2}mv^2$ gives

$$\frac{1}{2}mv^2 = W_{\text{net}} + \frac{1}{2}mv_0^2.$$

Thus,

$$\frac{1}{2}mv^2 = 92.0\ \text{J} + 3.75\ \text{J} = 95.75\ \text{J}.$$

Solving for the final speed as requested and entering known values gives

$$\begin{aligned} v &= \sqrt{\frac{2(95.75\ \text{J})}{m}} = \sqrt{\frac{191.5\ \text{kg}\cdot\text{m}^2/\text{s}^2}{30.0\ \text{kg}}} \\ &= 2.53\ \text{m/s}. \end{aligned}$$

**Discussion**

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on

the package. This means that the work indeed adds to the energy of the package.

## Example 4: Work and Energy Can Reveal Distance, Too

How far does the package in Figure 2 coast after the push, assuming friction remains constant? Use work and energy considerations.

**Strategy**

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package's kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

**Solution**

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so $\boldsymbol{\theta = 180°}$. To reduce the kinetic energy of the package to zero, the work $\mathbf{W_{fr}}$ by friction must be

minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus $W_{fr} = -95.75$ J. Furthermore, $W_{fr} = fd' \cos \theta = -fd'$, where $d'$ is the distance it takes to stop. Thus,

$$d' = -\frac{W_{fr}}{f} = -\frac{-95.75 \text{ J}}{5.00 \text{ N}},$$

and so

$$d' = 19.2 \text{ m}.$$

**Discussion**

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

## Section Summary

- The net work $W_{net}$ is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass $m$ moving

at speed $v$ is $\mathbf{KE} = \frac{1}{2}mv^2$.

- The work-energy theorem states that the net work $\mathbf{W_{net}}$ on a system changes its kinetic energy,

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

## *Conceptual Questions*

**1:** The person in Figure 3 does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?

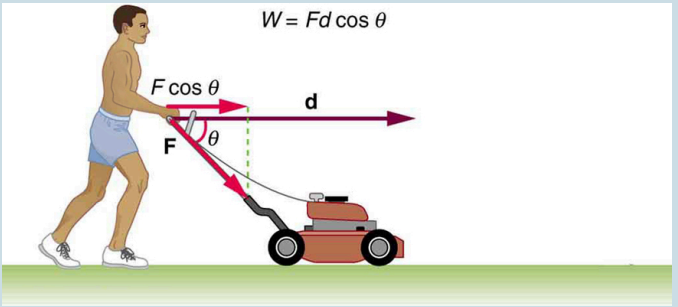



***Figure 3.***

**2:** Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

**3:** When solving for speed in Example 3, we kept only the positive root. Why?

## *Problems & Exercises*

**1:** Compare the kinetic energy of a 20,000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27,500 km/h.

**2:** (a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.

**3:** Confirm the value given for the kinetic energy of an aircraft carrier in Chapter 7.6 Table 1. You will need to look up the definition of a nautical mile (1 knot = 1 nautical mile/h).

**4:** (a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

**5:** A car's bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.

**6:** Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the gory old days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?

**7:** Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

## Glossary

**net work**
work done by the net force, or vector sum of all the forces, acting on an object

**work-energy theorem**
the result, based on Newton's laws, that the net work done on an object is equal to its change in kinetic energy

**kinetic energy**
the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of

an object of mass $m$ moving at speed $v$

## *Solutions*

**Problems & Exercises**

**1:** 1/250

**3:** $\mathbf{1.1 \times 10^{10}\ J}$

**5:** $\mathbf{2.8 \times 10^{3}\ N}$

**7:** net force = 72 N so the person force = 72 + 30 = 102 N

net force = change in kinetic energy = 1/2 m (v final)$^2$ – 1/2 m (v initial) $^2$

# 32. 6.3 Gravitational Potential Energy

*Summary*

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass $m$ at height $h$ on Earth is given by $PE_g = mgh$.
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

## Work Done Against Gravity

Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass $m$ through a height $h$, such as in Figure 1. If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight $mg$. The work done on the mass is then $W = Fd = mgh$. We

define this to be the **gravitational potential energy** ($\mathbf{PE_g}$)put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the $\mathbf{PE_g}$ gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word "system"? Potential energy is a property of a system rather than of a single object—due to its physical position. An object's gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth's surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

## Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to $\boldsymbol{mgh}$ on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of $\mathbf{PE_g}$ to **KE** without explicitly considering the intermediate step of work. (See Example 2.) This shortcut makes

it is easier to solve problems using energy (if possible) rather than explicitly using forces.

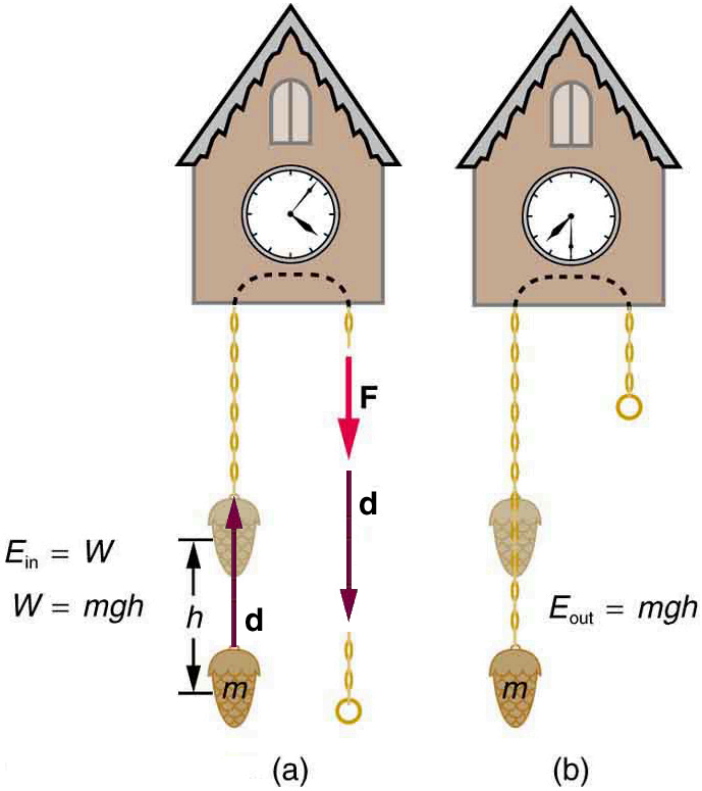



***Figure 1.*** *(a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.*

More precisely, we define the *change* in gravitational potential energy $\Delta\mathbf{PE_g}$ to be

$$\Delta\mathbf{PE_g} = \boldsymbol{mgh},$$

where, for simplicity, we denote the change in height by $\boldsymbol{h}$ rather than the usual $\boldsymbol{\Delta h}$. Note that $\boldsymbol{h}$ is positive when the final height

is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

$$\begin{aligned} mgh &= (0.500\ \text{kg})(9.80\ \text{m/s})(21.00\ \text{m}) \\ &= 4.90\ \text{kg}\cdot\text{m}^2/\text{s}^2 = 4.90\ \text{J}. \end{aligned}$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work.*

## Using Potential Energy to Simplify Calculations

The equation $\Delta PE_g = mgh$ applies for any path that has a change in height of $h$, not just when the mass is lifted straight up. (See Figure 2.) It is much easier to calculate $mgh$ (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position $h$ of a mass $m$ is accompanied by a change in gravitational potential energy $mgh$, and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.

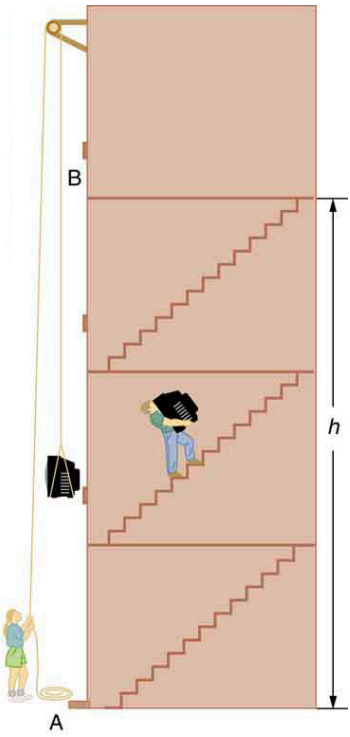



***Figure 2.*** *The change in gravitational potential energy* ***($\Delta PE_g$)*** *between points A and B is independent of the path.* ***$\Delta PE_g = mgh$*** *for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.*

## Work Done By The Gravitational Force

The concept of potential energy (and the change in potential energy) can also be used to quickly determine the **work** done by

the gravitational force. For example, in Figure 1a, when the mass moves upwards the gravitational force acts in the opposite direction to motion. This means that even though the gravitational potential energy increased, the work done by the gravitational force was negative. That is, the work done by the gravitational force is the negative of the change in potential energy,

$$W_{\text{g}} = -\Delta U_{\text{g}}$$

In Figure 1b, when the mass moves downwards, the gravitational force acts in the same direction as the motion. In this case, the change in potential energy is negative but the work down by gravity is positive (which follows from $W_{\text{g}} = -\Delta U_{\text{g}}$).

It turns out that this concept holds for *any* conservative force, not just the gravitational force (you will see another conservative force – the spring force – in the next section). In general, the work done by a conservative force ($W_{\text{cons}}$) is equal to the negative of the change in potential energy related to that force ($U$),

$$W_{\text{cons}} = -\Delta U$$

## Example 1: The Force to Stop Falling

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

**Strategy**

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial $\mathbf{PE_g}$ is transformed into **KE** as he falls.

The work done by the floor reduces this kinetic energy to zero.

**Solution**

The work done on the person by the floor as he stops is given by

$$W = Fd\cos\theta = -Fd,$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions (**cos $\theta$ = cos 180° = 1**). The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height **h**:

$$\mathrm{KE} = -\Delta\mathrm{PE_g} = -mgh,$$

The distance **d** that the person's knees bend is much smaller than the height **h** of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work **W** done by the floor on the person stops the person and brings the person's kinetic energy to zero:

$$W = -\mathrm{KE} = mgh.$$

Combining this equation with the expression for **W** gives

$$-Fd = mgh.$$

Recalling that **h** is negative because the person fell *down*, the force on the knee joints is given by

$$F = -\frac{mgh}{d} = -$$

$$\frac{(60.0\ \text{kg})(9.80\ \text{m/s}^2)(-3.00\ \text{m})}{5.00 \times 10^{-3}\ \text{m}} = 3.53 \times 10^5\ \text{N}.$$

**Discussion**

Such a large force (500 times more than the person's weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump.(See Figure 3.)



***Figure 3.*** *The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)*

## Example 2: Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in Figure 4 if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?

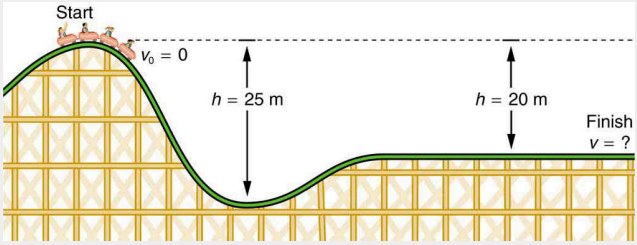



***Figure 4.*** *The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all* $\Delta\mathbf{PE_g}$ *is converted to* **KE**.

### Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by

gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance $h$ equals the *gain* in kinetic energy. This can be written in equation form as $-\Delta PE_g = \Delta KE$. Using the equations for $PE_g$ and $KE$, we can solve for the final speed $v$, which is the desired quantity.

**Solution for (a)**

Here the initial kinetic energy is zero, so that $\Delta KE = \frac{1}{2}mv^2$. The equation for change in potential energy states that $\Delta PE_g = mgh$. Since $h$ is negative in this case, we will rewrite this as $\Delta PE_g = -mg|h|$ to show the minus sign clearly. Thus,

$$-\Delta PE_g = \Delta KE$$

becomes

$$mg|h| = \frac{1}{2}mv^2.$$

Solving for $v$, we find that mass cancels and that

$$v = \sqrt{2g|h|}.$$

Substituting known values,

$$\begin{aligned} v &= \sqrt{2(9.80\ \text{m/s})(20.0\ \text{m})} \\ &= 19.8\ \text{m/s}. \end{aligned}$$

**Solution for (b)**

Again $-\Delta PE_g = \Delta KE$. In this case there is initial kinetic

energy, so $\mathbf{\Delta KE} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2$.

Thus,

$$mg|h| = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2.$$

Rearranging gives

$$\frac{1}{2}mv^2 = mg|h| + \frac{1}{2}mv_0{}^2.$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

$$v = \sqrt{2g|h| + v_0{}^2}.$$

This equation is very similar to the kinematics equation $v = \sqrt{v_0{}^2 + 2ad}$, but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

$$\begin{aligned} v &= \sqrt{2(9.80\ \text{m/s}^2)(20.0\ \text{m}) + (5.00\ \text{m/s})^2} \\ &= 20.4\ \text{m/s}. \end{aligned}$$

**Discussion and Implications**

First, note that mass cancels. This is quite consistent with observations made in Chapter 2.7 Falling Objects that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is

considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of $\boldsymbol{h}$ at the point of interest.

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

## MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—CONVERTING POTENTIAL TO

## KINETIC ENERGY

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see Figure 5). Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.

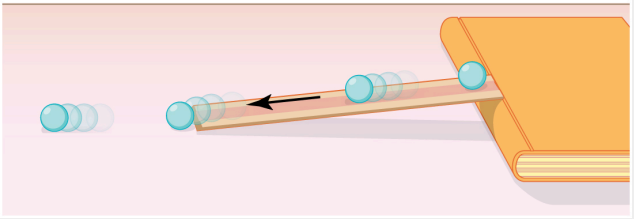


***Figure 5.*** *A marble rolls down a ruler, and its speed on the level surface is measured.*

## Section Summary

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy, $\Delta PE_g$, is $\Delta PEg = mgh$, with $h$ being the increase in height and $g$ the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, $\Delta PE_g$, have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that $\Delta KE = -\Delta PE_g$.

### *Conceptual Questions*

**1:** In Example 2, we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s *uphill* instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that it had the same final speed. Explain in terms of conservation of energy.

**2:** Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

## *Problems & Exercises*

**1:** A hydroelectric power facility (see Figure 6) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume 50.0 $km^3$ mass = $5.00 \times 10^{13}$ kg, given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.



***Figure 6.*** *Hydroelectric facility (credit: Denis Belevich, Wikimedia Commons)*

**2:** (a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about $7 \times 10^9$ kg

and its center of mass is 36.5 m above the surrounding ground? (b) How does this energy compare with the daily food intake of a person?

**3:** Suppose a 350-g kookaburra (a large kingfisher bird) picks up a 75-g snake and raises it 2.5 m from the ground to a branch. (a) How much work did the bird do on the snake? (b) How much work did it do to raise its own center of mass to the branch?

**4:** In Example 2, we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that $\Delta PE >> KE_i$. Confirm this statement by taking the ratio of $\mathbf{\Delta PE \text{ to } KE_i}$. (Note that mass cancels.)

**5:** A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in Figure 7. Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope, gaining 0.180 m in altitude.



***Figure 7.*** *A toy car moves up a sloped track. (credit: Leszek Leszczynski, Flickr)*

**6:** In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skies 70.0 m along a 30° slope neglecting friction. Hint: you have to use trigonometry to find the height of the hill first. (a) Starting from rest. (b) Starting with an initial speed of 2.50 m/s. (c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

## Glossary

**gravitational potential energy**
the energy an object has due to its position in a gravitational field

## *Solutions*

**Problems & Exercises**

**1:** (a) 1.96 x $10^{16}$ J (b) The ratio of gravitational potential energy in the lake to the energy stored in the bomb is 0.52. That is, the energy stored in the lake is approximately half that in a 9-megaton fusion bomb.

**3:** (a) 1.8 J (b) 8.6 J

**5:**

$$v_f = \sqrt{2gh + {v_0}^2} = \sqrt{2(9.80\ \text{m/s}^2)(-0.180\ \text{m}) + (2.00\ \text{m/s})^2} = 0.687\ \text{m/s}$$

6: h = 35.0 m = (70 m) sin $30.0^o$ m g h = 1/ 2 m $v^2$ so v = 26.2 m/s

# 33. 6.4 Conservative Forces and Potential Energy

*Summary*

- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke's law applies.
- Use the work-energy theorem to show how having only conservative forces implies conservation of mechanical energy.

## Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a **potential energy** (**PE**) for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy

contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

### POTENTIAL ENERGY AND CONSERVATIVE FORCES

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.

A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

We can define a potential energy (**PE**) for any conservative force. The work done against a conservative force to reach a final configuration depends on the configuration, not the path followed, and is the potential energy added.

## Potential Energy of a Spring

First, let us obtain an expression for the potential energy stored in a spring ($\mathbf{PE_s}$). We calculate the work done to stretch or compress

a spring that obeys Hooke's law. Hooke's Law states that the magnitude of force **F** on the spring and the resulting deformation **ΔL** are proportional, **F = kΔL**.) (See Figure 1.) For our spring, we will replace **ΔL** (the amount of deformation produced by a force **F**) by the distance **x** that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude **F = kx**, where **k** is the spring's force constant. The force increases linearly from 0 at the start to **kx** in the fully stretched position. The average force is **kx/2**. Thus the work done in stretching or compressing the spring is $W_s = Fd = (\frac{kx}{2})x = \frac{1}{2}kx^2$. Alternatively, we noted in Chapter 6.2 Kinetic Energy and the Work-Energy Theorem that the area under a graph of **F** vs. **x** is the work done by the force. In Figure 1(c) we see that this area is also $\frac{1}{2}kx^2$. We therefore define the **potential energy of a spring**, $PE_s$, to be

$$PE_s = \frac{1}{2}kx^2,$$

where **k** is the spring's force constant and **x** is the displacement from its undeformed position. The potential energy represents the work done *on* the spring and the energy stored in it as a result of stretching or compressing it a distance **x**. The potential energy of the spring $PE_s$ does not depend on the path taken; it depends only on the stretch or squeeze **x** in the final configuration.

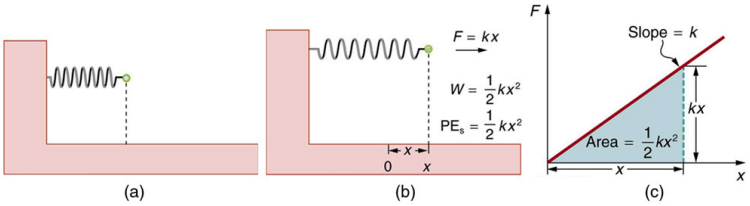



***Figure 1.*** *(a) An undeformed spring has no* $\mathbf{PE_s}$ *stored in it. (b) The force needed to stretch (or compress) the spring a distance* $\mathbf{x}$ *has a magnitude* $\mathbf{F=kx}$*, and the work done to stretch (or compress) it is* $\mathbf{1/2\ kx^2}$*. Because the force is conservative, this work is stored as potential energy* $\mathbf{(PE_s)}$ *in the spring, and it can be fully recovered. (c) A graph of* $\mathbf{F}$ *vs.* $\mathbf{x}$ *has a slope of* $\mathbf{k}$*, and the area under the graph is* $\mathbf{1/2\ kx^2}$*. Thus the work done or potential energy stored is* $\mathbf{1/2\ kx^2}$*.*

The equation $\mathbf{PE_s = \frac{1}{2}kx^2}$ has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of **potential energy** is energy due to position, shape, or configuration. For shape or position deformations, stored energy is $\mathbf{PE_s = \frac{1}{2}kx^2}$, where $\mathbf{k}$ is the force constant of the particular system and $\mathbf{x}$ is its deformation. Another example is seen in Figure 2 for a guitar string.

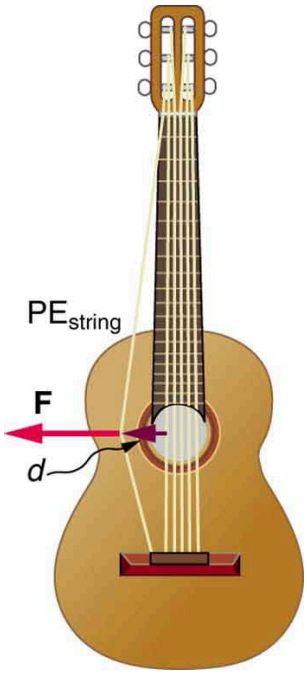



***Figure 2.*** *Work is done to deform the guitar string, giving it potential energy. When released, the potential energy is converted to kinetic energy and back to potential as the string oscillates back and forth. A very small fraction is dissipated as sound energy, slowly removing energy from the string.*

## Conservation of Mechanical Energy

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states

that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \Delta\text{KE}.$$

If only conservative forces act, then

$$W_{\text{net}} = W_{\text{c}},$$

where $W_{\text{c}}$ is the total work done by all conservative forces. Thus,

$$W_{\text{c}} = \Delta\text{KE}.$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is, $W_{\text{c}} = -\Delta\text{PE}$. Therefore,

$$-\Delta\text{PE} = \Delta\text{KE}$$

or

$$\Delta\text{KE} + \Delta\text{PE} = 0.$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

$$\text{KE} + \text{PE} = \text{constant}$$

$$\text{or} \qquad \Big\}$$

$$\text{KE}_{\text{i}} + \text{PE}_{\text{i}} = \text{KE}_{\text{f}} + \text{PE}_{\text{f}}$$

$$\text{(conservative forces only)},$$

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical energy**, (**KE+PE**). In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes

form between **KE** and the various types of **PE**, with the total energy remaining constant.

## Example 1: Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in Figure 3. The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.

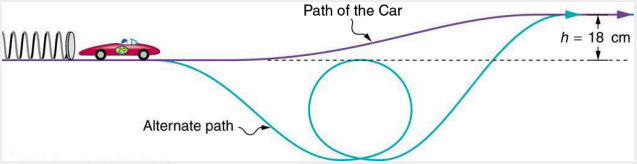



***Figure 3.*** *A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.*

**Strategy**

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

$$\mathbf{KE_i + PE_i = KE_f + PE_f}$$

or

$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2,$$

where $h$ is the height (vertical position) and $x$ is the compression of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

**Solution for (a)**

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both $h_i$ and $h_f$ are zero. Furthermore, the initial speed $v_i$ is zero and the final compression of the spring $x_f$ is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2.$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the

absence of friction. Solving for the final speed and entering known values yields

$$\begin{aligned} v_{\mathrm{f}} &= \sqrt{\frac{k}{m}}x_{\mathrm{i}} \\ &= \sqrt{\frac{250.0\ \mathrm{N/m}}{0.100\ \mathrm{kg}}}(0.0400\ \mathrm{m}) \\ &= 2.00\ \mathrm{m/s}. \end{aligned}$$

**Solution for (b)**

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

$$\frac{1}{2}k{x_{\mathrm{i}}}^2 = \frac{1}{2}m{v_{\mathrm{f}}}^2 + mgh_{\mathrm{f}}.$$

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for $v_{\mathrm{f}}$ and substituting known values gives

$$\begin{aligned} v_{\mathrm{f}} &= \sqrt{\frac{k{x_{\mathrm{i}}}^2}{m} - 2gh_{\mathrm{f}}} \\ &= \sqrt{(\frac{250.0\ \mathrm{N/m}}{0.100\ \mathrm{kg}})(0.0400\ \mathrm{m})^2 - 2(9.80\ \mathrm{m/s^2})(0.180\ \mathrm{m})} \\ &= 0.687\ \mathrm{m/s}. \end{aligned}$$

**Discussion**

Another way to solve this problem is to realize that

the car's kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in Example 1. Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.

## PHET EXPLORATIONS: ENERGY SKATE PARK

Learn about conservation of energy with a skater dude! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and friction as he moves. You can also take the skater to different planets or even space!

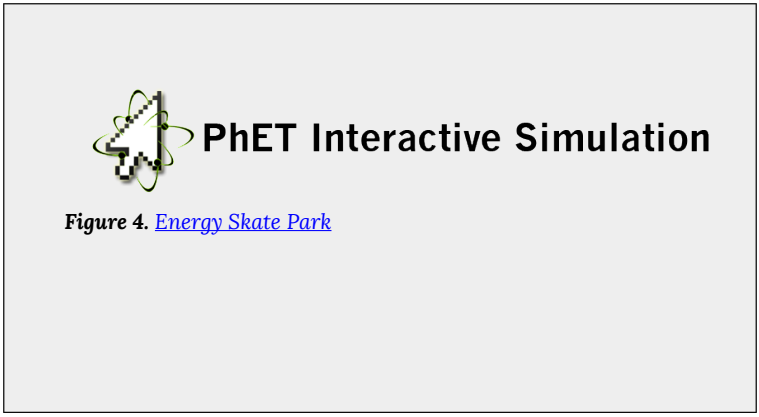



***Figure 4.*** *Energy Skate Park*

## Section Summary

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy **(PE)** for any conservative force, just as we defined $\mathbf{PE_g}$ for the gravitational force.
- The potential energy of a spring is $\mathbf{PE_s} = \frac{1}{2}kx^2$, where $k$ is the spring's force constant and $x$ is the displacement from its undeformed position.
- Mechanical energy is defined to be **KE+PE** for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,

$$\left.\begin{array}{l} \mathbf{KE + PE = constant} \\ \textbf{or} \\ \mathbf{KE_i + PE_i = KE_f + PE_f} \end{array}\right\}$$

where i and f denote initial and final values. This is known as the conservation of mechanical energy.

## *Conceptual Questions*

**1:** What is a conservative force?

**2:** The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.

**3:** Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?

**4:** What is the relationship of potential energy to conservative force?

## *Problems & Exercises*

**1:** A $5.00 \times 10^5$-kg subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the force constant $k$ of the spring?

**2:** A pogo stick has a spring with a force constant of $2.50 \times 10^4$ N/m, which can be compressed 12.0 cm. To what maximum height can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40.0 kg? Explicitly show how you follow the steps for problem solving.

## Glossary

**conservative force**
: a force that does the same work for any given initial and final configuration, regardless of the path followed

**potential energy**
: energy due to position, shape, or configuration

**potential energy of a spring**
: the stored energy of a spring as a function of its displacement; when Hooke's law applies, it is given by the expression $\frac{1}{2}kx^2$ where $x$ is the distance the spring is compressed or extended and $k$ is the spring constant

**conservation of mechanical energy**
: the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

**mechanical energy**
: the sum of kinetic energy and potential energy

## *Solutions*

**Problems & Exercises**

**1:** $7.81 \times 10^5$ N/m as kinetic energy = $^1/_2\, mv^2$ = elastic energy = $^1/_2\, kx^2$

**2:** gravitational potential energy = m g h = elastic spring energy = $^1/_2\, kx^2$ = $^1/_2\, (2.50 \times 10^4 \text{ N/m})\,(0.12 \text{ m})^2$

,

# 34. 6.5 Nonconservative Forces

*Summary*

- Define nonconservative forces and explain how they affect mechanical energy.
- Show how the principle of conservation of energy can be applied by treating the conservative forces in terms of their potential energies and any nonconservative forces in terms of the work they do.

## Nonconservative Forces and Friction

Forces are either conservative or nonconservative. Conservative forces were discussed in Chapter 6.4 Conservative Forces and Potential Energy. A **nonconservative force** is one for which work depends on the path taken. Friction is a good example of a nonconservative force. As illustrated in Figure 1, work done against friction depends on the length of the path between the starting and ending points. Because of this dependence on path, there is no potential energy associated with nonconservative forces. An important characteristic is that the work done by a nonconservative force *adds or removes mechanical energy from a system*. **Friction**, for example, creates **thermal energy** that dissipates, removing energy from the system. Furthermore, even if the thermal energy is

retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.

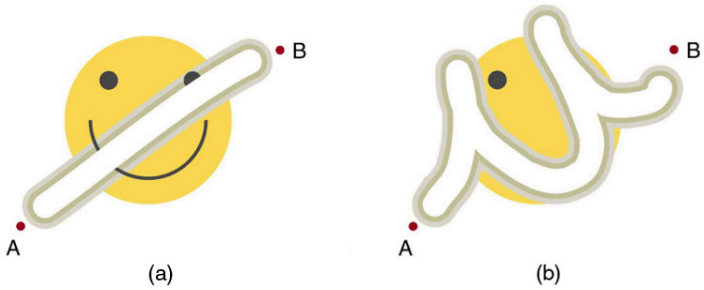



***Figure 1.*** *The amount of the happy face erased depends on the path taken by the eraser between points A and B, as does the work done against friction. Less work is done and less of the face is erased for the path in (a) than for the path in (b). The force here is friction, and most of the work goes into thermal energy that subsequently leaves the system (the happy face plus the eraser). The energy expended cannot be fully recovered.*

## How Nonconservative Forces Affect Mechanical Energy

*Mechanical* energy *may* not be conserved when nonconservative forces act. For example, when a car is brought to a stop by friction on level ground, it loses kinetic energy, which is dissipated as thermal energy, reducing its mechanical energy. Figure 2 compares the effects of conservative and nonconservative forces. We often choose to understand simpler systems such as that described in Figure 2(a) first before studying more complicated systems as in Figure 2(b).

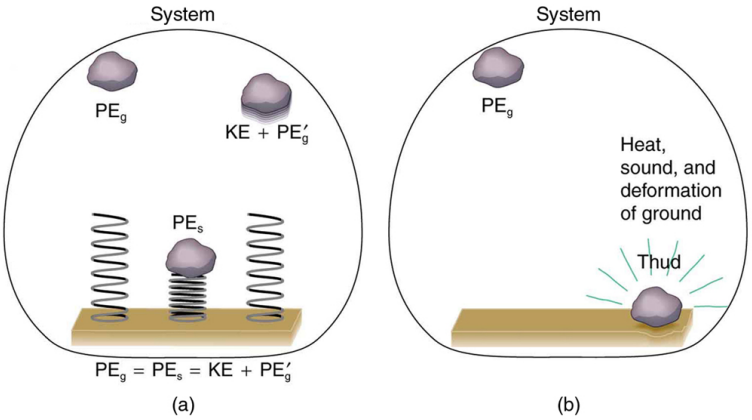



***Figure 2.*** *Comparison of the effects of conservative and nonconservative forces on the mechanical energy of a system. (a) A system with only conservative forces. When a rock is dropped onto a spring, its mechanical energy remains constant (neglecting air resistance) because the force in the spring is conservative. The spring can propel the rock back to its original height, where it once again has only potential energy due to gravity. (b) A system with nonconservative forces. When the same rock is dropped onto the ground, it is stopped by nonconservative forces that dissipate its mechanical energy as thermal energy, sound, and surface distortion. The rock has lost mechanical energy.*

## How the Work-Energy Theorem Applies

Now let us consider what form the work-energy theorem takes when both conservative and nonconservative forces act. We will see that the work done by nonconservative forces equals the change in the mechanical energy of a system. As noted in Chapter 6.2 Kinetic Energy and the Work-Energy Theorem, the work-energy theorem states that the net work on a system equals the change in its kinetic energy, or $\mathbf{W_{net} = \Delta KE}$. The net work is the sum of the work by nonconservative forces plus the work by conservative forces. That is,

$$\boldsymbol{W_{\text{net}} = W_{\text{nc}} + W_{\text{c}}},$$

so that

$$W_{nc} + W_c = \Delta KE,$$

where $W_{nc}$ is the total work done by all nonconservative forces and $W_c$ is the total work done by all conservative forces.

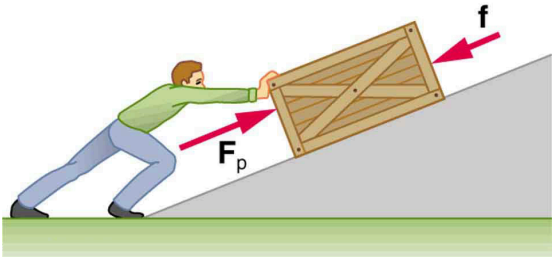



***Figure 3.*** *A person pushes a crate up a ramp, doing work on the crate. Friction and gravitational force (not shown) also do work on the crate; both forces oppose the person's push. As the crate is pushed up the ramp, it gains mechanical energy, implying that the work done by the person is greater than the work done by friction.*

Consider Figure 3, in which a person pushes a crate up a ramp and is opposed by friction. As in the previous section, we note that work done by a conservative force comes from a loss of gravitational potential energy, so that $W_c = -\Delta PE$. Substituting this equation into the previous one and solving for $W_{nc}$ gives

$$W_{nc} = \Delta KE + \Delta PE.$$

This equation means that the total mechanical energy **(KE+PE)** changes by exactly the amount of work done by nonconservative forces. In Figure 3, this is the work done by the person minus the work done by friction. So even if energy is not conserved for the system of interest (such as the crate), we know that an equal amount of work was done to cause the change in total mechanical energy.

We rearrange $W_{nc} = \Delta KE + \Delta PE$ to obtain

$$KE_i + PE_i + W_{nc} = KE_f + PE_f.$$

This means that the amount of work done by nonconservative forces adds to the mechanical energy of a system. If $\mathbf{W_{nc}}$ is positive, then mechanical energy is increased, such as when the person pushes the crate up the ramp in Figure 3. If $\mathbf{W_{nc}}$ is negative, then mechanical energy is decreased, such as when the rock hits the ground in Figure 2(b). If $\mathbf{W_{nc}}$ is zero, then mechanical energy is conserved, and nonconservative forces are balanced. For example, when you push a lawn mower at constant speed on level ground, your work done is removed by the work of friction, and the mower has a constant energy.

## Applying Energy Conservation with Nonconservative Forces

When no change in potential energy occurs, applying $\mathbf{KE_i + PE_i + W_{nc} = KE_f + PE_f}$ amounts to applying the work-energy theorem by setting the change in kinetic energy to be equal to the net work done on the system, which in the most general case includes both conservative and nonconservative forces. But when seeking instead to find a change in total mechanical energy in situations that involve changes in both potential and kinetic energy, the previous equation $\mathbf{KE_i + PE_i + W_{nc} = KE_f + PE_f}$ says that you can start by finding the change in mechanical energy that would have resulted from just the conservative forces, including the potential energy changes, and add to it the work done, with the proper sign, by any nonconservative forces involved.

## Example 1: Calculating Distance Traveled: How Far a Baseball Player Slides

Consider the situation shown in Figure 4, where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a constant 450 N.

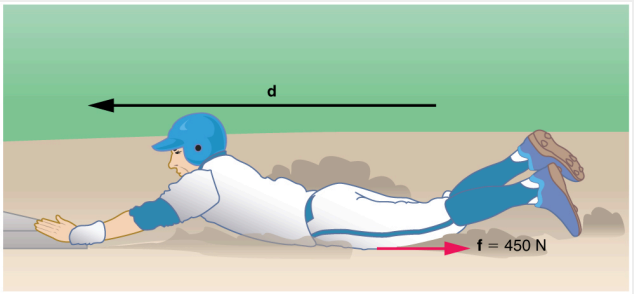



***Figure 4.*** *The baseball player slides to a stop in a distance* ***d****. In the process, friction removes the player's kinetic energy by doing an amount of work* ***fd*** *equal to the initial kinetic energy.*

**Strategy**

Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction, which is negative, is added to the initial kinetic

energy to reduce it to zero. The work done by friction is negative, because $f$ is in the opposite direction of the motion (that is, $\theta = 180°$, and so $\cos\theta = -1$). Thus $W_{nc} = -fd$. The equation simplifies to

$$\frac{1}{2}mv_i^2 - fd = 0$$

or

$$fd = \frac{1}{2}mv_i^2.$$

This equation can now be solved for the distance $d$.

**Solution**

Solving the previous equation for $d$ and substituting known values yields

$$\begin{aligned} d &= \frac{mv_i^2}{2f} \\ &= \frac{(65.0\ \text{kg})(6.00\ \text{m/s})^2}{(2)(450\ \text{N})} \\ &= 2.60\ \text{m}. \end{aligned}$$

**Discussion**

The most important point of this example is that the amount of nonconservative work equals the change in mechanical energy. For example, you must work harder to stop a truck, with its large mechanical energy, than to stop a mosquito.

## Example 2: Calculating Distance Traveled: Sliding Up an Incline

Suppose that the player from Example 1 is running up a hill having a **5.00°** incline upward with a surface similar to that in the baseball stadium. The player slides with the same initial speed. Determine how far he slides.

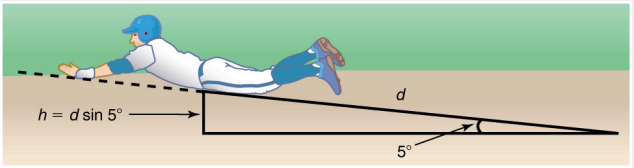



***Figure 5.*** *The same baseball player slides to a stop on a* $\mathbf{5.00^0}$ *slope.*

**Strategy**

In this case, the work done by the nonconservative friction force on the player reduces the mechanical energy he has from his kinetic energy at zero height, to the final mechanical energy he has by moving through distance $\boldsymbol{d}$ to reach height $\boldsymbol{h}$ along the hill, with $\boldsymbol{h = d \sin 5.00^0}$. This is expressed by the equation

$$\mathbf{KE_i + PE_i + \mathit{W}_{nc} = KE_f + PE_f}.$$

**Solution**

The work done by friction is again $\mathbf{W_{nc} = -\mathit{fd}}$; initially the potential energy is $\mathbf{PE_i = \mathit{mg} \cdot 0 = 0}$ and the kinetic

energy is $\mathbf{KE_i} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{mv_i^2}$; the final energy contributions are $\mathbf{KE_f = 0}$ for the kinetic energy and $\mathbf{PE_f = mgh = mgd \sin \theta}$ for the potential energy.

Substituting these values gives

$$\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{mv_i}^{\mathbf{2}} + \mathbf{0} + (-\boldsymbol{fd}) = \mathbf{0} + \boldsymbol{mgd} \sin \boldsymbol{\theta}.$$

Solve this for $\boldsymbol{d}$ to obtain

$$\begin{aligned} \boldsymbol{d} &= \frac{(\frac{1}{2})\boldsymbol{mv_i^2}}{\boldsymbol{f} + \boldsymbol{mg} \sin \boldsymbol{\theta}} \\ &= \frac{(0.5)(65.0 \text{ kg})(6.00 \text{ m/s})^2}{450 \text{ N} + (65.0 \text{ kg})(9.80 \text{ m/s}^2) \sin (5.00^0)} \\ &= \mathbf{2.31 \text{ m}}. \end{aligned}$$

**Discussion**

As might have been expected, the player slides a shorter distance by sliding uphill. Note that the problem could also have been solved in terms of the forces directly and the work energy theorem, instead of using the potential energy. This method would have required combining the normal force and force of gravity vectors, which no longer cancel each other because they point in different directions, and friction, to find the net force. You could then use the net force and the net work to find the distance $\boldsymbol{d}$ that reduces the kinetic energy to zero. By applying conservation of energy and using the potential energy instead, we need only consider the gravitational potential energy $\boldsymbol{mgh}$, without combining

and resolving force vectors. This simplifies the solution considerably.

## MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—DETERMINING FRICTION FROM STOPPING DISTANCE

This experiment involves the conversion of gravitational potential energy into thermal energy. You will need a ruler, book, marble, and a foam cup with a small hole in the side, as shown in Figure 6. From the 10-cm position on the ruler, let the marble roll into the cup positioned at the bottom of the ruler. Measure the distance $d$ the cup moves before stopping. What forces caused it to stop? What happened to the kinetic energy of the marble at the bottom of the ruler? Next, place the marble at the 20-cm and the 30-cm positions and again measure the distance the cup moves after the marble enters it. Plot the distance the cup moves versus the initial marble position on the ruler. Is this relationship linear?

With some simple assumptions, you can use these

data to find the coefficient of kinetic friction $\mu_k$ of the cup on the table. The force of friction $f$ on the cup is $\mu_k N$, where the normal force $N$ is just the weight of the cup plus the marble. The normal force and force of gravity do no work because they are perpendicular to the displacement of the cup, which moves horizontally. The work done by friction is $fd$. You will need the mass of the marble as well to calculate its initial kinetic energy.

It is interesting to do the above experiment also with a steel marble (or ball bearing). Releasing it from the same positions on the ruler as you did with the glass marble, is the velocity of this steel marble the same as the velocity of the marble at the bottom of the ruler? Is the distance the cup moves proportional to the mass of the steel and glass marbles?

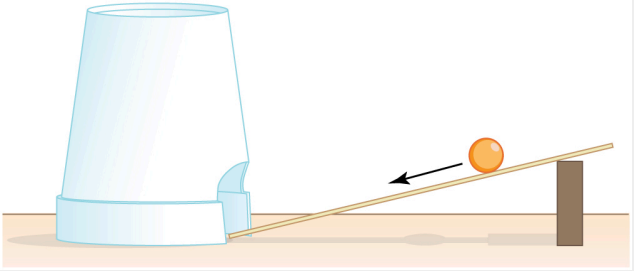


***Figure 6.*** *Rolling a marble down a ruler into a foam cup.*

PHET EXPLORATIONS: THE RAMP

Explore forces, energy and work as you push household objects up and down a ramp. Lower and raise the ramp to see how the angle of inclination affects the parallel forces acting on the file cabinet. Graphs show forces, energy and work.




*Figure 7.* *The Ramp*

## Section Summary

- A nonconservative force is one for which work depends on the path.
- Friction is an example of a nonconservative force that changes mechanical energy into thermal energy.
- Work $\mathbf{W_{nc}}$ done by a nonconservative force changes the mechanical energy of a system. In equation form, $\mathbf{W_{nc} = \Delta KE + \Delta PE}$ or, equivalently, $\mathbf{KE_i + PE_i + W_{nc} = KE_f + PE_f}$.
- When both conservative and nonconservative forces act, energy conservation can be applied and used to calculate

motion in terms of the known potential energies of the conservative forces and the work done by nonconservative forces, instead of finding the net work from the net force, or having to directly apply Newton's laws.

## *Problems & Exercises*

**1:** A 60.0-kg skier with an initial speed of 12.0 m/s coasts up a 2.50-m-high rise as shown in Figure 8. Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.0800. (Hint: Find the distance traveled up the incline assuming a straight-line path as shown in the figure.)

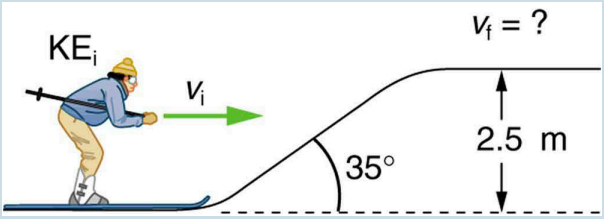



***Figure 8.*** *The skier's initial kinetic energy is partially used in coasting to the top of a rise.*

**2:** (a) How high a hill can a car coast up (engine disengaged) if work done by friction is negligible and its initial speed is 110 km/h? (b) If, in actuality, a 750-kg car with an initial speed of 110 km/h is observed to coast up a hill to a height 22.0 m above its starting point, how much thermal energy was generated by friction? (c) What is the

average force of friction if the hill has a slope 2.5° above the horizontal?

## Glossary

**nonconservative force**
a force whose work depends on the path followed between the given initial and final configurations

**friction**
the force between surfaces that opposes one sliding on the other; friction changes mechanical energy into thermal energy

## *Solutions*

**Problems & Exercises**

**1:** 9.46 N m/s

2: a) 48 m b)KE initial = 350 000 J PE final = so 160 000 J so thermal energy = 190 000 J c) 377 = 380 N as length of hill = 504 m

# 35. 6.6 Conservation of Energy

*Summary*

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

## Law of Conservation of Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of conservation of energy** can be stated as follows:

*Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.*

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy–mechanical energy **(KE + PE)** and energy transferred via work done by nonconservative forces **($W_{nc}$)**. But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all

of these before we can write an equation for the above general statement of the conservation of energy.

## Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called **other energy (OE)**. Then we can state the conservation of energy in equation form as

$$\mathbf{KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f}.$$

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is **KE**, work done by a conservative force is represented by **PE**, work done by nonconservative forces is $\mathbf{W_{nc}}$, and all other energies are included as **OE**. This equation applies to all previous examples; in those situations **OE** was constant, and so it subtracted out and was not directly considered.

### MAKING CONNECTIONS: USEFULNESS OF THE ENERGY CONSERVATION PRINCIPLE

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all

processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does **OE** play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of **OE**).

## Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast

in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

Table 1 gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

## PROBLEM SOLVING STRATEGIES FOR ENERGY

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on—continue to be relevant here.

**Step 1.** Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

**Step 2.** Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

**Step 3.** If you know the potential energies for the

forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is

$$\mathbf{KE_i + PE_i = KE_f + PE_f}.$$

**Step 4.** If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.

$$\mathbf{KE_i + PE_i} + W_{\mathbf{nc}} + \mathbf{OE_i = KE_f + PE_f + OE_f}.$$

In most problems, one or more of the terms is zero, simplifying its solution. Do not calculate $\mathbf{W_c}$, the work done by conservative forces; it is already incorporated in the **PE** terms.

**Step 5.** You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose $\boldsymbol{h = 0}$ at either the initial or final point, so that $\mathbf{PE_g}$ is zero there. Then solve for the unknown in the customary manner.

**Step 6.** *Check the answer to see if it is reasonable*. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value

obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

## Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see Figure 1) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.



***Figure 1.*** *Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)*

| Object/phenomenon | Energy in joules |
|---|---|
| Big Bang | $10^{68}$ |
| Energy released in a supernova | $10^{44}$ |
| Fusion of all the hydrogen in Earth's oceans | $10^{34}$ |
| Annual world energy use | $4 \times 10^{20}$ |
| Large fusion bomb (9 megaton) | $3.8 \times 10^{16}$ |
| 1 kg hydrogen (fusion to helium) | $6.4 \times 10^{14}$ |
| 1 kg uranium (nuclear fission) | $8.0 \times 10^{13}$ |
| Hiroshima-size fission bomb (10 kiloton) | $4.2 \times 10^{13}$ |
| 90,000-ton aircraft carrier at 30 knots | $1.1 \times 10^{10}$ |
| 1 barrel crude oil | $5.9 \times 10^{9}$ |
| 1 ton TNT | $4.2 \times 10^{9}$ |
| 1 gallon of gasoline | $1.2 \times 10^{8}$ |
| Daily home electricity use (developed countries) | $7 \times 10^{7}$ |
| Daily adult food intake (recommended) | $1.2 \times 10^{7}$ |
| 1000-kg car at 90 km/h | $3.1 \times 10^{5}$ |
| 1 g fat (9.3 kcal) | $3.9 \times 10^{4}$ |
| ATP hydrolysis reaction | $3.2 \times 10^{4}$ |
| 1 g carbohydrate (4.1 kcal) | $1.7 \times 10^{4}$ |
| 1 g protein (4.1 kcal) | $1.7 \times 10^{4}$ |
| Tennis ball at 100 km/h | $22$ |

| Object/phenomenon | Energy in joules |
| --- | --- |
| Mosquito $(10^{-2}\ \text{g at } 0.5\ \text{m/s})$ | $1.3 \times 10^{-6}$ |
| Single electron in a TV tube beam | $4.0 \times 10^{-15}$ |
| Energy to break one DNA strand | $10^{-19}$ |

**Table 1.** Energy of Various Objects and Phenomena.

## Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The **efficiency *Eff*** of an energy conversion process is defined as

$$\textbf{Efficiency}(\boldsymbol{Eff}) = \frac{\textbf{useful energy or work output}}{\textbf{total energy input}} = \frac{\boldsymbol{W}_{\textbf{out}}}{\boldsymbol{E}_{\textbf{in}}}.$$

Table 2 lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

| Activity/device | Efficiency (%)[1] |
|---|---|
| Cycling and climbing | 20 |
| Swimming, surface | 2 |
| Swimming, submerged | 4 |
| Shoveling | 3 |
| Weightlifting | 9 |
| Steam engine | 17 |
| Gasoline engine | 30 |
| Diesel engine | 35 |
| Nuclear power plant | 35 |
| Coal power plant | 42 |
| Electric motor | 98 |
| Compact fluorescent light | 20 |
| Gas heater (residential) | 90 |
| Solar cell | 10 |

**Table 2.** Efficiency of the Human Body and Mechanical Devices.

## PHET EXPLOTATIONS: MASSES AND SPRINGS

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energies for each spring.

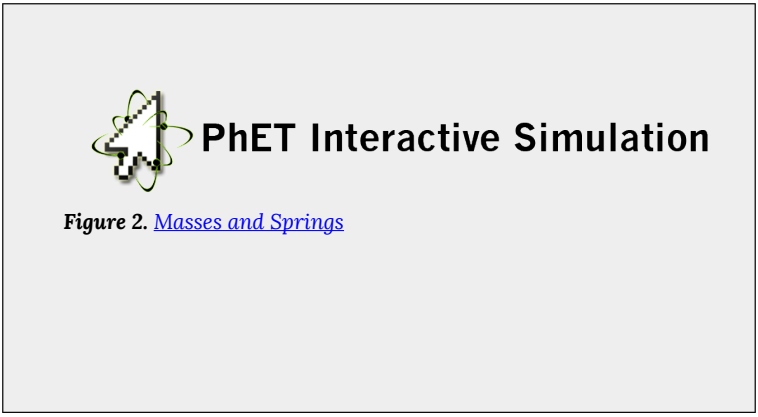



***Figure 2.*** *Masses and Springs*

## Section Summary

- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as $\mathbf{KE_i + PE_i + Wnc + OE_i = KE_f + PE_f + OE_f}$, where **OE** is all **other forms of energy** besides mechanical energy.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.
- The efficiency ***Eff*** of a machine or human is defined to be $\boldsymbol{Eff = \frac{W_{out}}{E_{in}}}$, where $\mathbf{W_{out}}$ is useful work output and

$\mathbf{E_{in}}$ is the energy consumed.

## *Conceptual Questions*

**1:** Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See Figure 3.)

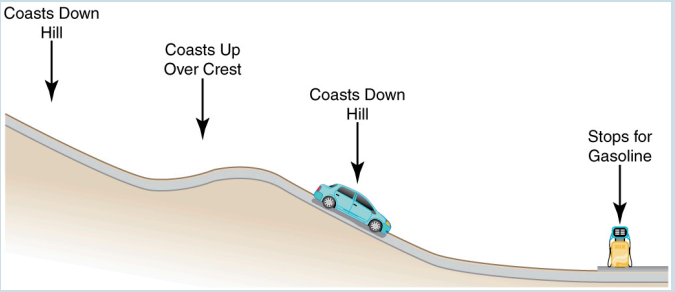



***Figure 3.*** *A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.*

**2:** Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks

up the javelin and ending when the javelin is stuck into the ground after being thrown.

**3:** Do devices with efficiencies of less than one violate the law of conservation of energy? Explain.

**4:** List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.

**5:** List the energy conversions that occur when riding a bicycle.

## *Problems & Exercises*

**1:** Using values from Table 1, how many DNA molecules could be broken by the energy carried by a single electron in the beam of an old-fashioned TV tube? (These electrons were not dangerous in themselves, but they did create dangerous x rays. Later model tube TVs had shielding that absorbed x rays before they escaped and exposed viewers.)

**2:** Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.

**3:** If the energy in fusion bombs were used to supply the

energy needs of the world, how many of the 9-megaton variety would be needed for a year's supply of energy (using data from Table 1)? This is not as far-fetched as it may sound—there are thousands of nuclear bombs, and their energy can be trapped in underground explosions and converted to electricity, as natural geothermal energy is.

**4:** (a) Use of hydrogen fusion to supply energy is a dream that may be realized in the next century. Fusion would be a relatively clean and almost limitless supply of energy, as can be seen from Table 1. To illustrate this, calculate how many years the present energy needs of the world could be supplied by one millionth of the oceans' hydrogen fusion energy. (b) How does this time compare with historically significant events, such as the duration of stable economic systems?

## Footnotes

1. 1 Representative values

## Glossary

**law of conservation of energy**
: the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

**electrical energy**
the energy carried by a flow of charge

**chemical energy**
the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

**radiant energy**
the energy carried by electromagnetic waves

**nuclear energy**
energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

**thermal energy**
the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

**efficiency**
a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

## *Solutions*

**Problems & Exercises**

**1:** 4 x $10^{4}$ molecules

**2:** Equating $\mathbf{\Delta PE_g}$ and $\mathbf{\Delta KE}$, we obtain

$$v = \sqrt{2gh + v_0^2} = \sqrt{2(9.80\ \text{m/s}^2)(20.0\ \text{m}) + (15.0\ \text{m/s})^2} = 24.8\ \text{m/s}$$

**4:** (a) 25 x $10^{6}$ years (b) This is much, much longer than human time scales.

# 36. 6.7 Power

*Summary*

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

## What is Power?

*Power*—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in Figure 1.



***Figure 1.*** *This powerful rocket on the Space Shuttle Endeavor did work and consumed energy at a very high rate. (credit: NASA)*

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** (**P**) as the rate at which work is done.

POWER

Power is the rate at which work is done.

$$P = \frac{W}{t}$$

The SI unit for power is the **watt** (**W**), where 1 watt equals 1 joule/second (**1 W = 1 J/s**).

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

## Calculating Power from Energy

### Example 1: Calculating the Power to Climb Stairs

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See Figure 2.)

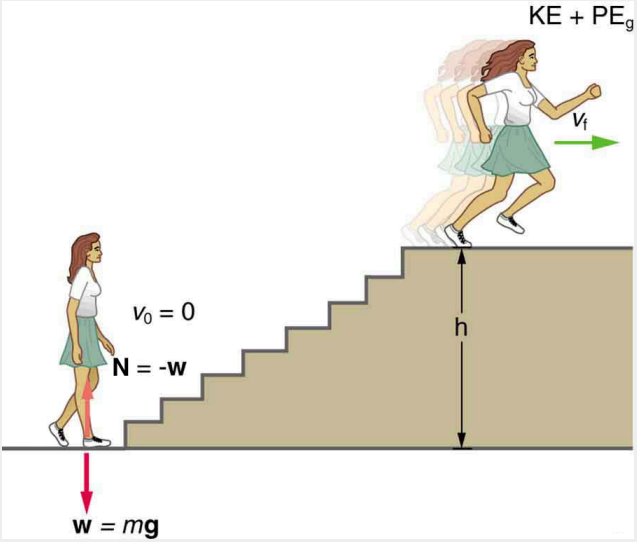



***Figure 2.*** *When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.*

**Strategy and Concept**

The work going into mechanical energy is **W = KE + PE**. At the bottom of the stairs, we take both **KE** and $\mathbf{PE_{gas}}$ initially zero; thus,

$$W = \mathrm{KE_f} + \mathrm{PE_g} = \frac{1}{2}mv_f^2 + mgh,$$

where ***h*** is the vertical height of the stairs. Because all terms are given, we can calculate **W** and then divide it by time to get power.

**Solution**

Substituting the expression for **W** into the definition of power given in the previous equation, **P = W/t** yields

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_f^2 + mgh}{t}.$$

Entering known values yields

$$\begin{aligned} P &= \frac{0.5(60.0\text{ kg})(2.00\text{ m/s})^2 + (60.0\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m})}{3.50\text{ s}} \\ &= \frac{120\text{ J} + 1764\text{ J}}{3.50\text{ s}} \\ &= 538\text{ W}. \end{aligned}$$

**Discussion**

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** (**1 hp = 746 W**)! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

### MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—MEASURE YOUR POWER RATING

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

## Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See Table 3 for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter ($kW/m^2$). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light,

with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is $\mathbf{10^6}$ **W** of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See Figure 3.)



***Figure 3.*** *Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)*

| Object or Phenomenon | Power in Watts |
|---|---|
| Supernova (at peak) | $5 \times 10^{37}$ |
| Milky Way galaxy | $10^{37}$ |
| Crab Nebula pulsar | $10^{28}$ |
| The Sun | $4 \times 10^{26}$ |
| Volcanic eruption (maximum) | $4 \times 10^{15}$ |
| Lightning bolt | $2 \times 10^{12}$ |
| Nuclear power plant (total electric and heat transfer) | $3 \times 10^{9}$ |
| Aircraft carrier (total useful and heat transfer) | $10^{8}$ |
| Dragster (total useful and heat transfer) | $2 \times 10^{6}$ |
| Car (total useful and heat transfer) | $8 \times 10^{4}$ |
| Football player (total useful and heat transfer) | $5 \times 10^{3}$ |
| Clothes dryer | $4 \times 10^{3}$ |
| Person at rest (all heat transfer) | 100 |
| Typical incandescent light bulb (total useful and heat transfer) | 60 |
| Heart, person at rest (total useful and heat transfer) | 8 |
| Electric clock | 3 |
| Pocket calculator | $10^{-3}$ |

**Table 3.** Power Output or Consumption.

## Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is **P = W/t = E/t**, where **E** is the energy supplied by the electricity company. So the energy consumed over a time $t$ is

$$E = Pt.$$

Electricity bills state the energy used in units of **kilowatt-hours** (**kW • h**), which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

### Example 2: Calculating Energy Costs

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is $0.120 per **kW · h**?

**Strategy**

Cost is based on energy consumed; thus, we must find **E** from **E = Pt** and then calculate the cost. Because electrical energy is expressed in **kW • h**, at the start of a problem such as this it is convenient to convert the units into **kW** and hours.

**Solution**

The energy consumed in **kW • h** is

$$\begin{aligned} E &= Pt = (0.200\ \text{kW})(6.00\ \text{h/d})(30.0\ \text{d}) \\ &= 36.0\ \text{kW} \cdot \text{h}, \end{aligned}$$

and the cost is simply given by

$$\text{cost} = (36.0\ \text{kW} \cdot \text{h})(\$0.120\ \text{per kW} \cdot \text{h}) = \$4.32\ \text{per month}.$$

**Discussion**

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of

energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail in Chapter 15 Thermodynamics, the potential for energy to produce useful work has been "degraded" in the energy transformation.

## Section Summary

- Power is the rate at which work is done, or in equation form, for the average power **P** for work **W** done over a time **t**, **P = W/t**.
- The SI unit for power is the watt (W), where **1 W = 1 J/s**.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where **1 hp = 746 W**.

### *Conceptual Questions*

**1:** Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

**2:** Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours

rather than joules. What is the relationship between these two energy units?

**3:** A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

## *Problems & Exercises*

**1:** The Crab Nebula (see Figure 4) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from Table 3, calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.

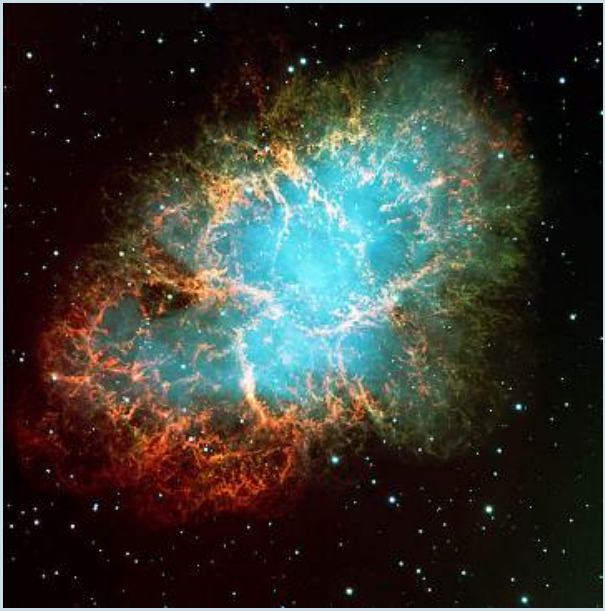


***Figure 4.*** *Crab Nebula (credit: ESO, via Wikimedia Commons)*

**2:** Suppose a star 1000 times brighter than our Sun (that is, emitting 1000 times the power) suddenly goes supernova. Using data from Table 3: (a) By what factor does its power output increase? (b) How many times brighter than our entire Milky Way galaxy is the supernova? (c) Based on your answers, discuss whether it should be possible to observe supernovas in distant galaxies. Note that there are on the order of $10^{11}$ observable galaxies, the average brightness of which is somewhat less than our own galaxy.

**3:** A person in good physical condition can put out 100 W

of useful power for several hours at a stretch, perhaps by pedalling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?

**4:** What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is \$0.0900 per kW • h?

**5:** A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is \$0.110 per kW • h?

**6:** (a) What is the average power consumption in watts of an appliance that uses 5.00 kW • h of energy per day? (b) How many joules of energy does this appliance consume in a year?

**7:** (a) What is the average useful power output of a person who does $6.00 \times 10^6$ J of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

**8:** A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

**9:** (a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp = 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m-high hill in the process?

**10:** (a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is \$0.0900 per kW • h?

**11:** (a) What is the available energy content, in joules, of a battery that operates a 2.00-W electric clock for 18 months? (b) How long can a battery that can supply $8.00 \times 10^4$ J run a pocket calculator that consumes energy at the rate of $1.00 \times 10^{-3}$ W?

**12:** (a) How long would it take a $1.50 \times 10^5$-kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)

**13:** Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N.

**14:** (a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be $4.00 \times 10^{26}$ W.) (b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of 1.30 kW/$m^2$ reaches Earth's surface. Calculate the area in $km^2$ of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs ($1.05 \times 10^{20}$ J)? Australia's energy needs ($5.4 \times 10^{18}$ J)? China's energy needs ($6.3 \times 10^{19}$ J)? (These energy consumption values are from 2006.)

## Glossary

**power**
: the rate at which work is done

**watt**
: (W) SI unit of power, with **1 W = 1 J/s**

**horsepower**
: an older non-SI unit of power, with **1 hp = 746 W**

**kilowatt-hour**
: (**kW • h**) unit used primarily for electrical energy provided by electric utility companies

## *Solutions*

**Problems & Exercises**

**1:** 2 x $10^{-10}$

**3:** (a) 40 (b) 8 million

**5:** $149

**7:** (a) 208 W (b) 141 s

**9:** (a) 3.20 s (b) 4.04 s

**11:** (a) $\mathbf{9.46 \times 10^7\ J}$ (b) $\mathbf{2.54\ y}$

**13:** Identify knowns:

$\boldsymbol{m = 950\ \text{kg},\ \text{slope angle}\ \theta = 2.00^0,\ v = 3.00\ \text{m/s},\ f = 600\ \text{N}}$

Identify unknowns: power **P** of the car, force **F** that car applies to road

Solve for unknown:

$$\boldsymbol{P = \frac{W}{t} = \frac{Fd}{t} = F(\frac{d}{t}) = Fv,}$$

where **F** is parallel to the incline and must oppose the resistive forces and the force of gravity:

$$\boldsymbol{F = f + w = 600\ \text{N} + mg \sin \theta}$$

Insert this into the expression for power and solve:

$$\begin{aligned} \boldsymbol{P} &= \boldsymbol{(f + mg \sin \theta)v} \\ &= \boldsymbol{[600\ \text{N} + (950\ \text{kg})(9.80\ \text{m/s}^2) \sin 2^0](30.0m/s)} \\ &= \boldsymbol{2.77 \times 10^4\ \text{W}} \end{aligned}$$

About 28 kW (or about 37 hp) is reasonable for a car to climb a gentle incline.

# 37. 6.8 Optional: Work, Energy, and Power in Humans

*Summary*

- Explain the human body's consumption of energy when at rest vs. when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

## Energy Conversion in Humans

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, and/or stored as chemical energy in fatty tissue. (See Figure 1.) The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.

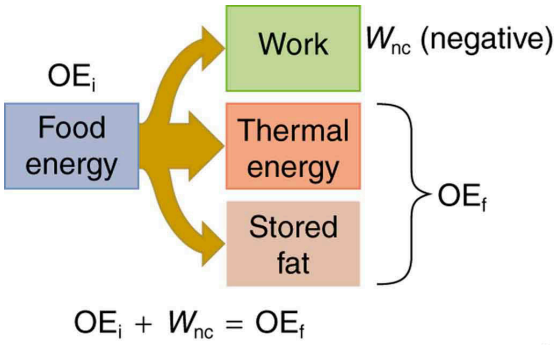



***Figure 1.*** *Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.*

## Power Consumed at Rest

The *rate* at which the body uses food energy to sustain life and to do different activities is called the **metabolic rate**. The total energy conversion rate of a person *at rest* is called the **basal metabolic rate** (BMR) and is divided among various systems in the body, as shown in Table 4. The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

| Organ | Power consumed at rest (W) | Oxygen consumption (mL/min) | Percent of BMR |
|---|---|---|---|
| Liver & spleen | 23 | 67 | 27 |
| Brain | 16 | 47 | 19 |
| Skeletal muscle | 15 | 45 | 18 |
| Kidney | 9 | 26 | 10 |
| Heart | 6 | 17 | 7 |
| Other | 16 | 48 | 19 |
| **Totals** | 85 W | 250 mL/min | 100% |

**Table 4.** Basal Metabolic Rates (BMR).

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. (See Figure 2.) Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food. Table 5 shows energy and oxygen consumption rates (power expended) for a variety of activities.

## Power of Doing Useful Work

Work done by a person is sometimes called **useful work**, which is *work done on the outside world*, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change

the mechanical energy **(KE + PE)** of the system worked upon, and this is often the goal. A baseball player throwing a ball, for example, increases both the ball's kinetic and potential energy.

If a person needs more energy than they consume, such as when doing vigorous work, the body must draw upon the chemical energy stored in fat. So exercise can be helpful in losing fat. However, the amount of exercise needed to produce a loss in fat, or to burn off extra calories consumed that day, can be large, as Example 1 illustrates.

### Example 1: Calculating Weight Loss from Exercising

If a person who normally requires an average of 12,000 kJ (3000 kcal) of food energy per day consumes 13,000 kJ per day, he will steadily gain weight. How much bicycling per day is required to work off this extra 1000 kJ?

**Solution**

Table 5 states that 400 W are used when cycling at a moderate speed. The time required to work off 1000 kJ at this rate is then

$$\textbf{Time} = \frac{\textbf{energy}}{\left(\frac{\textbf{energy}}{\textbf{time}}\right)} = \frac{\textbf{1000 kJ}}{\textbf{400 W}}$$
$$= \textbf{2500 s} = \textbf{42 min.}$$

**Discussion**

If this person uses more energy than he or she

consumes, the person's body will obtain the needed energy by metabolizing body fat. If the person uses 13,000 kJ but consumes only 12,000 kJ, then the amount of fat loss will be

$$\textbf{Fat loss} = (\textbf{1000 kJ})\left(\frac{\textbf{1.0 g fat}}{\textbf{39 kJ}}\right) = \textbf{26 g},$$

assuming the energy content of fat to be 39 kJ/g.

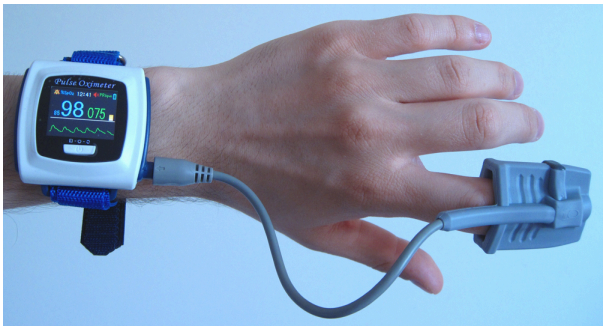



***Figure 2.*** *A pulse oxymeter is an apparatus that measures the amount of oxygen in blood. Oxymeters can be used to determine a person's metabolic rate, which is the rate at which food energy is converted to another form. Such measurements can indicate the level of athletic conditioning as well as certain medical problems. (credit: UusiAjaja, Wikimedia Commons)*

| Activity | Energy consumption in watts | Oxygen consumption in liters $O_2$/min |
|---|---|---|
| Sleeping | 83 | 0.24 |
| Sitting at rest | 120 | 0.34 |
| Standing relaxed | 125 | 0.36 |
| Sitting in class | 210 | 0.60 |
| Walking (5 km/h) | 280 | 0.80 |
| Cycling (13–18 km/h) | 400 | 1.14 |
| Shivering | 425 | 1.21 |
| Playing tennis | 440 | 1.26 |
| Swimming breaststroke | 475 | 1.36 |
| Ice skating (14.5 km/h) | 545 | 1.56 |
| Climbing stairs (116/min) | 685 | 1.96 |
| Cycling (21 km/h) | 700 | 2.00 |
| Running cross-country | 740 | 2.12 |
| Playing basketball | 800 | 2.28 |
| Cycling, professional racer | 1855 | 5.30 |
| Sprinting | 2415 | 6.90 |

**Table 5.** Energy and Oxygen Consumption Rates[1] (Power).

All bodily functions, from thinking to lifting weights, require energy. (See Figure 3.) The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all it that a full 25% of all energy consumed by the body

is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.

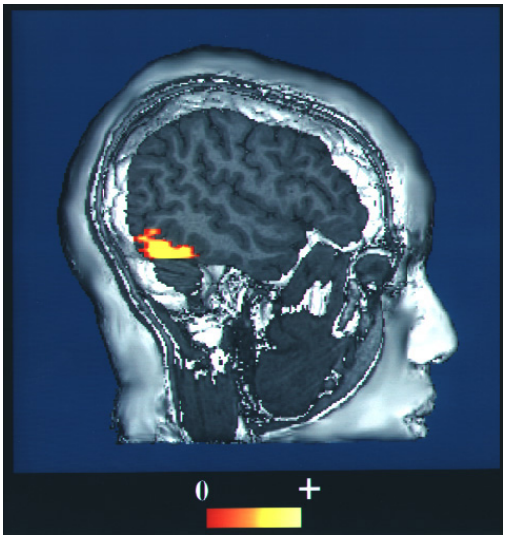



***Figure 3.*** *This fMRI scan shows an increased level of energy consumption in the vision center of the brain. Here, the patient was being asked to recognize faces. (credit: NIH via Wikimedia Commons)*

## Section Summary

- The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.
- The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic

rate (BMR)

- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

## *Conceptual Questions*

**1:** Explain why it is easier to climb a mountain on a zigzag path rather than one straight up the side. Is your increase in gravitational potential energy the same in both cases? Is your energy consumption the same in both?

**2:** Do you do work on the outside world when you rub your hands together to warm them? What is the efficiency of this activity?

**3:** Shivering is an involuntary response to lowered body temperature. What is the efficiency of the body when shivering, and is this a desirable value?

**4:** Discuss the relative effectiveness of dieting and exercise in losing weight, noting that most athletic activities consume food energy at a rate of 400 to 500 W, while a single cup of yogurt can contain 1360 kJ (325 kcal). Specifically, is it likely that exercise alone will be sufficient to lose weight? You may wish to consider that regular

exercise may increase the metabolic rate, whereas protracted dieting may reduce it.

## *Problems & Exercises*

**1:** (a) How long can you rapidly climb stairs (116 stairs/min) on the 93.0 kcal of energy in a 10.0-g pat of butter? (b) How many flights is this if each flight has 16 stairs? Table 5 tells us that climbing stairs at this rate requires 685 watts. Remember that 1 Food Calorie = 1000 calories. This question is written in the better form of kcal as opposed to Food Calories. Take 1 calorie to be 4.186 joules.

**2:** (a) What is the power output in watts and horsepower of a 70.0-kg sprinter who accelerates from rest to 10.0 m/s in 3.00 s? (b) Considering the amount of power generated, do you think a well-trained athlete could do this repetitively for long periods of time such as two hours?

**3:** Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)



***Figure 4.*** *Shot putter at the Dornoch Highland Gathering in 2007. (credit: John Haslam, Flickr)*

**4:** (a) What is the efficiency of an out-of-condition professor who does $2.10 \times 10^5$ J of useful work while metabolizing 500 kcal of food energy? (b) How many food calories would a well-conditioned athlete metabolize in doing the same work with an efficiency of 20%?

**5:** Energy that is not utilized for work or heat transfer is converted to the chemical energy of body fat containing about 39 kJ/g. How many grams of fat will you gain if you eat 10,000 kJ (about 2500 kcal) one day and do nothing but sit relaxed for 16.0 h and sleep for the other 8.00 h? Use data from Table 5 for the energy consumption rates of these activities.

**6:** Using data from Table 5, calculate the daily energy needs of a person who sleeps for 7.00 h, walks for 2.00 h, attends classes for 4.00 h, cycles for 2.00 h, sits relaxed for 3.00 h, and studies for 6.00 h. (Studying consumes energy at the same rate as sitting in class.)

**7:** What is the efficiency of a subject on a treadmill who puts out work at the rate of 100 W while consuming oxygen at the rate of 2.00 L/min? (Hint: See Table 5.)

**8:** Shoveling snow can be extremely taxing because the arms have such a low efficiency in this activity. Suppose a person shoveling a footpath metabolizes food at the rate of 800 W. (a) What is her useful power output? (b) How long will it take her to lift 3000 kg of snow 1.20 m? (This could be the amount of heavy snow on 20 m of footpath.) (c) How much waste heat transfer in kilojoules will she generate in the process?

**9:** Very large forces are produced in joints when a person jumps from some height to the ground. (a) Calculate the magnitude of the force produced if an 80.0-kg person jumps from a 0.600–m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.) (b) In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the magnitude of the force produced if the stopping distance is 0.300 m. (c) Compare both forces with the weight of the person.

**10:** Jogging on hard surfaces with insufficiently padded shoes produces large forces in the feet and legs. (a) Calculate the magnitude of the force needed to stop the

downward motion of a jogger's leg, if his leg has a mass of 13.0 kg, a speed of 6.00 m/s, and stops in a distance of 1.50 cm. (Be certain to include the weight of the 75.0-kg jogger's body.) (b) Compare this force with the weight of the jogger.

**11:** (a) Calculate the energy in kJ used by a 55.0-kg woman who does 50 deep knee bends in which her center of mass is lowered and raised 0.400 m. (She does work in both directions.) You may assume her efficiency is 20%. (b) What is the average power consumption rate in watts if she does this in 3.00 min?

**12:** Kanellos Kanellopoulos flew 119 km from Crete to Santorini, Greece, on April 23, 1988, in the *Daedalus* 88, an aircraft powered by a bicycle-type drive mechanism (see Figure 5). His useful power output for the 234-min trip was about 350 W. Using the efficiency for cycling from Table 2, calculate the food energy in kilojoules he metabolized during the flight.



***Figure 5.*** *The Daedalus 88 in flight. (credit: NASA photo by Beasley)*

**13:** The swimmer shown in Figure 6 exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke. (a) What is his work output in each stroke? (b) Calculate the power output of his arms if he does 120 strokes per minute.

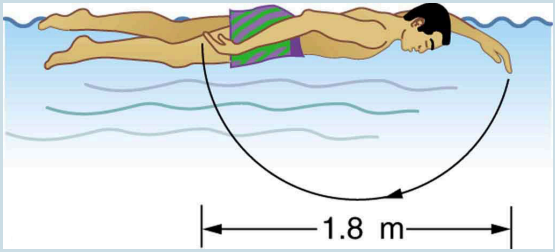



***Figure 6.***

**14:**Mountain climbers carry bottled oxygen when at very high altitudes. (a) Assuming that a mountain climber uses oxygen at twice the rate for climbing 116 stairs per minute (because of low air temperature and winds), calculate how many liters of oxygen a climber would need for 10.0 h of climbing. (These are liters at sea level.) Note that only 40% of the inhaled oxygen is utilized; the rest is exhaled. (b) How much useful work does the climber do if he and his equipment have a mass of 90.0 kg and he gains 1000 m of altitude? (c) What is his efficiency for the 10.0-h climb?

**15:** The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about $7 \times 10^9$ kg. (The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20,000 workers spent 20 years to construct it, working 12-hour days, 330 days per year. (a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height. (b) Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see

Figure 7), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own bodies? (c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)

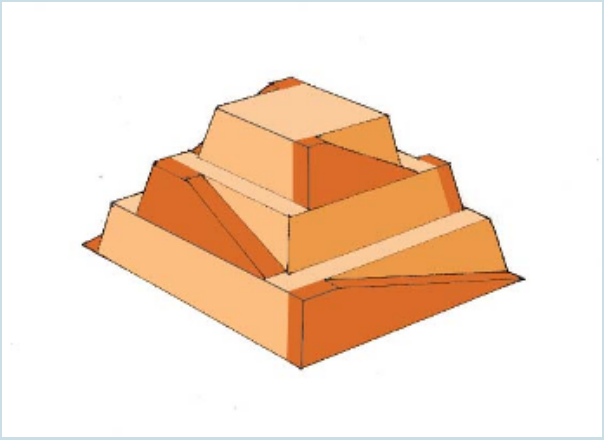


***Figure 7.*** *Ancient pyramids were probably constructed using ramps as simple machines. (credit: Franck Monnier, Wikimedia Commons)*

**16:** (a) How long can you play tennis on the 800 kJ (about 200 kcal) of energy in a candy bar? (b) Does this seem like a

long time? Discuss why exercise is necessary but may not be sufficient to cause a person to lose weight.

## Footnotes

1. 1 for an average 76-kg male

## Glossary

**metabolic rate**
: the rate at which the body uses food energy to sustain life and to do different activities

**basal metabolic rate**
: the total energy conversion rate of a person at rest

**useful work**
: work done on an external system

## *Solutions*

**Problems & Exercises**

**1:** (a) 3.89 x$10^5$ J at 685 watts gives 568 seconds = 9.47 minutes (b) 9.47 minutes at 116 stairs/minutes = 1098 stairs = 68.7 flights of stairs

**2:** a) Net work = ½ m $v_{final}^2$ – ½ m $v_{initial}^2$

Net work = ½ (70.0 kg) (10.0 m/s) $^2$ – 0 J = 3500 J

Power = work/time = 1170 W = 1.56 hp No. Imagine sprinting for hours

**3:** 641 W , 0.860 hp

**5:** 31 g

**7:** 14.3%

**9:** (a) 3.21 x $10^4$ N (b) 2.35 x $10^3$ N (c) Ratio of net force to weight of person is 41.0 in part (a); 3.00 in part (b)

**11:** (a)108 kJ b) 559 W

**13:** (a) 144 J b) 288 W

**15:** (a) 2.50 x $10^{12}$ J (b) 2.52 % (c) 1.4 x $10^4$ kg (14 metric tons)

# 38. 6.9 Optional: World Energy Use

*Summary*

- Describe the distinction between renewable and nonrenewable energy sources.
- Explain why the inevitable conversion of energy to less useful forms makes it necessary to conserve energy resources.

Energy is an important ingredient in all phases of society. We live in a very interdependent world, and access to adequate and reliable energy resources is crucial for economic growth and for maintaining the quality of our lives. But current levels of energy consumption and production are not sustainable. About 40% of the world's energy comes from oil, and much of that goes to transportation uses. Oil prices are dependent as much upon new (or foreseen) discoveries as they are upon political events and situations around the world. The U.S., with 4.5% of the world's population, consumes 24% of the world's oil production per year; 66% of that oil is imported!

## Renewable and Nonrenewable Energy Sources

The principal energy resources used in the world are shown in Figure 1. The fuel mix has changed over the years but now is dominated by oil, although natural gas and solar contributions are increasing. **Renewable forms of energy** are those sources that cannot be used up, such as water, wind, solar, and biomass. About 85% of our energy comes from nonrenewable **fossil fuels**—oil, natural gas, coal. The likelihood of a link between global warming and fossil fuel use, with its production of carbon dioxide through combustion, has made, in the eyes of many scientists, a shift to non-fossil fuels of utmost importance—but it will not be easy.

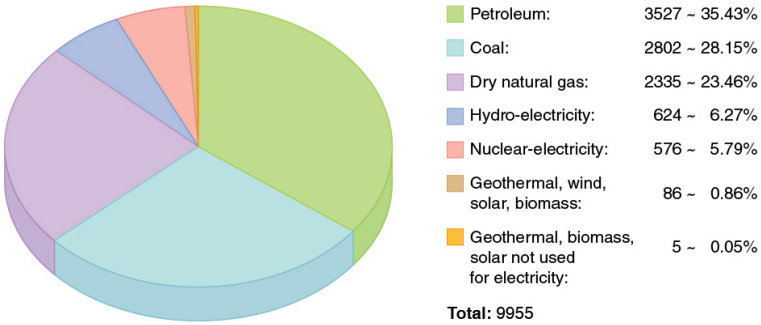



***Figure 1.*** *World energy consumption by source, in billions of kilowatt-hours: 2006. (credit: KVDP)*

## The World's Growing Energy Needs

World energy consumption continues to rise, especially in the developing countries. (See Figure 2.) Global demand for energy has tripled in the past 50 years and might triple again in the next 30 years. While much of this growth will come from the rapidly

booming economies of China and India, many of the developed countries, especially those in Europe, are hoping to meet their energy needs by expanding the use of renewable sources. Although presently only a small percentage, renewable energy is growing very fast, especially wind energy. For example, Germany plans to meet 20% of its electricity and 10% of its overall energy needs with renewable resources by the year 2020. (See Figure 3.) Energy is a key constraint in the rapid economic growth of China and India. In 2003, China surpassed Japan as the world's second largest consumer of oil. However, over 1/3 of this is imported. Unlike most Western countries, coal dominates the commercial energy resources of China, accounting for 2/3 of its energy consumption. In 2009 China surpassed the United States as the largest generator of $CO_2$. In India, the main energy resources are biomass (wood and dung) and coal. Half of India's oil is imported. About 70% of India's electricity is generated by highly polluting coal. Yet there are sizeable strides being made in renewable energy. India has a rapidly growing wind energy base, and it has the largest solar cooking program in the world.

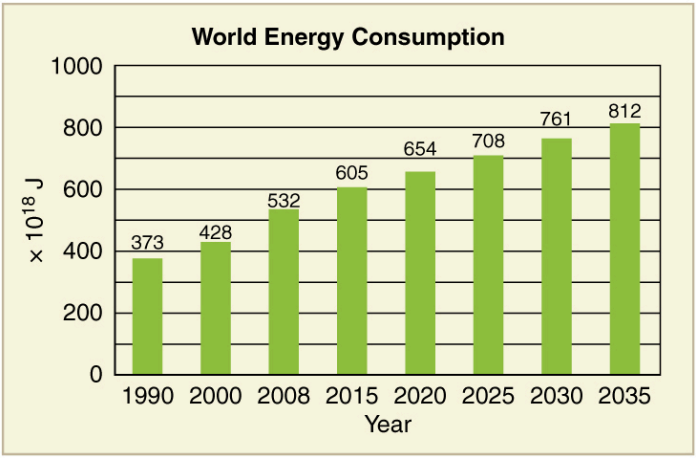



***Figure 2.*** *Past and projected world energy use (source: Based on data from U.S. Energy Information Administration, 2011)*

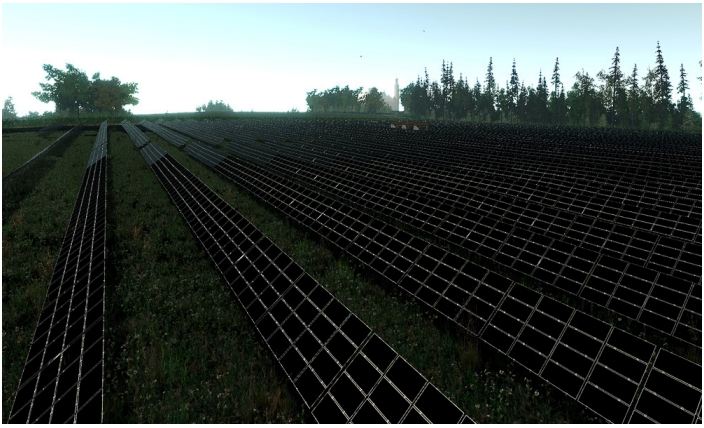


***Figure 3.*** *Solar cell arrays at a power plant in Steindorf, Germany (credit: Michael Betke, Flickr)*

Table 6 displays the 2006 commercial energy mix by country for some of the prime energy users in the world. While non-renewable sources dominate, some countries get a sizeable percentage of their electricity from renewable resources. For example, about 67% of New Zealand's electricity demand is met by hydroelectric. Only 10% of the U.S. electricity is generated by renewable resources, primarily hydroelectric. It is difficult to determine total contributions of renewable energy in some countries with a large rural population, so these percentages in this table are left blank.

| Country | Consumption, in EJ ($10^{18}$ J) | Oil | Natural Gas | Coal | Nuclear | Hydro | Other Renewables | Electr Use pe capita (kWh/ |
|---|---|---|---|---|---|---|---|---|
| Australia | 5.4 | 34% | 17% | 44% | 0% | 3% | 1% | 10000 |
| Brazil | 9.6 | 48% | 7% | 5% | 1% | 35% | 2% | 2000 |
| China | 63 | 22% | 3% | 69% | 1% | 6% | | 1500 |
| Egypt | 2.4 | 50% | 41% | 1% | 0% | 6% | | 990 |
| Germany | 16 | 37% | 24% | 24% | 11% | 1% | 3% | 6400 |
| India | 15 | 34% | 7% | 52% | 1% | 5% | | 470 |
| Indonesia | 4.9 | 51% | 26% | 16% | 0% | 2% | 3% | 420 |
| Japan | 24 | 48% | 14% | 21% | 12% | 4% | 1% | 7100 |
| New Zealand | 0.44 | 32% | 26% | 6% | 0% | 11% | 19% | 8500 |
| Russia | 31 | 19% | 53% | 16% | 5% | 6% | | 5700 |
| U.S. | 105 | 40% | 23% | 22% | 8% | 3% | 1% | 12500 |
| **World** | **432** | **39%** | **23%** | **24%** | **6%** | **6%** | **2%** | **2600** |

**Table 6.** Energy Consumption–Selected Countries (2006).

## Energy and Economic Well-being

The last two columns in this table examine the energy and electricity use per capita. Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (gross domestic product) per capita, are matched by higher levels of energy consumption per capita. This is borne out in Figure 4. Increased efficiency of energy use will change this dependency. A global problem is balancing energy resource development against the harmful effects upon the environment in its extraction and use.

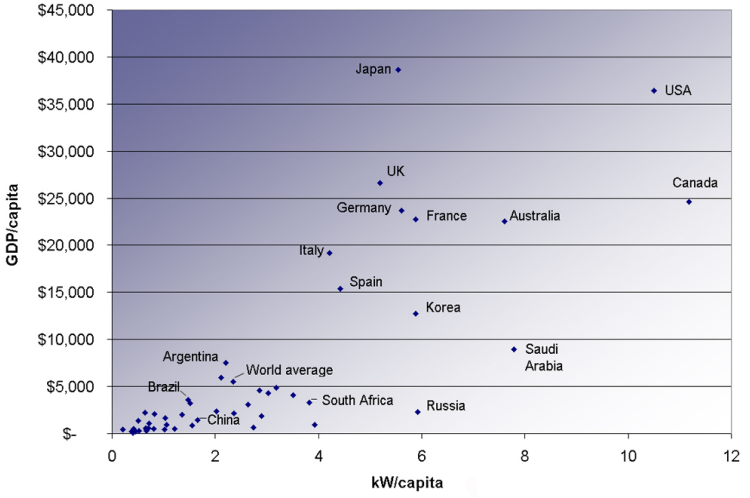



***Figure 4.*** *Power consumption per capita versus GDP per capita for various countries. Note the increase in energy usage with increasing GDP. (2007, credit: Frank van Mierlo, Wikimedia Commons)*

# Conserving Energy

As we finish this chapter on energy and work, it is relevant to draw some distinctions between two sometimes misunderstood terms in the area of energy use. As has been mentioned elsewhere, the "law of the conservation of energy" is a very useful principle in analyzing physical processes. It is a statement that cannot be proven from basic principles, but is a very good bookkeeping device, and no exceptions have ever been found. It states that the total amount of energy in an isolated system will always remain constant. Related to this principle, but remarkably different from it, is the important philosophy of energy conservation. This concept has to do with seeking to decrease the amount of energy used by an individual or group through (1) reduced activities (e.g., turning down thermostats, driving fewer kilometers) and/or (2) increasing conversion

efficiencies in the performance of a particular task—such as developing and using more efficient room heaters, cars that have greater miles-per-gallon ratings, energy-efficient compact fluorescent lights, etc.

Since energy in an isolated system is not destroyed or created or generated, one might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat transfer to the environment and conversion to energy forms no longer useful for doing work. To state it in another way, the potential for energy to produce useful work has been "degraded" in the energy transformation.

## Section Summary

- The relative use of different fuels to provide energy has changed over the years, but fuel use is currently dominated by oil, although natural gas and solar contributions are increasing.
- Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources.
- The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power.
- Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (Gross Domestic Product) per capita, are matched by higher levels of energy consumption per capita.
- Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes.

## *Conceptual Questions*

**1:** What is the difference between energy conservation and the law of conservation of energy? Give some examples of each.

**2:** If the efficiency of a coal-fired electrical generating plant is 35%, then what do we mean when we say that energy is a conserved quantity?

## *Problems & Exercises*

**1: Integrated Concepts**

(a) Calculate the force the woman in Figure 5 exerts to do a push-up at constant speed, taking all data to be known to three digits. (b) How much work does she do if her center of mass rises 0.240 m? (c) What is her useful power output if she does 25 push-ups in 1 min? (Should work done lowering her body be included? See the discussion of useful work in Chapter 7.8 Work, Energy, and Power in Humans.

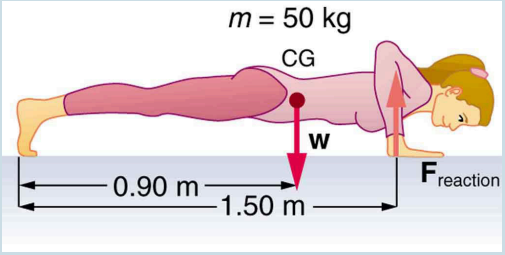



***Figure 5.*** *Forces involved in doing push-ups. The woman's weight acts as a force exerted downward on her center of gravity (CG).*

**2: Integrated Concepts**

A 75.0-kg cross-country skier is climbing a 3.0° slope at a constant speed of 2.00 m/s and encounters air resistance of 25.0 N. Find his power output for work done against the gravitational force and air resistance. (b) What average force does he exert backward on the snow to accomplish this? (c) If he continues to exert this force and to experience the same air resistance when he reaches a level area, how long will it take him to reach a velocity of 10.0 m/s?

**3: Integrated Concepts**

The 70.0-kg swimmer in Chapter 7.8 Work, Energy, and Power in Humans starts a race with an initial velocity of 1.25 m/s and exerts an average force of 80.0 N backward with his arms during each 1.80 m long stroke. (a) What is his initial acceleration if water resistance is 45.0 N? (b) What is the subsequent average resistance force from the water during the 5.00 s it takes him to reach his top velocity of

2.50 m/s? (c) Discuss whether water resistance seems to increase linearly with velocity.

**4: Integrated Concepts**

A toy gun uses a spring with a force constant of 300 N/m to propel a 10.0-g steel ball. If the spring is compressed 7.00 cm and friction is negligible: (a) How much force is needed to compress the spring? (b) To what maximum height can the ball be shot? (c) At what angles above the horizontal may a child aim to hit a target 3.00 m away at the same height as the gun? (d) What is the gun's maximum range on level ground?

**5: Integrated Concepts**

(a) What force must be supplied by an elevator cable to produce an acceleration of 0.800 $m/s^2$ against a 200-N frictional force, if the mass of the loaded elevator is 1500 kg? (b) How much work is done by the cable in lifting the elevator 20.0 m? (c) What is the final speed of the elevator if it starts from rest? (d) How much work went into thermal energy?

**6: Unreasonable Results**

A car advertisement claims that its 900-kg car accelerated from rest to 30.0 m/s and drove 100 km, gaining 3.00 km in altitude, on 1.0 gal of gasoline. The average force of friction including air resistance was 700 N. Assume all values are known to three significant figures. (a) Calculate the car's efficiency. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

**7: Unreasonable Results**

Body fat is metabolized, supplying 9.30 kcal/g, when dietary intake is less than needed to fuel metabolism. The manufacturers of an exercise bicycle claim that you can lose 0.500 kg of fat per day by vigorously exercising for 2.00 h per day on their machine. (a) How many kcal are supplied by the metabolization of 0.500 kg of fat? (b) Calculate the kcal/min that you would have to utilize to metabolize fat at the rate of 0.500 kg in 2.00 h. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

**8: Construct Your Own Problem**

Consider a person climbing and descending stairs. Construct a problem in which you calculate the long-term rate at which stairs can be climbed considering the mass of the person, his ability to generate power with his legs, and the height of a single stair step. Also consider why the same person can descend stairs at a faster rate for a nearly unlimited time in spite of the fact that very similar forces are exerted going down as going up. (This points to a fundamentally different process for descending versus climbing stairs.)

**9: Construct Your Own Problem**

Consider humans generating electricity by pedaling a device similar to a stationary bicycle. Construct a problem in which you determine the number of people it would take to replace a large electrical generation facility. Among the things to consider are the power output that is reasonable using the legs, rest time, and the need for electricity 24 hours per day. Discuss the practical implications of your results.

**10: Integrated Concepts**

A 105-kg basketball player crouches down 0.400 m while waiting to jump. After exerting a force on the floor through this 0.400 m, his feet leave the floor and his center of gravity rises 0.950 m above its normal standing erect position. (a) Using energy considerations, calculate his velocity when he leaves the floor. (b) What average force did he exert on the floor? (Do not neglect the force to support his weight as well as that to accelerate him.) (c) What was his power output during the acceleration phase?

## Glossary

**renewable forms of energy**
: those sources that cannot be used up, such as water, wind, solar, and biomass

**fossil fuels**
: oil, natural gas, and coal

## *Solutions*

**Problems & Exercises**

**1:** (a) 294 N b) 118 J c) 49.0 W

**3:** (a) 0.500 m $/s^2$ (b) 62.5 N (c) Assuming the acceleration of the swimmer decreases linearly with time

over the 5.00 s interval, the frictional force must therefore be increasing linearly with time, since **f=F-ma** . If the acceleration decreases linearly with time, the velocity will contain a term dependent on time squared ($t^2$). Therefore, the water resistance will not depend linearly on the velocity.

**5:** (a) $\mathbf{16.1 \times 10^3\ N}$ (b) $\mathbf{3.22 \times 10^5\ J}$ (c) 5.66 m/s (d) 4.00 kJ

**7:** (a) 4.65 x $10^3$ kcal (b) 38.8 kcal/min

(c) This power output is higher than the highest value on Table 5, which is about 35 kcal/min (corresponding to 2415 watts) for sprinting.

(d) It would be impossible to maintain this power output for 2 hours (imagine sprinting for 2 hours!).

10: (a) 4.32 m/s b) 3.47 x $10^3$ N c) 8.93 kW

PART VIII

# CHAPTER 7 LINEAR MOMENTUM AND COLLISIONS

We use the term momentum in various ways in everyday language, and most of these ways are consistent with its precise scientific definition. We speak of sports teams or politicians gaining and maintaining the momentum to win. We also recognize that momentum has something to do with collisions. For example, looking at the rugby players in the photograph colliding and falling to the ground, we expect their momenta to have great effects in the resulting collisions. Generally, momentum implies a tendency to continue on course—to move in the same direction—and is associated with great mass and speed.

Momentum, like energy, is important because it is conserved. Only a few physical quantities are conserved in nature, and studying them yields fundamental insight into how nature works, as we shall see in our study of momentum.

# 39. 7.1 Linear Momentum and Force

*Summary*

- Define linear momentum.
- Explain the relationship between momentum and force.
- State Newton's second law of motion in terms of momentum.
- Calculate momentum given mass and velocity.

## Linear Momentum

The scientific definition of linear momentum is consistent with most people's intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. **Linear momentum** is defined as the product of a system's mass multiplied by its velocity. In symbols, linear momentum is expressed as

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}.$$

Momentum is directly proportional to the object's mass and also its velocity. Thus the greater an object's mass or the greater its velocity, the greater its momentum. Momentum $\vec{p}$ is a vector having the

same direction as the velocity $\vec{v}$. The SI unit for momentum is **kg · m/s**.

## LINEAR MOMENTUM

Linear momentum is defined as the product of a system's mass multiplied by its velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}.$$

## Example 1: Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

**Strategy**

No information is given regarding direction, and so we can calculate only the magnitude of the momentum, $p$. (As usual, a symbol that is in italics is a magnitude, whereas one that has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be

calculated directly from the definition of momentum given in the equation, which becomes

$$p = mv$$

when only magnitudes are considered.

**Solution for (a)**

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation.

$$p_{\text{player}} = (110\ \text{kg})(8.00\ \text{m/s}) = 880\ \text{kg}\cdot\text{m/s}$$

**Solution for (b)**

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

$$p_{\text{ball}} = (0.410\ \text{kg})(25.0\ \text{m/s}) = 10.3\ \text{kg}\cdot\text{m/s}$$

The ratio of the player's momentum to that of the ball is

$$\frac{p_{\text{player}}}{p_{\text{ball}}} = \frac{880}{10.3} = 85.9.$$

**Discussion**

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

## Momentum and Newton's Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the "quantity of motion." Newton actually stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

$$\vec{\mathbf{F}}_{\mathbf{net}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t},$$

where $\vec{\mathbf{F}}_{\mathbf{net}}$ is the net external force, $\Delta \vec{\mathbf{p}}$ is the change in momentum, and $\Delta t$ is the change in time.

### NEWTON'S SECOND LAW OF MOTION IN TERMS OF MOMENTUM

The net external force equals the change in momentum of a system divided by the time over which it changes.

$$\vec{\mathbf{F}}_{\mathbf{net}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t}$$

## MAKING CONNECTIONS: FORCE AND MOMENTUM

Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in its most broadly applicable form in terms of momentum. Momentum continues to be a key concept in the study of atomic and subatomic particles in quantum mechanics.

This statement of Newton's second law of motion includes the more familiar $\vec{\mathbf{F}}_{\mathbf{net}} = m\vec{\mathbf{a}}$ as a special case. We can derive this form as follows. First, note that the change in momentum $\Delta\vec{\mathbf{p}}$ is given by

$$\Delta\vec{\mathbf{p}} = \Delta(m\vec{\mathbf{v}}).$$

If the mass of the system is constant, then

$$\Delta(m\vec{\mathbf{v}}) = m\Delta\vec{\mathbf{v}}.$$

So that for constant mass, Newton's second law of motion becomes

$$\vec{\mathbf{F}}_{\mathbf{net}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{m\Delta\vec{\mathbf{v}}}{\Delta t}.$$

Because $\dfrac{\Delta\vec{\mathbf{v}}}{\Delta t} = \vec{\mathbf{a}}$, we get the familiar equation

$$\vec{\mathbf{F}}_{\mathbf{net}} = m\vec{\mathbf{a}}$$

*when the mass of the system is constant.*

Newton's second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass. We will consider systems with varying mass in some detail; however, the relationship between momentum and force remains useful when mass is constant, such as in the following example.

## Example 2: Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

**Strategy**

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as

$$\boldsymbol{F_{\text{net}} = \frac{\Delta p}{\Delta t}}.$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta p = m\Delta v = m(v_{\text{f}} - v_{\text{i}}).$$

In this example, the velocity just after impact and the change in time are given; thus, once $\Delta p$ is calculated, $F_{\text{net}} = \frac{\Delta p}{\Delta t}$ can be used to find the force.

**Solution**

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

$$\begin{aligned} \Delta p &= m(v_{\text{f}} - v_{\text{i}}) \\ &= (0.057\ \text{kg})(58\ \text{m/s} - 0\ \text{m/s}) \\ &= 3.306\ \text{kg}\cdot\text{m/s} \approx 3.3\ \text{kg}\cdot\text{m/s} \end{aligned}$$

Now the magnitude of the net external force can determined by using $F_{\text{net}} = \frac{\Delta p}{\Delta t}$:

$$\begin{aligned} F_{\text{net}} &= \frac{\Delta p}{\Delta t} = \frac{3.306\ \text{kg}\cdot\text{m/s}}{5.0\times10^{-3}\ \text{s}} \\ &= 661\ \text{N} \approx 660\ \text{N}, \end{aligned}$$

where we have retained only two significant figures in the final step.

**Discussion**

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the

racquet). This problem could also be solved by first finding the acceleration and then using $\mathbf{F_{net} = ma}$, but one additional step would be required compared with the strategy used in this example.

## Section Summary

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum $\vec{\mathbf{p}}$ is defined to be

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}},$$

  where $\boldsymbol{m}$ is the mass of the system and $\vec{\mathbf{v}}$ is its velocity.
- The SI unit for momentum is $\mathbf{kg \cdot m/s}$.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be

$$\vec{\mathbf{F}}_{\mathbf{net}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t},$$

  $\vec{\mathbf{F}}_{\mathbf{net}}$ is the net external force, $\Delta\vec{\mathbf{p}}$ is the change in momentum, and $\Delta\mathbf{t}$ is the change time.

## *Conceptual Questions*

**1:** An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

**2:** An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

**3: Professional Application**

Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

**4:** How can a small force impart the same momentum to an object as a large force?

## *Problems & Exercises*

**1:** (a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of 7.50 m/s. (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of 600 m/s. (c) What is the

momentum of the 90.0-kg hunter running at 7.40 m/s after missing the elephant?

**2:** (a) What is the mass of a large ship that has a momentum of $1.60 \times 10^9$ kg · m/s, when the ship is moving at a speed of 48.0 km/h? (b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of 1200 m/s.

**3:** (a) At what speed would a $2.00 \times 10^4$-kg airplane have to fly to have a momentum of $1.60 \times 10^9$ kg · m/s (the same as the ship's momentum in the problem above)? (b) What is the plane's momentum when it is taking off at a speed of 60.0 m/s? (c) If the ship is an aircraft carrier that launches these airplanes with a catapult, discuss the implications of your answer to (b) as it relates to recoil effects of the catapult on the ship.

**4:** (a) What is the momentum of a garbage truck that is $1.20 \times 10^4$ kg and is moving at 10.0 m/s? (b) At what speed would an 8.00-kg trash can have the same momentum as the truck?

**5:** A runaway train car that has a mass of 15,000 kg travels at a speed of 5.4 m/s down a track. Compute the time required for a force of 1500 N to bring the car to rest.

**6:** The mass of Earth is $5.972 \times 10^{24}$ kg and its orbital radius is an average of $1.496 \times 10^{11}$ m. Calculate its linear momentum.

## Glossary

**linear momentum**
: the product of mass and velocity

**second law of motion**
: physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

## *Solutions*

**Problems & Exercises**

**1:** (a) $\mathbf{1.50 \times 10^4\ kg \cdot m/s}$ (b) **625 to 1** (c) $\mathbf{6.66 \times 10^2 kg \cdot m/s}$

**3:** (a) $\mathbf{8.00 \times 10^4\ m/s}$ (b) $\mathbf{1.20 \times 10^6\ kg \cdot m/s}$ (c) Because the momentum of the airplane is 3 orders of magnitude smaller than of the ship, the ship will not recoil very much. The recoil would be $\mathbf{-0.0100\ m/s}$, which is probably not noticeable.

**5:** 54 s

# 40. 7.2 Impulse

*Summary*

- Define impulse.
- Describe effects of impulses in everyday life.
- Determine the average effective force using graphical representation.
- Calculate average force and impulse given mass, velocity, and time.

The effect of a force on an object depends on how long it acts, as well as how great the force is. In the game of tennis, a very large force acting for a short time had a great effect on the momentum of the tennis ball. A small force could cause the same **change in momentum**, but it would have to act for a much longer time. For example, if the ball were thrown upward, the gravitational force (which is much smaller than the tennis racquet's force) would eventually reverse the momentum of the ball. Quantitatively, the effect we are talking about is the change in momentum $\boldsymbol{\Delta\vec{p}}$.

By rearranging the equation $\vec{\mathbf{F}}_{\mathbf{net}} = \dfrac{\boldsymbol{\Delta\vec{p}}}{\boldsymbol{\Delta t}}$ to be

$$\boldsymbol{\Delta\vec{p}} = \vec{\mathbf{F}}_{\mathbf{net}}\,\boldsymbol{\Delta t},$$

we can see how the change in momentum equals the average net external force multiplied by the time this force acts. The quantity $\vec{\mathbf{F}}_{\mathbf{net}}\,\boldsymbol{\Delta t}$ is given the name **impulse**. Impulse is the same as the change in momentum.

## IMPULSE: CHANGE IN MOMENTUM

Change in momentum equals the average net external force multiplied by the time this force acts.

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}_{\mathbf{net}}\,\Delta t$$

The quantity $\vec{\mathbf{F}}_{\mathbf{net}}\,\Delta t$ is given the name impulse.

There are many ways in which an understanding of impulse can save lives, or at least limbs. The dashboard padding in a car, and certainly the airbags, allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant, whether an air bag is deployed or not, but the force (to bring the occupant to a stop) will be much less if it acts over a larger time. Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

Bones in a body will fracture if the force on them is too large. If you jump onto the floor from a table, the

force on your legs can be immense if you land stiff-legged on a hard surface. Rolling on the ground after jumping from the table, or landing with a parachute, extends the time over which the force (on you from the ground) acts.

## Example 1: Calculating Magnitudes of Impulses: Two Billiard Balls Striking a Rigid Wall

Two identical billiard balls strike a rigid wall with the same speed, and are reflected without any change of speed. The first ball strikes perpendicular to the wall. The second ball strikes the wall at an angle of **30°** from the perpendicular, and bounces off at an angle of **30°** from perpendicular to the wall.

(a) Determine the direction of the force on the wall due to each ball.

(b) Calculate the ratio of the magnitudes of impulses on the two balls by the wall.

**Strategy for (a)**

In order to determine the force on the wall, consider the force on the ball due to the wall using Newton's second law and then apply Newton's third law to

determine the direction. Assume the $x$-axis to be normal to the wall and to be positive in the initial direction of motion. Choose the $y$-axis to be along the wall in the plane of the second ball's motion. The momentum direction and the velocity direction are the same.

**Solution for (a)**

The first ball bounces directly into the wall and exerts a force on it in the **+x** direction. Therefore the wall exerts a force on the ball in the **−x** direction. The second ball continues with the same momentum component in the **y** direction, but reverses its $x$-component of momentum, as seen by sketching a diagram of the angles involved and keeping in mind the proportionality between velocity and momentum.

These changes mean the change in momentum for both balls is in the **−x** direction, so the force of the wall on each ball is along the **−x** direction.

**Strategy for (b)**

Calculate the change in momentum for each ball, which is equal to the impulse imparted to the ball.

**Solution for (b)**

Let $\boldsymbol{u}$ be the speed of each ball before and after collision with the wall, and $\boldsymbol{m}$ the mass of each ball. Choose the $x$-axis and $y$-axis as previously described, and consider the change in momentum of the first ball which strikes perpendicular to the wall.

$$\boldsymbol{p_{\mathrm{xi}} = mu;\ p_{\mathrm{yi}} = 0}$$
$$\boldsymbol{p_{\mathrm{xf}} = -mu;\ p_{\mathrm{yf}} = 0}$$

Impulse is the change in momentum vector. Therefore the $x$-component of impulse is equal to $\mathbf{-2mu}$ and the $y$-component of impulse is equal to zero.

Now consider the change in momentum of the second ball.

$$p_{xi} = mu \cos 30^0;\ p_{yi} = -mu \sin 30^0$$
$$p_{xf} = -mu \cos 30^0;\ p_{yf} = -mu \sin 30^0$$

It should be noted here that while $p_x$ changes sign after the collision, $p_y$ does not. Therefore the $x$-component of impulse is equal to $\mathbf{-2mu \cos 30°}$ and the $y$-component of impulse is equal to zero.

The ratio of the magnitudes of the impulse imparted to the balls is

$$\frac{2mu}{2mu \cos 30^0} = \frac{2}{\sqrt{3}} = 1.155.$$

**Discussion**

The direction of impulse and force is the same as in the case of (a); it is normal to the wall and along the negative $x$-direction. Making use of Newton's third law, the force on the wall due to each ball is normal to the wall along the positive $x$-direction.

Our definition of impulse includes an assumption that the force is constant over the time interval $\mathbf{\Delta t}$. *Forces are usually not constant.* Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force $\mathbf{F_{eff}}$ that produces the same result as the corresponding time-varying force. Figure 1 shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area

under the curve has units of momentum and is equal to the impulse or change in momentum between times $t_1$ and $t_2$. That area is equal to the area inside the rectangle bounded by $F_{eff}$, $t_1$, and $t_2$. Thus the impulses and their effects are the same for both the actual and effective forces.

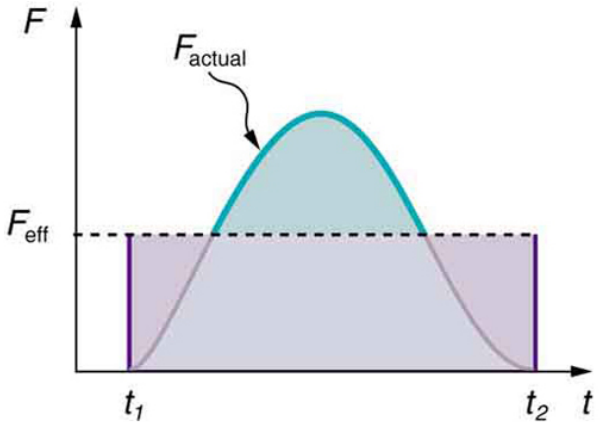



***Figure 1.*** *A graph of force versus time with time along the* $x$*-axis and force along the* $y$*-axis for an actual force and an equivalent effective force. The areas under the two curves are equal.*

## MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—HAND MOVEMENT AND IMPULSE

Try catching a ball while "giving" with the ball, pulling your hands toward your body. Then, try catching a ball while keeping your hands still. Hit water in a tub with your full palm. After the water has settled, hit the water again by diving your hand with your fingers first into the water. (Your full palm represents a swimmer doing a belly flop and your diving hand represents a swimmer doing a dive.) Explain what happens in each case and why. Which orientations would you advise people to avoid and why?

## MAKING CONNECTIONS: CONSTANT FORCE AND CONSTANT ACCELERATION

The assumption of a constant force in the definition

of impulse is analogous to the assumption of a constant acceleration in kinematics. In both cases, nature is adequately described without the use of calculus.

## Section Summary

- Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts:

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}_{\mathbf{net}}\,\Delta t.$$

- Forces are usually not constant over a period of time.

### *Conceptual Questions*

**1: Professional Application**

Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care centre.

**2:** While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?

**3: Professional Application**

Tennis racquets have "sweet spots." If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

## *Problems & Exercises*

**1:** A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder. What is the average force exerted on a 0.0300-kg bullet to accelerate it to a speed of 600 m/s in a time of 2.00 ms (milliseconds)?

**2: Professional Application**

A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seat belt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.

**3:** A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s. (a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg? (b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.

**4: Professional Application**

A professional boxer hits his opponent with a 1000-N horizontal blow that lasts for 0.150 s. (a) Calculate the

impulse imparted by this blow. (b) What is the opponent's final velocity, if his mass is 105 kg and he is motionless in midair when struck near his center of mass? (c) Calculate the recoil velocity of the opponent's 10.0-kg head if hit in this manner, assuming the head does not initially transfer significant momentum to the boxer's body. (d) Discuss the implications of your answers for parts (b) and (c).

**5: Professional Application**

Suppose a child drives a bumper car head on into the side rail, which exerts a force of 4000 N on the car for 0.200 s. (a) What impulse is imparted by this force? (b) Find the final velocity of the bumper car if its initial velocity was 2.80 m/s and the car plus driver have a mass of 200 kg. You may neglect friction between the car and floor.

**6: Professional Application**

One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of $4.00 \times 10^3$ m/s, given the collision lasts $6.00 \times 10^{-8}$ s.

**7: Professional Application**

A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment. (a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm. (b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.

**8: Professional Application**

Military rifles have a mechanism for reducing the recoil forces of the gun on the person firing it. An internal part recoils over a relatively large distance and is stopped by damping mechanisms in the gun. The larger distance reduces the average force needed to stop the internal part. (a) Calculate the recoil velocity of a 1.00-kg plunger that directly interacts with a 0.0200-kg bullet fired at 600 m/s from the gun. (b) If this part is stopped over a distance of 20.0 cm, what average force is exerted upon it by the gun? (c) Compare this to the force exerted on the gun if the bullet is accelerated to its velocity in 10.0 ms (milliseconds).

**9:** A cruise ship with a mass of $1.00 \times 10^7$ kg strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)

**10:** Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of $1.76 \times 10^4$ N.

**11:** Water from a fire hose is directed horizontally against a wall at a rate of 50.0 kg/s and a speed of 42.0 m/s. Calculate the magnitude of the force exerted on the wall, assuming the water's horizontal momentum is reduced to zero.

**12:** A 0.450-kg hammer is moving horizontally at 7.00 m/s

when it strikes a nail and comes to rest after driving the nail 1.00 cm into a board. (a) Calculate the duration of the impact. (b) What was the average force exerted on the nail?

## Glossary

**change in momentum**
the difference between the final and initial momentum; the mass times the change in velocity

**impulse**
the average net external force times the time it acts; equal to the change in momentum

## *Solutions*

**Problems & Exercises**

**1:** $9.00 \times 10^3$ N

**3:** (a) $2.40 \times 10^3$ N toward the leg (b) The force on each hand would have the same magnitude as that found in part (a) (but in opposite directions by Newton's third law) because the change in momentum and the time interval are the same.

**5:** (a) $\mathbf{800\ kg \cdot m/s}$ away from the wall (b) $\mathbf{1.20\ m/s}$ away from the wall

**7:** (a) $1.5 \times 10^6$ N away from the dashboard because a = 20,000 $m/s^2$ or t = 0.0010 seconds

(b) $1.0 \times 10^5$ N away from the dashboard because a = 1,333 $m/s^2$ or t = 0.015 seconds

**9:** $4.69 \times 10^5$ N in the boat's original direction of motion

**11:** $2.10 \times 10^3$ N away from the wall

# 41. 7.3 Conservation of Momentum

*Summary*

- Describe the principle of conservation of momentum.
- Derive an expression for the conservation of momentum.
- Explain conservation of momentum with examples.
- Explain the principle of conservation of momentum as it relates to atomic and subatomic particles.

Momentum is an important quantity because it is conserved. Yet it was not conserved in the examples in Chapter 7.2 Impulse and Chapter 7.1 Linear Momentum and Force, where large changes in momentum were produced by forces acting on the system of interest. Under what circumstances is momentum conserved?

The answer to this question entails considering a sufficiently large system. It is always possible to find a larger system in which total momentum is constant, even if momentum changes for components of the system. If a football player runs into the goalpost in the end zone, there will be a force on him that causes him to bounce backward. However, the Earth also recoils –conserving momentum–because of the force applied to it through the goalpost. Because Earth is many orders of magnitude more massive than the player, its recoil is immeasurably small and can be neglected in any practical sense, but it is real nevertheless.

Consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—for example, one car bumping into another, as shown in Figure 1. Both cars are coasting in the same direction when the lead car (labeled $m_2$) is bumped by the trailing car (labeled $m_1$). The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.

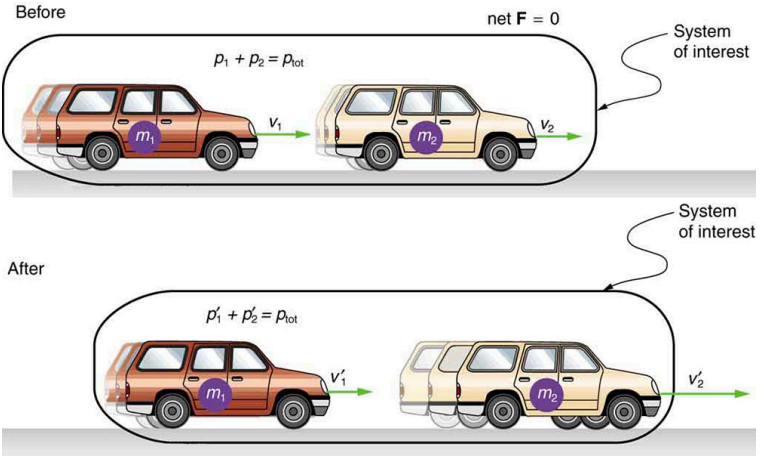



***Figure 1.*** *A car of mass $m_1$ moving with a velocity of $v_1$ bumps into another car of mass $m_2$ and velocity $v_2$ that it is following. As a result, the first car slows down to a velocity of $v'_1$ and the second speeds up to a velocity of $v'_2$. The momentum of each car is changed, but the total momentum $p_{tot}$ of the two cars is the same before and after the collision (if you assume friction is negligible).*

Using the definition of impulse, the change in momentum of car 1 is given by

$$\Delta\vec{p}_1 = \vec{F}_1 \Delta t,$$

where $\vec{F}_1$ is the force on car 1 due to car 2, and $\Delta t$ is the time

the force acts (the duration of the collision). Intuitively, it seems obvious that the collision time is the same for both cars, but it is only true for objects traveling at ordinary speeds. This assumption must be modified for objects travelling near the speed of light, without affecting the result that momentum is conserved.

Similarly, the change in momentum of car 2 is

$$\Delta\vec{\mathbf{p}}_2 = \vec{\mathbf{F}}_2 \Delta t,$$

where $\vec{\mathbf{F}}_2$ is the force on car 2 due to car 1, and we assume the duration of the collision $\Delta t$ is the same for both cars. We know from Newton's third law that $\vec{\mathbf{F}}_1 = -\vec{\mathbf{F}}_2$, and so

$$\Delta\vec{\mathbf{p}}_2 = -\vec{\mathbf{F}}_1 \Delta t = -\Delta\vec{\mathbf{p}}_1.$$

Thus, the changes in momentum are equal and opposite, and

$$\Delta\vec{\mathbf{p}}_1 + \Delta\vec{\mathbf{p}}_2 = 0.$$

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \text{constant},$$
$$\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \vec{\mathbf{p}}_1' + \vec{\mathbf{p}}_2',$$

where $\vec{\mathbf{p}}_1'$ and $\vec{\mathbf{p}}_2'$ are the momenta of cars 1 and 2 after the collision. (We often use primes to denote the final state.)

This result—that momentum is conserved—has validity far beyond the preceding one-dimensional case. It can be similarly shown that total momentum is conserved for any isolated system, with any number of objects in it. In equation form, the **conservation of momentum principle** for an isolated system is written

$$\vec{\mathbf{p}}_{\text{tot}} = \text{constant},$$

or

$$\vec{\mathbf{p}}_{\text{tot}} = \vec{\mathbf{p}}_{\text{tot}}',$$

where $\vec{\mathbf{p}}_{\mathbf{tot}}$ is the total momentum (the sum of the momenta of the individual objects in the system) and $\vec{\mathbf{p}}'_{\mathbf{tot}}$ is the total momentum some time later. (The total momentum can be shown to be the momentum of the center of mass of the system.) An **isolated system** is defined to be one for which the net external force is zero $(\vec{\mathbf{F}}_{\mathbf{net}} = \mathbf{0})$.

### CONSERVATION OF MOMENTUM PRINCIPLE

$$\vec{\mathbf{p}}_{\mathbf{tot}} = \textbf{constant}$$
$$\vec{\mathbf{p}}_{\mathbf{tot}} = \vec{\mathbf{p}}'_{\mathbf{tot}} \textbf{ (isolated system)}$$

### ISOLATED SYSTEM

An isolated system is defined to be one for which the net external force is zero $(\vec{\mathbf{F}}_{\mathbf{net}} = \mathbf{0})$.

Perhaps an easier way to see that momentum is conserved for an isolated system is to consider Newton's second law in terms

of momentum, $\vec{\mathbf{F}}_{\mathbf{net}} = \frac{\Delta \vec{\mathbf{p}}_{\mathbf{tot}}}{\Delta t}$. For an isolated system, $\vec{\mathbf{F}}_{\mathbf{net}} = \mathbf{0}$; thus, $\Delta \vec{\mathbf{p}}_{\mathbf{tot}} = \mathbf{0}$, and $\vec{\mathbf{p}}_{\mathbf{tot}}$ is constant.

We have noted that the three length dimensions in nature–$x$, $y$, and $z$–are independent, and it is interesting to note that momentum can be conserved in different ways along each dimension. For example, during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero and momentum is unchanged. But along the vertical direction, the net vertical force is not zero and the momentum of the projectile is not conserved. (See Figure 2.) However, if the momentum of the projectile-Earth system is considered in the vertical direction, we find that the total momentum is conserved.

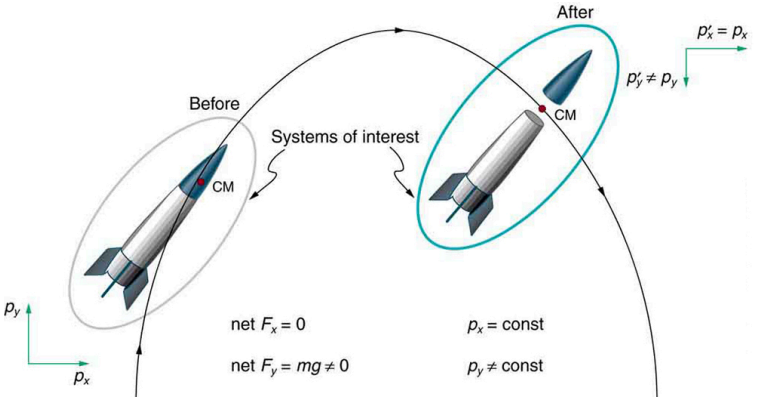



***Figure 2.*** *The horizontal component of a projectile's momentum is conserved if air resistance is negligible, even in this case where a space probe separates. The forces causing the separation are internal to the system, so that the net external horizontal force* $\mathbf{F_{x\text{-}net}}$ *is still zero. The vertical component of the momentum is not conserved, because the net vertical force* $\mathbf{F_{y\text{-}net}}$ *is not zero. In the vertical direction, the space probe-Earth system needs to be considered and we find that the total momentum is conserved. The center of mass of the space probe takes the same path it would if the separation did not occur.*

The conservation of momentum principle can be applied to systems as different as a comet striking Earth and a gas containing huge numbers of atoms and molecules. Conservation of momentum is violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

## MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—DROP OF TENNIS BALL AND A BASEBALL

Hold a tennis ball side by side and in contact with a basketball. Drop the balls together. (Be careful!) What happens? Explain your observations. Now hold the tennis ball above and in contact with the basketball. What happened? Explain your observations. What do you think will happen if the basketball ball is held above and in contact with the tennis ball?

## MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—TWO TENNIS BALLS IN A BALLISTIC TRAJECTORY

Tie two tennis balls together with a string about a foot long. Hold one ball and let the other hang down and throw it in a ballistic trajectory. Explain your observations. Now mark the center of the string with bright ink or attach a brightly colored sticker to it and throw again. What happened? Explain your observations.

Some aquatic animals such as jellyfish move around based on the principles of conservation of momentum. A jellyfish fills its umbrella section with water and then pushes the water out resulting in motion in the opposite direction to that of the jet of water. Squids propel themselves in a similar manner but, in contrast with jellyfish, are able to control the direction in which they move by aiming their nozzle forward or backward. Typical squids can move at speeds of 8 to 12 km/h.

The ballistocardiograph (BCG) was a diagnostic tool used in the second half of the 20th century to study the strength of the heart. About once a second, your heart beats, forcing blood into the aorta. A force in the opposite direction is exerted on the rest of your body

(recall Newton's third law). A ballistocardiograph is a device that can measure this reaction force. This measurement is done by using a sensor (resting on the person) or by using a moving table suspended from the ceiling. This technique can gather information on the strength of the heart beat and the volume of blood passing from the heart. However, the electrocardiogram (ECG or EKG) and the echocardiogram (cardiac ECHO or ECHO; a technique that uses ultrasound to see an image of the heart) are more widely used in the practice of cardiology.

## MAKING CONNECTIONS: CONSERVATION OF MOMENTUM AND COLLISION

Conservation of momentum is quite useful in describing collisions. Momentum is crucial to our understanding of atomic and subatomic particles because much of what we know about these particles comes from collision experiments.

## Subatomic Collisions and Momentum

The conservation of momentum principle not only applies to the macroscopic objects, it is also essential to our explorations of atomic and subatomic particles. Giant machines hurl subatomic particles at one another, and researchers evaluate the results by assuming conservation of momentum (among other things).

On the small scale, we find that particles and their properties are invisible to the naked eye but can be measured with our instruments, and models of these subatomic particles can be constructed to describe the results. Momentum is found to be a property of all subatomic particles including massless particles such as photons that compose light. Momentum being a property of particles hints that momentum may have an identity beyond the description of an object's mass multiplied by the object's velocity. Indeed, momentum relates to wave properties and plays a fundamental role in what measurements are taken and how we take these measurements. Furthermore, we find that the conservation of momentum principle is valid when considering systems of particles. We use this principle to analyze the masses and other properties of previously undetected particles, such as the nucleus of an atom and the existence of quarks that make up particles of nuclei. Figure 3 below illustrates how a particle scattering backward from another implies that its target is massive and dense. Experiments seeking evidence that **quarks** make up protons (one type of particle that makes up nuclei) scattered high-energy electrons off of protons (nuclei of hydrogen atoms). Electrons occasionally scattered straight backward in a manner that implied a very small and very dense particle makes up the proton—this observation is considered nearly direct evidence of quarks. The analysis was based partly on the same conservation of momentum principle that works so well on the large scale.

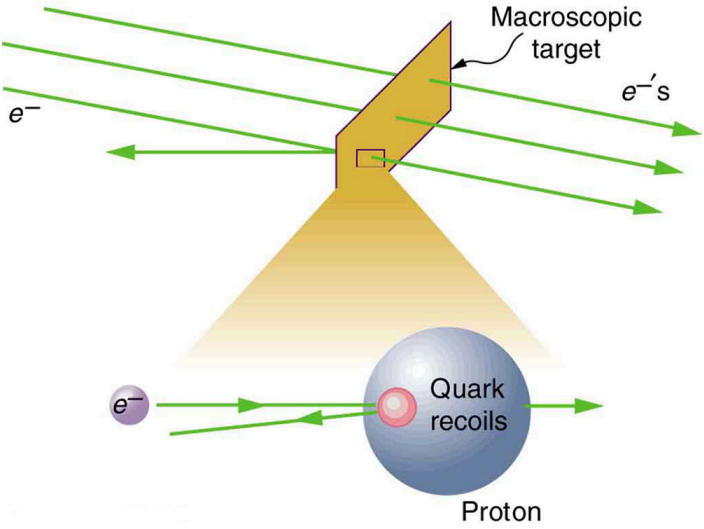



***Figure 3.*** *A subatomic particle scatters straight backward from a target particle. In experiments seeking evidence for quarks, electrons were observed to occasionally scatter straight backward from a proton.*

## Section Summary

- The conservation of momentum principle is written

$$\vec{\mathbf{p}}_{\mathbf{tot}} = \mathbf{constant}$$

or

$$\vec{\mathbf{p}}_{\mathbf{tot}} = \vec{\mathbf{p}}'_{\mathbf{tot}} \ \mathbf{(isolated\ system),}$$

$\vec{\mathbf{p}}_{\mathbf{tot}}$ is the initial total momentum and $\vec{\mathbf{p}}'_{\mathbf{tot}}$ is the total momentum some time later.

- An isolated system is defined to be one for which the net external force is zero $(\vec{\mathbf{F}}_{\mathbf{net}} = \mathbf{0})$.
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because

horizontal forces are zero.

- Conservation of momentum applies only when the net external force is zero.
- The conservation of momentum principle is valid when considering systems of particles.

## *Conceptual Questions*

**1: Professional Application**

If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.

**2:** Under what circumstances is momentum conserved?

**3:** Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?

**4:** Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

**5: Professional Application**

Explain in terms of momentum and Newton's laws how a car's air resistance is due in part to the fact that it pushes air in its direction of motion.

**6:** Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

**7:** Must the total energy of a system be conserved whenever its momentum is conserved? Explain why or why not.

## *Problems & Exercises*

**1: Professional Application**

Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of -0.120 m/s. (The minus indicates direction of motion.) What is their final velocity?

**2:** Suppose a clay model of a koala bear has a mass of 0.200 kg and slides on ice at a speed of 0.750 m/s. It runs into another clay model, which is initially motionless and has a mass of 0.350 kg. Both being soft clay, they naturally stick together. What is their final velocity?

**3: Professional Application**

Consider the following question: A *car moving at* 10 *m/s crashes into a tree and stops in* 0.26 *s. Calculate the force the seatbelt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is* 70 *kg.* Would the answer to this question be different if the car with the 70-kg passenger had collided with a car that has a mass equal to

and is traveling in the opposite direction and at the same speed? Explain your answer.

**4:** What is the velocity of a 900-kg car initially moving at 30.0 m/s, just after it hits a 150-kg deer initially running at 12.0 m/s in the same direction? Assume the deer remains on the car.

**5:** A 1.80-kg falcon catches a 0.650-kg dove from behind in midair. What is their velocity after impact if the falcon's velocity is initially 28.0 m/s and the dove's velocity is 7.00 m/s in the same direction?

## Glossary

**conservation of momentum principle**
: when the net external force is zero, the total momentum of the system is conserved or constant

**isolated system**
: a system in which the net external force is zero

**quark**
: fundamental constituent of matter and an elementary particle

## *Solutions*

**Problems & Exercises**

**1:** 0.122 m/s

**3:** In a collision with an identical car, momentum is conserved. Afterwards $v_f = 0$ for both cars. The change in momentum will be the same as in the crash with the tree. However, the force on the body is not determined since the time is not known. A padded stop will reduce injurious force on body.

**5:** 22.4 m/s in the same direction as the original motion

# 42. 7.4 Elastic Collisions in One Dimension

*Summary*

- Describe an elastic collision of two objects in one dimension.
- Define internal kinetic energy.
- Derive an expression for conservation of internal kinetic energy in a one dimensional collision.
- Determine the final velocities in an elastic collision given masses and initial velocities.

Let us consider various types of two-object collisions. These collisions are the easiest to analyze, and they illustrate many of the physical principles involved in collisions. The conservation of momentum principle is very useful here, and it can be used whenever the net external force on a system is zero.

We start with the elastic collision of two objects moving along the same line—a one-dimensional problem. In a one-dimensional problem we can omit the vector arrows as we are dealing solely with magnitudes. An **elastic collision** is one that also conserves internal kinetic energy. **Internal kinetic energy** is the sum of the kinetic energies of the objects in the system. Figure 1 illustrates an elastic collision in which internal kinetic energy and momentum are conserved.

Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions

can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

## ELASTIC COLLISION

An **elastic collision** is one that conserves internal kinetic energy.

## INTERNAL KINETIC ENERGY

**Internal kinetic energy** is the sum of the kinetic energies of the objects in the system.

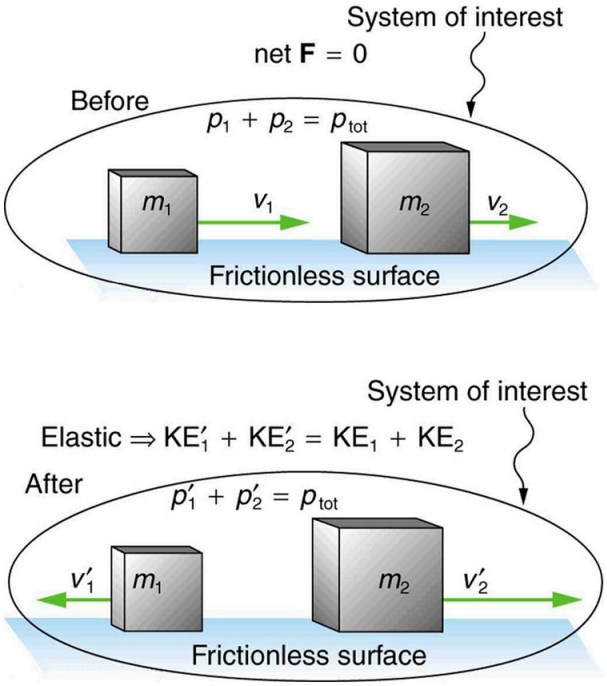



***Figure 1.*** *An elastic one-dimensional two-object collision. Momentum and internal kinetic energy are conserved.*

Now, to solve problems involving one-dimensional elastic collisions between two objects we can use the equations for conservation of momentum and conservation of internal kinetic energy. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$\boldsymbol{p_1 + p_2 = p'_1 + p'_2 \ \ (F_{net} = 0)}$$

or

$$\boldsymbol{m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \ \ (F_{net} = 0),}$$

where the primes (') indicate values after the collision. By definition, an elastic collision conserves internal kinetic energy, and so the

sum of kinetic energies before the collision equals the sum after the collision. Thus,

$$\frac{1}{2}m_1{v_1}^2+\frac{1}{2}m_2{v_2}^2=\frac{1}{2}m_1{v'_1}^2+\frac{1}{2}m_2{v'_2}^2 \quad \textbf{(two-object elastic collision)}$$

expresses the equation for conservation of internal kinetic energy in a one-dimensional collision.

## Example 1: Calculating Velocities Following an Elastic Collision

Calculate the velocities of two objects following an elastic collision, given that

$m_1 = 0.500\ \text{kg},\ m_2 = 3.50\ \text{kg},\ v_1 = 4.00\ \text{m/s},\ \text{and}\ v_2 = 0.$

**Strategy and Concept**

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation shown in Figure 1 where both objects are initially moving. We are asked to find two unknowns (the final velocities $v'_1$ and $v'_2$). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus $v_2 = 0$. Once we simplify these equations, we combine them algebraically to solve for the unknowns.

**Solution**

For this problem, note that $v_2 = 0$ and use conservation of momentum. Thus,

$$p_1 = p_1' + p_2'$$

or

$$m_1 v_1 = m_1 v_1' + m_2 v_2'.$$

Using conservation of internal kinetic energy and that $v_2 = 0$,

$$\frac{1}{2} m_1 {v_1}^2 = \frac{1}{2} m_1 {v_1'}^2 + \frac{1}{2} m_2 {v_2'}^2.$$

Solving the first equation (momentum equation) for $v'_2$, we obtain

$$v_2' = \frac{m_1}{m_2}(v_1 - v_1').$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable $v'_2$, leaving only $v'_1$ as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are

$$v_1' = 4.00 \text{ m/s}$$

and

$$v_1' = -3.00 \text{ m/s}.$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution ($v'_1 = -3.00$ m/s) is negative, meaning

that the first object bounces backward. When this negative value of $v'_1$ is used to find the velocity of the second object after the collision, we get

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1) = \frac{0.500\ \text{kg}}{3.50\ \text{kg}}[4.00 - (-3.00\ \text{m/s})]$$

or

$$v'_2 = 1.00\ \text{m/s}.$$

**Discussion**

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but with a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy as written above can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.

## MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—ICE CUBES AND ELASTIC COLLISION

Find a few ice cubes which are about the same size and a smooth kitchen tabletop or a table with a glass top. Place the ice cubes on the surface several centimeters away from each other. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes. Have you created approximately elastic collisions? Explain the speeds and directions of the ice cubes using momentum.

## PHET EXPLORATIONS: COLLISION LAB

Investigate collisions on an air hockey table. Set up your own experiments: vary the number of discs, masses and initial conditions. Is momentum conserved? Is

kinetic energy conserved? Vary the elasticity and see what happens.

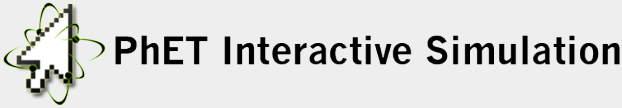



***Figure 2.*** Collision Lab

## Section Summary

- An elastic collision is one that conserves internal kinetic energy.
- Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions.

### *Conceptual Questions*

**1:** What is an elastic collision?

## *Problems & Exercises*

**1:** Two identical objects (such as billiard balls) have a one-dimensional collision in which one is initially motionless. After the collision, the moving object is stationary and the other moves with the same speed as the other originally had. Show that both momentum and kinetic energy are conserved.

**2: Professional Application**

Two manned satellites approach one another at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00 \times 10^3$ kg, and the second a mass of $7.50 \times 10^3$ kg. If the two satellites collide elastically rather than dock, what is their final relative velocity?

**3:** A 70.0-kg ice hockey goalie, originally at rest, catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. Suppose the goalie and the ice puck have an elastic collision and the puck is reflected back in the direction from which it came. What would their final velocities be in this case?

## Glossary

**elastic collision**
a collision that also conserves internal kinetic energy

**internal kinetic energy**
the sum of the kinetic energies of the objects in a system

## *Solutions*

**Problems & Exercises**

**2:** 0.250 m/s

# 43. 7.5 Inelastic Collisions in One Dimension

*Summary*

- Define inelastic collision.
- Explain perfectly inelastic collision.
- Apply an understanding of collisions to sports.
- Determine recoil velocity and loss in kinetic energy given mass and initial velocity.

We have seen that in an elastic collision, internal kinetic energy is conserved. An **inelastic collision** is one in which the internal kinetic energy changes (it is not conserved). This lack of conservation means that the forces between colliding objects may remove or add internal kinetic energy. Work done by internal forces may change the forms of energy within a system. For inelastic collisions, such as when colliding objects stick together, this internal work may transform some internal kinetic energy into heat transfer. Or it may convert stored energy into internal kinetic energy, such as when exploding bolts separate a satellite from its launch vehicle.

## INELASTIC COLLISION

An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).

Figure 1 below shows an example of an inelastic collision. Two objects that have equal masses head toward one another at equal speeds and then stick together. Their total internal kinetic energy is initially $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$. The two objects come to rest after sticking together, conserving momentum. But the internal kinetic energy is zero after the collision. A collision in which the objects stick together is sometimes called a **perfectly inelastic collision** because it reduces internal kinetic energy more than does any other type of inelastic collision. In fact, such a collision reduces internal kinetic energy to the minimum it can have while still conserving momentum.

## PERFECTLY INELASTIC COLLISION

A collision in which the objects stick together is sometimes called “perfectly inelastic.”

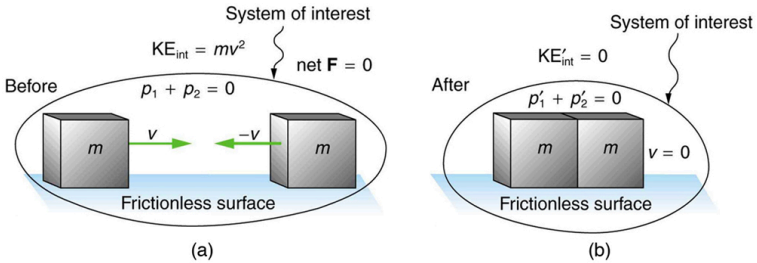



***Figure 1.*** *An inelastic one-dimensional two-object collision. Momentum is conserved, but internal kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward one another at the same speed. (b) The objects stick together (a perfectly inelastic collision), and so their final velocity is zero. The internal kinetic energy of the system changes in any inelastic collision and is reduced to zero in this example.*

## Example 1: Calculating Velocity and

## Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie

(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See Figure 2 )

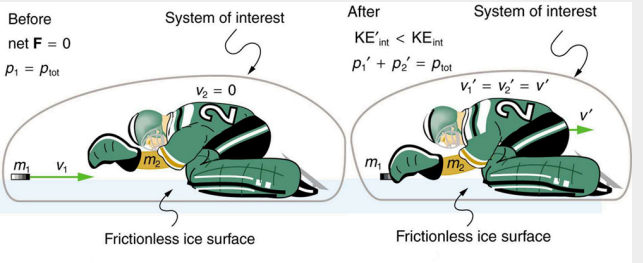



***Figure 2.*** *An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.*

### Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of

the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

**Solution for (a)**

Momentum is conserved because the net external force on the puck-goalie system is zero.

Conservation of momentum is

$$p_1 + p_2 = p_1' + p_2'$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'.$$

Because the goalie is initially at rest, we know $v_2 = 0$. Because the goalie catches the puck, the final velocities are equal, or $v'_1 = v'_2 = v'$. Thus, the conservation of momentum equation simplifies to

$$m_1 v_1 = (m_1 + m_2) v'.$$

Solving for $v'$ yields

$$v' = \frac{m_1}{m_1 + m_2} v_1.$$

Entering known values in this equation, we get

$$v' = \left(\frac{0.150\ \text{kg}}{70.0\ \text{kg} + 0.150\ \text{kg}}\right)(35.0\ \text{m/s}) = 7.48 \times 10^{-2}\ \text{m/s}.$$

**Discussion for (a)**

This recoil velocity is small and in the same direction as the puck's original velocity, as we might expect.

**Solution for (b)**

Before the collision, the internal kinetic energy $\mathbf{KE_{int}}$ of the system is that of the hockey puck, because the goalie is initially at rest. Therefore, $\mathbf{KE_{int}}$ is initially

$$\begin{aligned} \mathbf{KE_{int}} &= \tfrac{1}{2}mv^2 = \tfrac{1}{2}(\mathbf{0.150\ kg})(\mathbf{35.0\ m/s})^2 \\ &= \mathbf{91.9\ J.} \end{aligned}$$

After the collision, the internal kinetic energy is

$$\begin{aligned} \mathbf{KE'_{int}} &= \tfrac{1}{2}(m+M)v^2 = \tfrac{1}{2}(\mathbf{70.15\ kg})(\mathbf{7.48 \times 10^{-2}\ m/s})^2 \\ &= \mathbf{0.196\ J.} \end{aligned}$$

The change in internal kinetic energy is thus

$$\begin{aligned} \mathbf{KE'_{int} - KE_{int}} &= \mathbf{0.196\ J - 91.9\ J} \\ &= \mathbf{-91.7\ J} \end{aligned}$$

where the minus sign indicates that the energy was lost.

**Discussion for (b)**

Nearly all of the initial internal kinetic energy is lost in this perfectly inelastic collision. $\mathbf{KE_{int}}$ is mostly converted to thermal energy and sound.

During some collisions, the objects do not stick together and less of the internal kinetic energy is removed—such as happens in most automobile accidents. Alternatively, stored energy may be converted into internal kinetic energy during a collision. Figure 3 shows a one-dimensional example in which two carts on an air track collide, releasing potential energy from a compressed spring. Example 2 deals with data from such a collision.

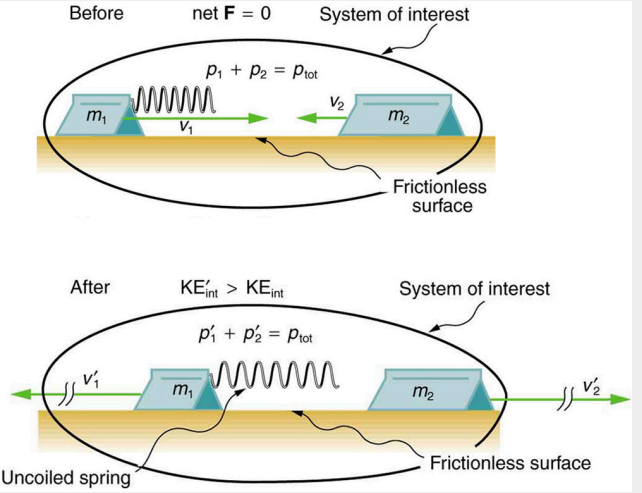



***Figure 3.*** *An air track is nearly frictionless, so that momentum is conserved. Motion is one-dimensional. In this collision, examined in Example 2, the potential energy of a compressed spring is released during the collision and is converted to internal kinetic energy.*

Collisions are particularly important in sports and the sporting and leisure industry utilizes elastic and inelastic collisions. Let us look briefly at tennis. Recall that in a collision, it is momentum and not force that is important. So, a heavier tennis racquet will have the advantage over a lighter one. This conclusion also holds true for other sports—a lightweight bat (such as a softball bat) cannot hit a hardball very far.

The location of the impact of the tennis ball on the racquet is also important, as is the part of the stroke during which the impact occurs. A smooth motion

results in the maximizing of the velocity of the ball after impact and reduces sports injuries such as tennis elbow. A tennis player tries to hit the ball on the "sweet spot" on the racquet, where the vibration and impact are minimized and the ball is able to be given more velocity. Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

## TAKE-HOME EXPERIMENT—BOUNCING OF TENNIS BALL

1. Find a racquet (a tennis, badminton, or other racquet will do). Place the racquet on the floor and stand on the handle. Drop a tennis ball on the strings from a measured height. Measure how high the ball bounces. Now ask a friend to hold the racquet firmly by the handle and drop a tennis ball from the same measured height above the racquet. Measure how high the ball bounces and observe what happens to your friend's hand during the collision. Explain your observations and measurements.
2. The coefficient of restitution (**c**) is a measure of

the elasticity of a collision between a ball and an object, and is defined as the ratio of the speeds after and before the collision. A perfectly elastic collision has a $\boldsymbol{c}$ of 1. For a ball bouncing off the floor (or a racquet on the floor), $\boldsymbol{c}$ can be shown to be $\boldsymbol{c = (h/H)^{1/2}}$ where $\boldsymbol{h}$ is the height to which the ball bounces and $\boldsymbol{H}$ is the height from which the ball is dropped. Determine $\boldsymbol{c}$ for the cases in Part 1 and for the case of a tennis ball bouncing off a concrete or wooden floor ($\boldsymbol{c = 0.85}$ for new tennis balls used on a tennis court).

## Example 2: Calculating Final Velocity and Energy Release: Two Carts Collide

In the collision pictured in Figure 3, two carts collide inelastically. Cart 1 (denoted $\boldsymbol{m_1}$ carries a spring which is initially compressed. During the collision, the spring releases its potential energy and converts it to internal kinetic energy. The mass of cart 1 and the spring is 0.350 kg, and the cart and the spring together have an initial velocity of **2.00 m/s**. Cart 2 (denoted $\boldsymbol{m_2}$ in Figure 3) has a mass of 0.500 kg and an initial velocity of **-0.500 m/s**. After the collision, cart 1 is observed to recoil with a

velocity of **-4.00 m/s**. (a) What is the final velocity of cart 2? (b) How much energy was released by the spring (assuming all of it was converted into internal kinetic energy)?

**Strategy**

We can use conservation of momentum to find the final velocity of cart 2, because $\mathbf{F_{net} = 0}$ (the track is frictionless and the force of the spring is internal). Once this velocity is determined, we can compare the internal kinetic energy before and after the collision to see how much energy was released by the spring.

**Solution for (a)**

As before, the equation for conservation of momentum in a two-object system is

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'.$$

The only unknown in this equation is $v_2'$. Solving for $v_2'$ and substituting known values into the previous equation yields

$$\begin{aligned} v_2' &= \frac{m_1 v_1 + m_2 v_2 - m_1 v_1'}{m_2} \\ &= \frac{(0.350\ \text{kg})(2.00\ \text{m/s}) + (0.500\ \text{kg})(-0.500\ \text{m/s})}{0.500\ \text{kg}} - \frac{(0.350\ \text{kg})(-4.00\ \text{m/s})}{0.500\ \text{kg}} \\ &= \mathbf{3.70\ m/s.} \end{aligned}$$

**Solution for (b)**

The internal kinetic energy before the collision is

$$\begin{aligned} \mathbf{KE}_{\text{int}} &= \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 \\ &= \tfrac{1}{2}(0.350\ \text{kg})(2.00\ \text{m/s})^2 + \tfrac{1}{2}(0.500\ \text{kg})(-0.500\ \text{m/s})^2 \\ &= \mathbf{0.763\ J.} \end{aligned}$$

After the collision, the internal kinetic energy is

$$\begin{aligned} \mathbf{KE}'_{\mathbf{int}} &= \tfrac{1}{2}m_1 {v'_1}^2 + \tfrac{1}{2}m_2 {v'_2}^2 \\ &= \tfrac{1}{2}(\mathbf{0.350\ kg})(\mathbf{-4.00\ m/s})^2 + \tfrac{1}{2}(\mathbf{0.500\ kg})(\mathbf{3.70\ m/s})^2 \\ &= \mathbf{6.22\ J.} \end{aligned}$$

The change in internal kinetic energy is thus

$$\begin{aligned} \mathbf{KE}'_{\mathbf{int}} - \mathbf{KE}_{\mathbf{int}} &= \mathbf{6.22\ J - 0.763\ J} \\ &= \mathbf{5.46\ J.} \end{aligned}$$

**Discussion**

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision. The internal kinetic energy in this collision increases by 5.46 J. That energy was released by the spring.

## Section Summary

- An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).
- A collision in which the objects stick together is sometimes called perfectly inelastic because it reduces internal kinetic energy more than does any other type of inelastic collision.
- Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

## *Conceptual Questions*

**1:** What is an inelastic collision? What is a perfectly inelastic collision?

**2:** Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?

**3:** A small pickup truck that has a camper shell slowly coasts toward a red light with negligible friction. Two dogs in the back of the truck are moving and making various inelastic collisions with each other and the walls. What is the effect of the dogs on the motion of the centre of mass of the system (truck plus entire load)? What is their effect on the motion of the truck?

## *Problems & Exercises*

**1:** A 0.240-kg billiard ball that is moving at 3.00 m/s strikes the bumper of a pool table and bounces straight back at 2.40 m/s (80% of its original speed). The collision lasts 0.0150 s. (a) Calculate the average force exerted on the ball by the bumper. (b) How much kinetic energy in joules is

lost during the collision? (c) What percent of the original energy is left?

**2:** During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater. (a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s? (b) How much kinetic energy is lost?

**3: Professional Application**




*Football player catching a football*

Use mass and speed data from the example earlier in this chapter. A 110.0-kg football player running at 8.00 m/s. A hard-thrown 0.410-kg football has a speed of 25.0 m/s. Assume that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate: (a) the final velocity if the ball and player are going in the same direction and (b) the loss of kinetic energy in this case. Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite

directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?

**4:** A battleship that is $6.00 \times 10^7$ kg and is originally at rest fires a 1100-kg artillery shell horizontally with a velocity of 575 m/s. (a) If the shell is fired straight aft (toward the rear of the ship), there will be negligible friction opposing the ship's recoil. Calculate its recoil velocity. (b) Calculate the increase in internal kinetic energy (that is, for the ship and the shell). This energy is less than the energy released by the gun powder—significant heat transfer occurs.

**5: Professional Application**

Two manned satellites approaching one another, at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00 \times 10^3$ kg, and the second a mass of $7.50 \times 10^3$ kg. (a) Calculate the final velocity (after docking) by using the frame of reference in which the first satellite was originally at rest. (b) What is the loss of kinetic energy in this inelastic collision? (c) Repeat both parts by using the frame of reference in which the second satellite was originally at rest. Explain why the change in velocity is different in the two frames, whereas the change in kinetic energy is the same in both.

**6: Professional Application**

A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it. (a) What is the final velocity of the loaded freight car? (b) How much kinetic energy is lost?

**7: Professional Application**

Space probes may be separated from their launchers by

exploding bolts. (They bolt away from one another.) Suppose a 4800-kg satellite uses this method to separate from the 1500-kg remains of its launcher, and that 5000 J of kinetic energy is supplied to the two parts. What are their subsequent velocities using the frame of reference in which they were at rest before separation?

**8:** A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder. (a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder. (b) How much kinetic energy does the rifle gain? (c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg? (d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation. (e) Calculate the momentum of a 110-kg football player running at 8.00 m/s. Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s. Discuss its relationship to this problem.

**9: Professional Application**

One of the waste products of a nuclear reactor is plutonium-239 ($^{239}$ Pu).] This nucleus is radioactive and decays by splitting into a helium-4 nucleus and a uranium-235 nucleus ($^{4}$He + $^{235}$U), the latter of which is also radioactive and will itself decay some time later. The energy emitted in the plutonium decay is $8.40 \times 10^{-13}$ J and is entirely converted to kinetic energy of the helium and uranium nuclei. The mass of the helium nucleus is 6.68 ×

$10^{-27}$ kg, while that of the uranium is $3.92 \times 10^{-25}$ kg (note that the ratio of the masses is 4 to 235). (a) Calculate the velocities of the two nuclei, assuming the plutonium nucleus is originally at rest. (b) How much kinetic energy does each nucleus carry away? Note that the data given here are accurate to three digits only.

**10: Professional Application**

The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of $5.00 \times 10^{12}$ kg (about a kilometer across) strikes the Moon at a speed of 15.0 km/s. (a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is $7.36 \times 10^{22}$ kg) ? (b) How much kinetic energy is lost in the collision? Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon. (c) In October 2009, NASA crashed a rocket into the Moon, and analyzed the plume produced by the impact. (Significant amounts of water were detected.) Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000 kg and its speed upon impact was 9000 km/h. How does the plume produced alter these results?

**11: Professional Application**

Two football players collide head-on in midair while trying to catch a thrown football. The first player is 95.0 kg and has an initial velocity of 6.00 m/s to the south, while the second player is 115 kg and has an initial velocity of 3.50 m/s to the north. a) What is their velocity just after impact if they cling together? b) What if anything was the change in the kinetic energy of the system?

**12:** What is the speed of a garbage truck that is $1.20 \times 10^4$ kg and is initially moving at 25.0 m/s just after it hits and adheres to a trash can that is 80.0 kg and is initially at rest?

**13:** During a circus act, an elderly performer thrills the crowd by catching a cannon ball shot at him. The cannon ball has a mass of 10.0 kg and the horizontal component of its velocity is 8.00 m/s when the 65.0-kg performer catches it. If the performer is on nearly frictionless roller skates, what is his recoil velocity?

**14:** (a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a velocity of 10.0 m/s, what is the mass of the barbell? (b) How much kinetic energy is gained by this maneuver? (c) Where does the kinetic energy come from?

## Glossary

**inelastic collision**
a collision in which internal kinetic energy is not conserved

**perfectly inelastic collision**
a collision in which the colliding objects stick together

## *Exercises*

**Problems & Exercises**

**1:** (a) 86.4 N perpendicularly away from the bumper (b) 0.389 J (c) 64.0 %

**2:** a) v final = 1.78 m/s because (60.0 kg)(4.00 m/s) = (60.0 kg + 75.0 kg)( v final) KE before = 480 J KE after = 213 J so change in KE = − 194 J

**3:** (a) 8.06 m/s (b) -56.0 J (c) 7.88 m/s (d) − 223 J

**5:** (a) 0.163 m/s in the direction of motion of the more massive satellite (b) 81.6 J (c) $\mathbf{8.70 \times 10^{-2}\ m/s}$ in the direction of motion of the less massive satellite, 81.5 J. Because there are no external forces, the velocity of the centre of mass of the two-satellite system is unchanged by the collision. The two velocities calculated above are the velocity of the centre of mass in each of the two different individual reference frames. The loss in KE is the same in both reference frames because the KE lost to internal forces (heat, friction, etc.) is the same regardless of the coordinate system chosen.

**7:** 0.704 m/s and -2.25 m/s

**8:** (a) 4.58 m/s away from the bullet (b) 31.5 J (c) -0.491 m/s (d) 3.38 J

**10:** (a) $\mathbf{1.02 \times 10^{-6}\ m/s}$ (b) $\mathbf{5.63 \times 10^{20}\ J}$ (almost all KE lost) (c) Recoil speed is $\mathbf{6.79 \times 10^{-17}\ m/s,}$ energy lost is $\mathbf{6.25 \times 10^{9}\ J.}$ The plume will not affect the

momentum result because the plume is still part of the Moon system. The plume may affect the kinetic energy result because a significant part of the initial kinetic energy may be transferred to the kinetic energy of the plume particles.

**11:** a) 0.798 m/s to the south, or positive if you make south positive.

(95.0 kg)(+6.00 m/s) + (115 kg)(-3.50 m/s) = (95.0 kg + 115 kg)(v final)

b) KE before = 2414 = 2410 J  KE after = 66.8 J  so change in KE = − 2350 J if you round off at end, or − 2340 if you round off at each step

**12:** 24.8 m/s

**14:** (a) 4.00 kg (b) 210 J  (c) The clown does work to throw the barbell, so the kinetic energy comes from the muscles of the clown. The muscles convert the chemical potential energy of ATP into kinetic energy.

PART IX

# CHAPTER 8 STATICS AND TORQUE

What might desks, bridges, buildings, trees, and mountains have in common—at least in the eyes of a physicist? The answer is that they are ordinarily motionless relative to the Earth. Furthermore, their acceleration is zero because they remain motionless. That means they also have something in common with a car moving at a constant velocity, because anything with a constant velocity also has an acceleration of zero. Now, the important part—Newton's second law states that net $\vec{F} = m\vec{a}$, and so the net external force is zero for all stationary objects and for all objects moving at constant velocity. There are forces acting, but they are balanced. That is, they are in *equilibrium*.

## STATICS

Statics is the study of forces in equilibrium, a large group of situations that makes up a special case of Newton's second law. We have already considered a few such situations; in this chapter, we cover the topic more thoroughly, including consideration of such possible effects as the rotation and deformation of an object by the forces acting on it.

How can we guarantee that a body is in equilibrium and what can we learn from systems that are in equilibrium? There are actually two conditions that must be satisfied to achieve equilibrium. These conditions are the topics of the first two sections of this chapter.

# 44. 8.1 The First Condition for Equilibrium

*Summary*

- State the first condition of equilibrium.
- Explain static equilibrium.
- Explain dynamic equilibrium.

The first condition necessary to achieve equilibrium is the one already mentioned: the net external force on the system must be zero. Expressed as an equation, this is simply

$$\textbf{net } \vec{\mathbf{F}} = 0$$

Note that if net **F** is zero, then the net external force in *any* direction is zero. For example, the net external forces along the typical $x$– and $y$-axes are zero. This is written as

$$\textbf{net } F_x = 0 \textbf{ and } F_y = 0$$

Figure 1 and Figure 2 illustrate situations where **net F = 0** for both **static equilibrium** (motionless), and **dynamic equilibrium** (constant velocity).

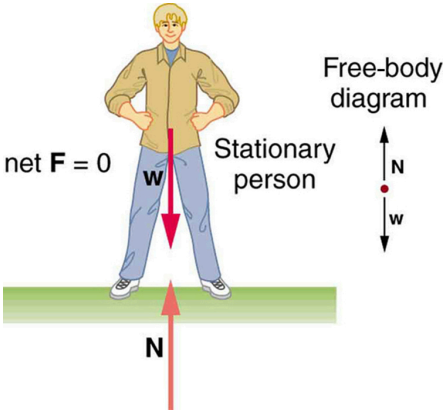



***Figure 1.*** *This motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.*

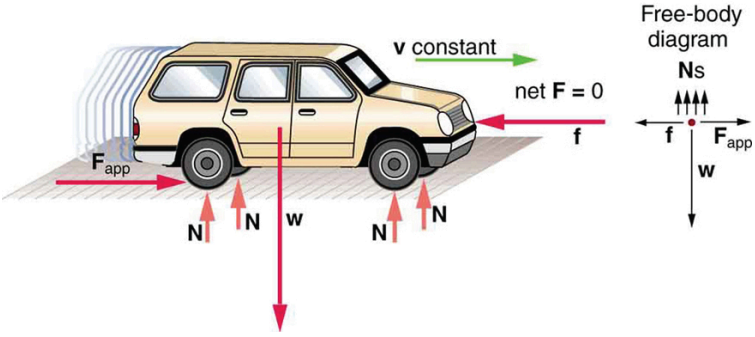



***Figure 2.*** *This car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force* $\mathbf{F_{app}}$ *between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.*

However, it is not sufficient for the net external force of a system to be zero for a system to be in equilibrium. Consider the two situations illustrated in Figure 3 and Figure 4 where forces are

applied to an ice hockey stick lying flat on ice. The net external force is zero in both situations shown in the figure; but in one case, equilibrium is achieved, whereas in the other, it is not. In Figure 3, the ice hockey stick remains motionless. But in Figure 4, with the same forces applied in different places, the stick experiences accelerated rotation. Therefore, we know that the point at which a force is applied is another factor in determining whether or not equilibrium is achieved. This will be explored further in the next section.

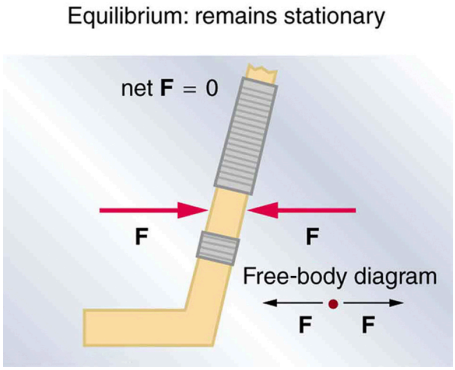



***Figure 3.*** *An ice hockey stick lying flat on ice with two equal and opposite horizontal forces applied to it. Friction is negligible, and the gravitational force is balanced by the support of the ice (a normal force). Thus,* ***net F = 0****. Equilibrium is achieved, which is static equilibrium in this case.*

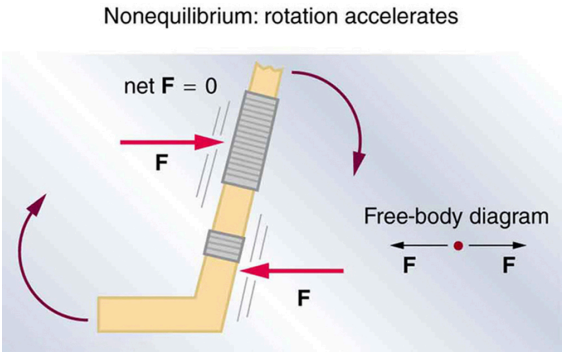



***Figure 4.*** *The same forces are applied at other points and the stick rotates—in fact, it experiences an accelerated rotation. Here* ***net F = 0*** *but the system is not at equilibrium. Hence, the* ***net F = 0*** *is a necessary—but not sufficient—condition for achieving equilibrium.*

## PHET EXPLORATIONS: TORQUE

Investigate how torque causes an object to rotate. Discover the relationships between angular acceleration, moment of inertia, angular momentum and torque.

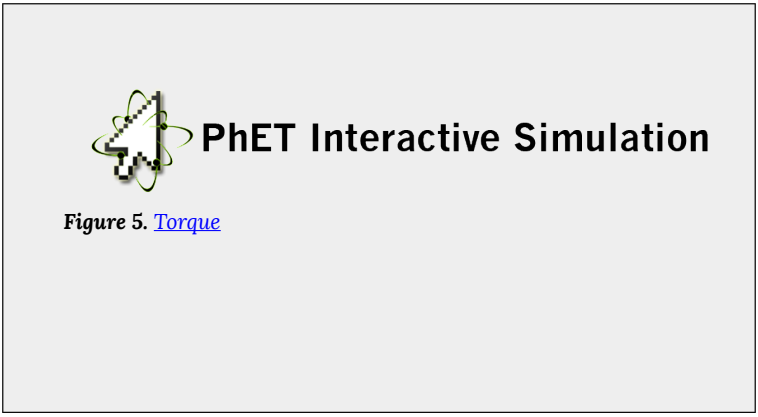



***Figure 5.*** *Torque*

## Section Summary

- Statics is the study of forces in equilibrium.
- Two conditions must be met to achieve equilibrium, which is defined to be motion without linear or rotational acceleration.
- The first condition necessary to achieve equilibrium is that the net external force on the system must be zero, so that **net *F* = 0**.

### *Conceptual Questions*

**1:** What can you say about the velocity of a moving body that is in dynamic equilibrium? Draw a sketch of such a body using clearly labeled arrows to represent all external forces on the body.

**2:** Under what conditions can a rotating body be in equilibrium? Give an example.

## Glossary

**static equilibrium**
: a state of equilibrium in which the net external force and torque acting on a system is zero

**dynamic equilibrium**
: a state of equilibrium in which the net external force and torque on a system moving with constant velocity are zero

# 45. 8.2 The Second Condition for Equilibrium

*Summary*

- State the second condition that is necessary to achieve equilibrium.
- Explain torque and the factors on which it depends.
- Describe the role of torque in rotational mechanics

## TORQUE

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a constant angular velocity. A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it. To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.

Several familiar factors determine how effective you are in opening

the door. See Figure 1. First of all, the larger the force, the more effective it is in opening the door—obviously, the harder you push, the more rapidly the door opens. Also, the point at which you push is crucial. If you apply your force too close to the hinges, the door will open slowly, if at all. Most people have been embarrassed by making this mistake and bumping up against a door when it did not open as quickly as expected. Finally, the direction in which you push is also important. The most effective direction is perpendicular to the door—we push in this direction almost instinctively.

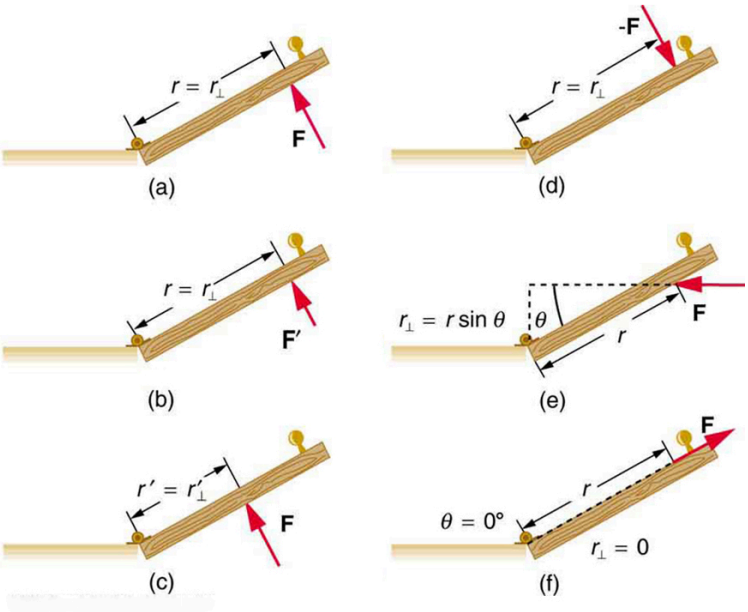



***Figure 1.*** *Torque is the turning or twisting effectiveness of a force, illustrated here for door rotation on its hinges (as viewed from overhead). Torque has both magnitude and direction. (a) Counterclockwise torque is produced by this force, which means that the door will rotate in a counterclockwise due to* **F**. *Note that* $\boldsymbol{r}_\perp$ *is the perpendicular distance of the pivot from the line of action of the force. (b) A smaller counterclockwise torque is produced by a smaller force* **F′** *acting at the same distance from the hinges (the pivot point). (c) The same force as in (a) produces a smaller counterclockwise torque when applied at a smaller distance from the hinges. (d) The same force as in (a), but acting in the opposite direction, produces a clockwise torque. (e) A smaller counterclockwise torque is produced by the same magnitude force acting at the same point but in a different direction. Here,* $\boldsymbol{\theta}$ *is less than* **90°**. *(f) Torque is zero here since the force just pulls on the hinges, producing no rotation. In this case,* $\boldsymbol{\theta} = \mathbf{0°}$.

The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque. **Torque** is the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation

(changing the angular velocity over a period of time). In equation form, the magnitude of torque is defined to be

$$\tau = rF \sin \theta$$

where $\tau$ (the Greek letter tau) is the symbol for torque, $r$ is the distance from the pivot point to the point where the force is applied, $F$ is the magnitude of the force, and $\theta$ is the angle between the force and the vector directed from the point of application to the pivot point, as seen in Figure 1 and Figure 2. An alternative expression for torque is given in terms of the **perpendicular lever arm** $r_\perp$ as shown in Figure 1 and Figure 2, which is defined as

$$r_\perp = r \sin \theta$$

so that

$$\tau = r_\perp F.$$

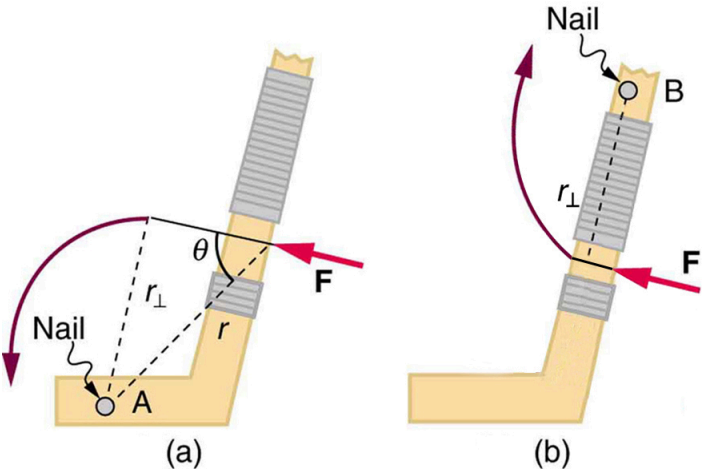



***Figure 2.*** *A force applied to an object can produce a torque, which depends on the location of the pivot point. (a) The three factors* **r**, **F**, *and* $\theta$ *for pivot point* A *on a body are shown here—***r** *is the distance from the chosen pivot point to the point where the force* **F** *is applied, and* $\theta$ *is the angle between* **F** *and the vector directed from the point of application to the pivot point. If the object can rotate around point* A*, it will rotate counterclockwise. This means that torque is counterclockwise relative to pivot* A*. (b) In this case, point* B *is the pivot point. The torque from the applied force will cause a clockwise rotation around point* B*, and so it is a clockwise torque relative to* B*.*

The perpendicular lever arm $r_\perp$ is the shortest distance from the pivot point to the line along which $\vec{\mathbf{F}}$ acts; it is shown as a dashed line in Figure 1 and Figure 2. This line is know was the **line of action** of the force. Note that the line segment that defines the distance $r_\perp$ is perpendicular to $\vec{\mathbf{F}}$, as its name implies. It is sometimes easier to find or visualize $r_\perp$ than to find both $r$ and $\theta$. In such cases, it may be more convenient to use $\tau = r_\perp F$ rather than $\tau = rF \sin\theta$ for torque, but both are equally valid.

The **SI unit of torque** is newtons times meters, usually written as **N·m**. For example, if you push perpendicular to the door with a force of 40 N at a distance of 0.800 m from the hinges, you exert a torque of **32 N·m (0.800 m × 40 N × sin 90°)** relative to the hinges. If you reduce the force to 20 N, the torque is reduced to **16 N·m**, and so on.

The torque is always calculated with reference to some chosen pivot point. For the same applied force, a different choice for the location of the pivot will give you a different value for the torque, since both $r$ and $\theta$ depend on the location of the pivot. Any point in any object can be chosen to calculate the torque about that point. The object may not actually pivot about the chosen "pivot point."

For rotation in a plane, torque has two possible directions. Torque is either clockwise or counterclockwise relative to the chosen pivot point, as illustrated for points B and A, respectively, in Figure 2. If the object can rotate about point A, it will rotate counterclockwise, which means that the torque for the force is shown as counterclockwise relative to A. But if the object can rotate about point B, it will rotate clockwise, which means the torque for the force shown is clockwise relative to B. Also, the magnitude of the torque is greater when the lever arm is longer.

Now, *the second condition necessary to achieve equilibrium* is that *the net external torque on a system must be zero*. An external torque is one that is created by an external force. You can choose the point around which the torque is calculated. The point can be the physical pivot point of a system or any other point in space—but it must be the same point for all torques. If the second condition (net external

torque on a system is zero) is satisfied for one choice of pivot point, it will also hold true for any other choice of pivot point in or out of the system of interest. (This is true only in an inertial frame of reference.) The second condition necessary to achieve equilibrium is stated in equation form as

$$\textbf{net } \boldsymbol{\tau} = \mathbf{0}$$

where net means total. Torques, which are in opposite directions are assigned opposite signs. A common convention is to call counterclockwise (ccw) torques positive and clockwise (cw) torques negative.

## Center of Mass and Center of Gravity

In the previous chapter on Force and Newton's Laws, we made the assumption that all objects were point masses. Clearly, that assumption is no longer valid in problems involving torques and extended objects (i.e., objects that are not considered point masses). As we have seen above, apply/douglasphys1107/part/chapter-4-dynamics-force-and-newtons-laws-of-motion/ing a force typically results in a torque. However, if a force is applied to the **center of mass** of an object (or the line of action of the force runs through the center of mass), the force will only result in linear motion (i.e., the force will not result in any torque). For objects with a uniform mass density, the center of mass is the geometric center of the object. The center of mass is a hypothetical point where it can be assumed all the mass of the object is concentrated to help visualize the linear motion of the object.

A closely related concept is the **center of gravity** of an object, which is the point where it is assumed that the gravitational force acts on the object. *For objects where the acceleration due to gravity does not change over the size of the object, the center of gravity is located at the same point as the center of mass* (this will be true for all

problems you encounter in this book). Depending on the location of the center of gravity in relation to the pivot point, it is possible for the gravitational force to exert a torque. For example, in Figure 3a the center of gravity of the pencil is directly above the pivot point, so it does not exert a force. On the other hand, in Figure 3b the gravitational force exerts a counter clockwise torque. However, in both cases the net force on the pencil is zero.

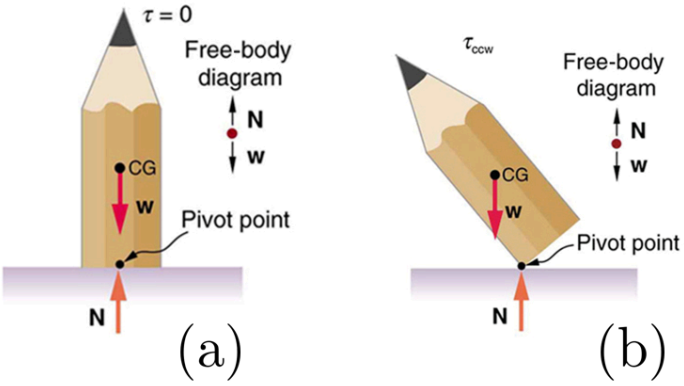



***Figure 3.*** *In both cases, the net force on the pencil is zero. In Figure 3a the center of gravity of the pencil is directly above the pivot point, so it does not exert a torque. In this case, the pencil is in equilibrium. In Figure 3b the gravitational force exerts a counter clockwise torque so the pencil is not in equilibrium.*

## Example 1: She Saw Torques On A

## Seesaw

When two children balance a seesaw as shown in Figure 4, they satisfy the two conditions for equilibrium. Most people have perfect intuition about seesaws, knowing that the lighter child must sit farther from the pivot and that a heavier child can keep a lighter one off the ground indefinitely. The two children shown in Figure 4 are balanced on a seesaw of negligible mass. (This assumption is made to keep the example simple—more involved examples will follow.) The first child has a mass of 26.0 kg and sits 1.60 m from the pivot.(a) If the second child has a mass of 32.0 kg, how far is she from the pivot? (b) What is $\mathbf{F_p}$, the supporting force exerted by the pivot?

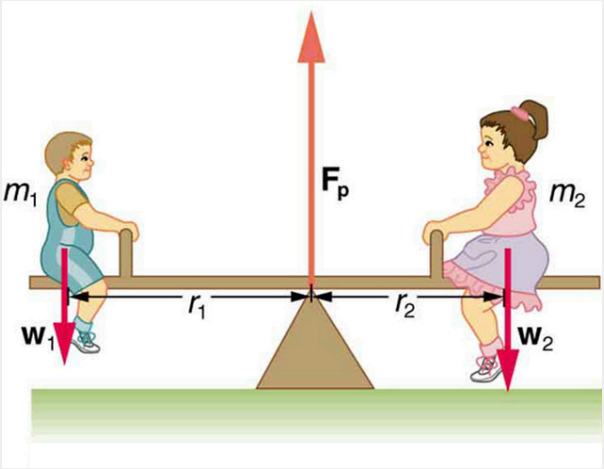



***Figure 4.*** *Two children balancing a seesaw satisfy both conditions for equilibrium. The lighter child sits farther from the pivot to create a torque equal in magnitude to that of the heavier child.*

**Strategy**

Both conditions for equilibrium must be satisfied. In part (a), we are asked for a distance; thus, the second condition (regarding torques) must be used, since the first (regarding only forces) has no distances in it. To apply the second condition for equilibrium, we first identify the system of interest to be the seesaw plus the two children. We take the supporting pivot to be the point about which the torques are calculated. We then identify all external forces acting on the system.

**Solution (a)**

The three external forces acting on the system are the

weights of the two children and the supporting force of the pivot. Let us examine the torque produced by each. Torque is defined to be

$$\tau = rF \sin \theta.$$

Here $\theta = 90°$, so that $\sin \theta = 1$ for all three forces. That means $r_\perp = r$ for all three. The torques exerted by the three forces are first,

$$\tau_1 = r_1 w_1$$

second,

$$\tau_2 = -r_2 w_2$$

and third,

$$\begin{aligned} \tau_p &= r_p F_p \\ &= 0 \cdot F_p \\ &= 0. \end{aligned}$$

Note that a minus sign has been inserted into the second equation because this torque is clockwise and is therefore negative by convention. Since $F_p$ acts directly on the pivot point, the distance $r_p$ is zero. A force acting on the pivot cannot cause a rotation, just as pushing directly on the hinges of a door will not cause it to rotate. Now, the second condition for equilibrium is that the sum of the torques on both children is zero. Therefore

$$\tau_2 = -\tau_1,$$

or

$$r_2 w_2 = r_1 w_1.$$

Weight is mass times the acceleration due to gravity. Entering $mg$ for $w$, we get

$$r_2 m_2 g = r_1 m_1 g.$$

Solve this for the unknown $r_2$:

$$r_2 = r_1 \frac{m_1}{m_2}.$$

The quantities on the right side of the equation are known; thus, $r_2$ is

$$r_2 = (1.60\ \text{m}) \frac{26.0\ \text{kg}}{32.0\ \text{kg}} = 1.30\ \text{m}.$$

As expected, the heavier child must sit closer to the pivot (1.30 m versus 1.60 m) to balance the seesaw.

**Solution (b)**

This part asks for a force $\mathbf{F_p}$. The easiest way to find it is to use the first condition for equilibrium, which is

$$\text{net } \vec{\mathbf{F}} = 0.$$

The forces are all vertical, so that we are dealing with a one-dimensional problem along the vertical axis; hence, the condition can be written as

$$\text{net } F_y = 0$$

where we again call the vertical axis the $y$-axis. Choosing upward to be the positive direction, and using plus and minus signs to indicate the directions of the forces, we see that

$$F_\text{p} - w_1 - w_2 = 0.$$

This equation yields what might have been guessed at the beginning:

$$F_\text{p} = w_1 + w_2.$$

So, the pivot supplies a supporting force equal to the total weight of the system:

$$F_{\mathrm{p}} = m_1 g + m_2 g.$$

Entering known values gives

$$\begin{aligned} F_{\mathrm{p}} &= (26.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) + (32.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) \\ &= 568\ \mathrm{N}. \end{aligned}$$

**Discussion**

The two results make intuitive sense. The heavier child sits closer to the pivot. The pivot supports the weight of the two children. Part (b) can also be solved using the second condition for equilibrium, since both distances are known, but only if the pivot point is chosen to be somewhere other than the location of the seesaw's actual pivot!

Several aspects of the preceding example have broad implications. First, the choice of the pivot as the point around which torques are calculated simplified the problem. Since $\mathbf{F_p}$ is exerted on the pivot point, its lever arm is zero. Hence, the torque exerted by the supporting force $\mathbf{F_p}$ is zero relative to that pivot point. The second condition for equilibrium holds for any choice of pivot point, and so we choose the pivot point to simplify the solution of the problem.

Second, the acceleration due to gravity canceled in this problem, and we were left with a ratio of masses. *This will not always be the case*. Always enter the correct forces–do not jump ahead to enter some ratio of masses.

Third, the weight of each child is distributed over an area of the seesaw, yet we treated the weights as if each force were exerted at a single point. This is not an approximation–the distances $r_1$ and $r_2$ are the distances to points directly below the **center of gravity** of

each child. As we shall see in the next section, the mass and weight of a system can act as if they are located at a single point.

### TAKE-HOME EXPERIMENT

Take a piece of modeling clay and put it on a table, then mash a cylinder down into it so that a ruler can balance on the round side of the cylinder while everything remains still. Put a penny 8 cm away from the pivot. Where would you need to put two pennies to balance? Three pennies?

## Section Summary

- The second condition assures those torques are also balanced. Torque is the rotational equivalent of a force in producing a rotation and is defined to be

$$\boldsymbol{\tau = rF \sin \theta}$$

where $\boldsymbol{\tau}$ is torque, $\boldsymbol{r}$ is the distance from the pivot point to the point where the force is applied, $\boldsymbol{F}$ is the magnitude of the force, and $\boldsymbol{\theta}$ is the angle between $\boldsymbol{F}$ and the vector directed from the point where the force acts to the pivot point. The perpendicular lever arm $\boldsymbol{r_\perp}$ is defined to be

$$\boldsymbol{r_\perp = r \sin \theta}$$

so that

$$\tau = r_{\perp} F.$$

- The perpendicular lever arm $r_{\perp}$ is the shortest distance from the pivot point to the line along which $\mathbf{F}$ acts. The SI unit for torque is newton-meter ($\mathbf{N \cdot m}$). The second condition necessary to achieve equilibrium is that the net external torque on a system must be zero:

$$\mathbf{net}\ \tau = 0$$

  By convention, counterclockwise torques are positive, and clockwise torques are negative.

## *Conceptual Questions*

**1:** What three factors affect the torque created by a force relative to a specific pivot point?

**2:** A wrecking ball is being used to knock down a building. One tall unsupported concrete wall remains standing. If the wrecking ball hits the wall near the top, is the wall more likely to fall over by rotating at its base or by falling straight down? Explain your answer. How is it most likely to fall if it is struck with the same force at its base? Note that this depends on how firmly the wall is attached at its base.

**3:** Mechanics sometimes put a length of pipe over the handle of a wrench when trying to remove a very tight bolt. How does this help? (It is also hazardous since it can break the bolt.)

## *Problems & Exercises*

**1:** (a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?

**2:** When tightening a bolt, you push perpendicularly on a wrench with a force of 165 N at a distance of 0.140 m from the center of the bolt. (a) How much torque are you exerting in newton × meters (relative to the center of the bolt)? (b) Convert this torque to footpounds.

**3:** Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 17.5 N at a distance of 0.600 m from the hinges, and the second child pushes at a distance of 0.450 m. What force must the second child exert to keep the door from moving? Assume friction is negligible.

**4:** Use the second condition for equilibrium (net $\tau = 0$) to calculate $F_p$ in Example 1, employing any data given or solved for in part (a) of the example.

**5:** Repeat the seesaw problem in Example 1 with the center of mass of the seesaw 0.160 m to the left of the pivot (on the side of the lighter child) and assuming a mass of 12.0 kg for the seesaw. The other data given in the example remain unchanged. Explicitly show how you follow the

steps in the Problem-Solving Strategy for static equilibrium.

## Glossary

**torque**
turning or twisting effectiveness of a force

**perpendicular lever arm**
the shortest distance from the pivot point to the line along which FF lies

**SI units of torque**
newton times meters, usually written as N·m**center of mass**
a hypothetical point where it can be assumed all the mass of the object is concentrated to help visualize the linear motion of the object.

**center of gravity**
the point where the total weight of the body is assumed to be concentrated, which is the point where the force of gravity acts

## *Solutions*

**Problems & Exercises**

**1:** (a) $\mathbf{46.8\ N \cdot m}$ (b) It does not matter at what height you push. The torque depends on only the magnitude of the force applied and the perpendicular distance of the force's

application from the hinges. (Children don't have a tougher time opening a door because they push lower than adults, they have a tougher time because they don't push far enough from the hinges.)

**3:** 23.3 N

**5:** Given:

$$\begin{aligned} m_1 &= \mathbf{26.0\ kg},\ m_2 = \mathbf{32.0\ kg},\ m_s = \mathbf{12.0\ kg}, \\ r_1 &= \mathbf{1.60\ m},\ r_s = \mathbf{0.160\ m},\ \textbf{find (a)}r_2,\ \textbf{(b)}F_{\text{p}} \end{aligned}$$

a) Since children are balancing:

$$\begin{aligned} \textbf{net}\ \tau_{\text{cw}} &= -\textbf{net}\ \tau_{\text{ccw}} \\ \Rightarrow w_1 r_1 + m_{\text{s}} g r_{\text{s}} &= w_2 r_2 \end{aligned}$$

So, solving for $r_2$ gives:

$$\begin{aligned} r_2 &= \frac{w_1 r_1 + m_{\text{s}} g r_{\text{s}}}{w_2} = \frac{m_1 g r_1 + m_{\text{s}} g r_{\text{s}}}{m_2 g} = \frac{m_1 r_1 + m_{\text{s}} r_{\text{s}}}{m_2} \\ &= \frac{(\mathbf{26.0\ kg})(\mathbf{1.60\ m}) + (\mathbf{12.0\ kg})(\mathbf{0.160\ m})}{\mathbf{32.0\ kg}} \\ &= \mathbf{1.36\ m} \end{aligned}$$

b) Since the children are not moving:

$$\begin{aligned} \textbf{net}\ \mathbf{F} = 0 &= F_{\text{p}} - w_1 - w_2 - w_s \\ \Rightarrow F_{\text{p}} &= w_1 + w_2 + w_{\text{s}} \end{aligned}$$

So that

$$\begin{aligned} F_{\text{p}} &= (\mathbf{26.0\ kg} + \mathbf{32.0\ kg} + \mathbf{12.0\ kg})(\mathbf{9.80\ m/s^2}) \\ &= \mathbf{686\ N} \end{aligned}$$

# 46. 8.3 Applications of Statics, Including Problem-Solving Strategies

*Summary*

- Discuss the applications of Statics in real life.
- State and discuss various problem-solving strategies in Statics.

Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We begin with a discussion of problem-solving strategies specifically used for statics. Since statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in Chapter 4.7 Problem-Solving Strategies, still apply.

## PROBLEM-SOLVING STRATEGY: STATIC EQUILIBRIUM SITUATIONS

1. The first step is to determine whether or not the system is in **static equilibrium**. This condition is always the case when the *acceleration of the system is zero and accelerated rotation does not occur*.
2. It is particularly important to *draw a free body diagram for the system of interest*. Carefully label all forces, and note their relative magnitudes, directions, and points of application whenever these are known.
3. Solve the problem by applying either or both of the conditions for equilibrium (represented by the equations **net $F = 0$** and **net $\tau = 0$**, depending on the list of known and unknown factors. If the second condition is involved, *choose the pivot point to simplify the solution*. Any pivot point can be chosen, but the most useful ones cause torques by unknown forces to be zero. (Torque is zero if the force is applied at the pivot (then $r = 0$), or along a line through the pivot point (then $\theta = 0$). Always choose a convenient coordinate system for projecting forces.
4. *Check the solution to see if it is reasonable* by examining the magnitude, direction, and units of the answer. The importance of this last step never

diminishes, although in unfamiliar applications, it is usually more difficult to judge reasonableness. These judgments become progressively easier with experience.

Now let us apply this problem-solving strategy for the pole vaulter shown in the three figures below. The pole is uniform and has a mass of 5.00 kg. In Figure 1, the pole's cg lies halfway between the vaulter's hands. It seems reasonable that the force exerted by each hand is equal to half the weight of the pole, or 24.5 N. This obviously satisfies the first condition for equilibrium (**net $F = 0$**). The second condition (**net $\tau = 0$**) is also satisfied, as we can see by choosing the cg to be the pivot point. The weight exerts no torque about a pivot point located at the cg, since it is applied at that point and its lever arm is zero. The equal forces exerted by the hands are equidistant from the chosen pivot, and so they exert equal and opposite torques. Similar arguments hold for other systems where supporting forces are exerted symmetrically about the cg. For example, the four legs of a uniform table each support one-fourth of its weight.

In Figure 1, a pole vaulter holding a pole with its cg halfway between his hands is shown. Each hand exerts a force equal to half the weight of the pole, $\mathbf{F_R = F_L = w/2}$. (b) The pole vaulter moves the pole to his left, and the forces that the hands exert are no longer equal. See Figure 1. If the pole is held with its cg to the left of the person, then he must push down with his right hand and up with his left. The forces he exerts are larger here because they are in opposite directions and the cg is at a long distance from either hand.

Similar observations can be made using a meter stick held at different locations along its length.

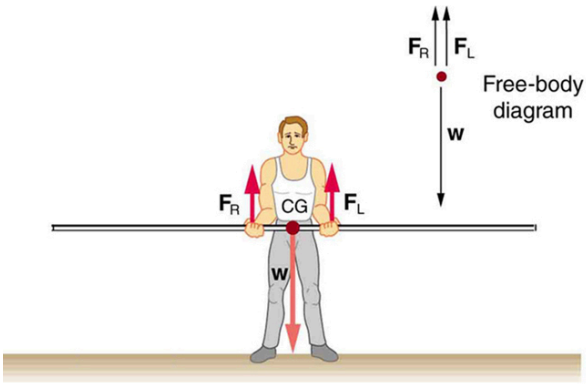



***Figure 1.*** *A pole vaulter holds a pole horizontally with both hands.*

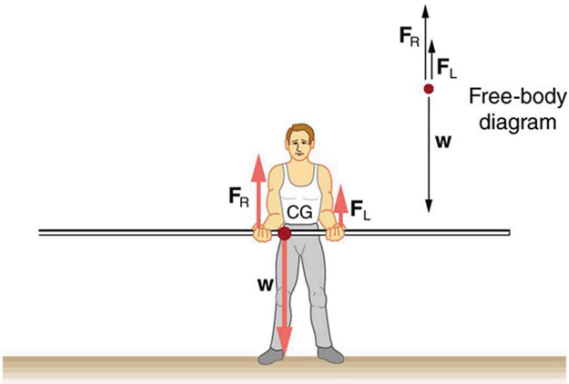



***Figure 2.*** *A pole vaulter is holding a pole horizontally with both hands. The center of gravity is near his right hand.*

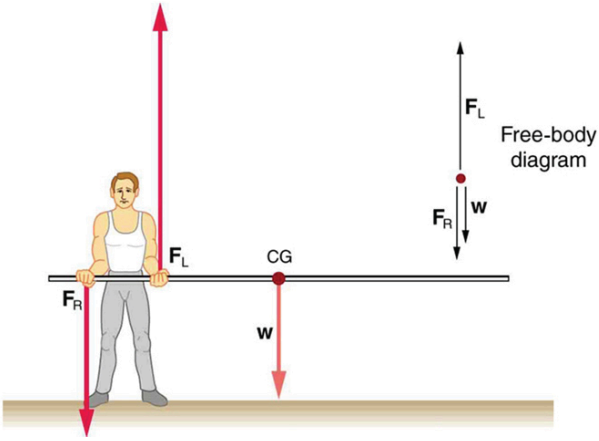



***Figure 3.*** *A pole vaulter is holding a pole horizontally with both hands. The center of gravity is to the left side of the vaulter.*

If the pole vaulter holds the pole as shown in Figure 2, the situation is not as simple. The total force he exerts is still equal to the weight of the pole, but it is not evenly divided between his hands. (If $\mathbf{F_L} = \mathbf{F_R}$, then the torques about the cg would not be equal since the lever arms are different.) Logically, the right hand should support more weight, since it is closer to the cg. In fact, if the right hand is moved directly under the cg, it will support all the weight. This situation is exactly analogous to two people carrying a load; the one closer to the cg carries more of its weight. Finding the forces $\mathbf{F_L}$ and $\mathbf{F_R}$ is straightforward, as the next example shows.

If the pole vaulter holds the pole from near the end of the pole (Figure 3), the direction of the force applied by the right hand of the vaulter reverses its direction.

## Example 1: What Force Is Needed to Support a Weight Held Near Its CG?

For the situation shown in Figure 2, calculate: (a) $\mathbf{F_R}$, the force exerted by the right hand, and (b) $\mathbf{F_L}$, the force exerted by the left hand. The hands are 0.900 m apart, and the cg of the pole is 0.600 m from the left hand.

**Strategy**

Figure 2 includes a free body diagram for the pole, the system of interest. There is not enough information to use the first condition for equilibrium (**net F = 0**), since two of the three forces are unknown and the hand forces cannot be assumed to be equal in this case. There is enough information to use the second condition for equilibrium (**net $\tau$ = 0**) if the pivot point is chosen to be at either hand, thereby making the torque from that hand zero. We choose to locate the pivot at the left hand in this part of the problem, to eliminate the torque from the left hand.

**Solution for (a)**

There are now only two nonzero torques, those from the gravitational force ($\boldsymbol{\tau_w}$) and from the push or pull of the right hand ($\boldsymbol{\tau_R}$). Stating the second condition in terms of clockwise and counterclockwise torques,

$$\textbf{net } \tau_{\textbf{cw}} = -\textbf{ net } \tau_{\textbf{ccw}}.$$

or the algebraic sum of the torques is zero.

Here this is

$$\tau_{\mathrm{R}} = -\tau_{\mathrm{w}}$$

since the weight of the pole creates a counterclockwise torque and the right hand counters with a clockwise torque. Using the definition of torque, $\tau = rF \sin \theta$, noting that $\theta = 90°$, and substituting known values, we obtain

$$(0.900\ \mathrm{m})(F_{\mathrm{R}}) = (0.600\ \mathrm{m})(mg)$$

Thus,

$$\begin{aligned} F_{\mathrm{R}} &= (0.667)(5.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) \\ &= 32.7\ \mathrm{N}. \end{aligned}$$

**Solution for (b)**

The first condition for equilibrium is based on the free body diagram in the figure. This implies that by Newton's second law:

$$F_{\mathrm{L}} + F_{\mathrm{R}} - mg = 0$$

From this we can conclude:

$$F_{\mathrm{L}} + F_{\mathrm{R}} = w = mg$$

Solving for $F_{\mathrm{L}}$, we obtain

$$\begin{aligned} F_{\mathrm{L}} &= mg - F_{\mathrm{R}} \\ &= mg - 32.7\ \mathrm{N} \\ &= (5.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) - 32.7\ \mathrm{N} \\ &= 16.3\ \mathrm{N} \end{aligned}$$

**Discussion**

$F_{\mathrm{L}}$ is seen to be exactly half of $F_{\mathrm{R}}$, as we might have guessed, since $F_{\mathrm{L}}$ is applied twice as far from the cg as $F_{\mathrm{R}}$.

If the pole vaulter holds the pole as he might at the start of a run, shown in Figure 3, the forces change again. Both are considerably greater, and one force reverses direction.

## TAKE-HOME EXPERIMENT

This is an experiment to perform while standing in a bus or a train. Stand facing sideways. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Now stand facing forward. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Why is it easier and safer to stand facing sideways rather than forward? Note: For your safety (and those around you), make sure you are holding onto something while you carry out this activity!

## PHET EXPLORATIONS: BALANCING ACT

Play with objects on a teeter totter to learn about balance. Test what you've learned by trying the Balance Challenge game.

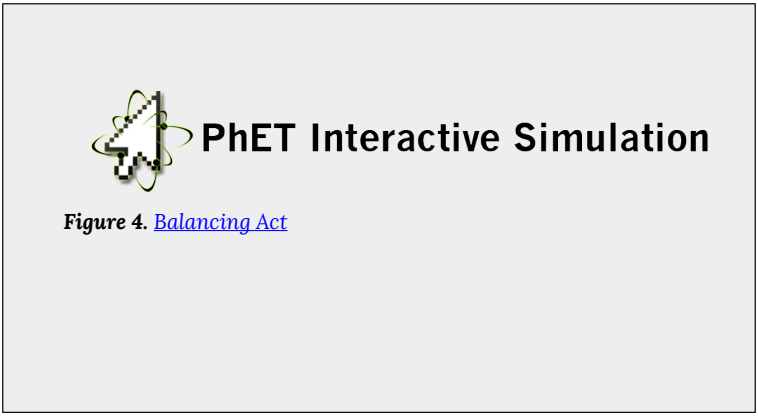



***Figure 4.*** *Balancing Act*

## Summary

- Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We have discussed the problem-solving strategies specifically useful for statics. Statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in Chapter 4.7 Problem-Solving Strategies, still apply.

### *Conceptual Questions*

**1:** When visiting some countries, you may see a person balancing a load on the head. Explain why the center of mass of the load needs to be directly above the person's neck vertebrae.

## *Problems & Exercises*

**1:** To get up on the roof, a person (mass 70.0 kg) places a 6.00-m aluminum ladder (mass 10.0 kg) against the house on a concrete pad with the base of the ladder 2.00 m from the house. The ladder rests against a plastic rain gutter, which we can assume to be frictionless. The center of mass of the ladder is 2 m from the bottom. The person is standing 3 m from the bottom. What are the magnitudes of the forces on the ladder at the top and bottom?

**2:** In Figure 3, the cg of the pole held by the pole vaulter is 2.00 m from the left hand, and the hands are 0.700 m apart. Calculate the force exerted by (a) his right hand and (b) his left hand. (c) If each hand supports half the weight of the pole in Figure 1, show that the second condition for equilibrium (net $\tau = 0$) is satisfied for a pivot other than the one located at the center of gravity of the pole. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium described above.

# Glossary

**static equilibrium**
: equilibrium in which the acceleration of the system is zero and accelerated rotation does not occur

# 47. 8.4 Forces and Torques in Muscles and Joints

*Summary*

- Explain the forces exerted by muscles.
- State how a bad posture causes back strain.
- Discuss the benefits of skeletal muscles attached close to joints.
- Discuss various complexities in the real system of muscles, bones, and joints.

Muscles, bones, and joints are some of the most interesting applications of statics. There are some surprises. Muscles, for example, exert far greater forces than we might think. Figure 1 shows a forearm holding a book and a schematic diagram of an analogous lever system. The schematic is a good approximation for the forearm, which looks more complicated than it is, and we can get some insight into the way typical muscle systems function by analyzing it.

Muscles can only contract, so they occur in pairs. In the arm, the biceps muscle is a flexor—that is, it closes the limb. The triceps muscle is an extensor that opens the limb. This configuration is typical of skeletal muscles, bones, and joints in humans and other vertebrates. Most skeletal muscles exert much larger forces within the body than the limbs apply to the outside world. The reason is clear once we realize that most muscles are attached to bones via tendons close to joints, causing these systems to have mechanical

advantages much less than one. Viewing them as simple machines, the input force is much greater than the output force, as seen below.

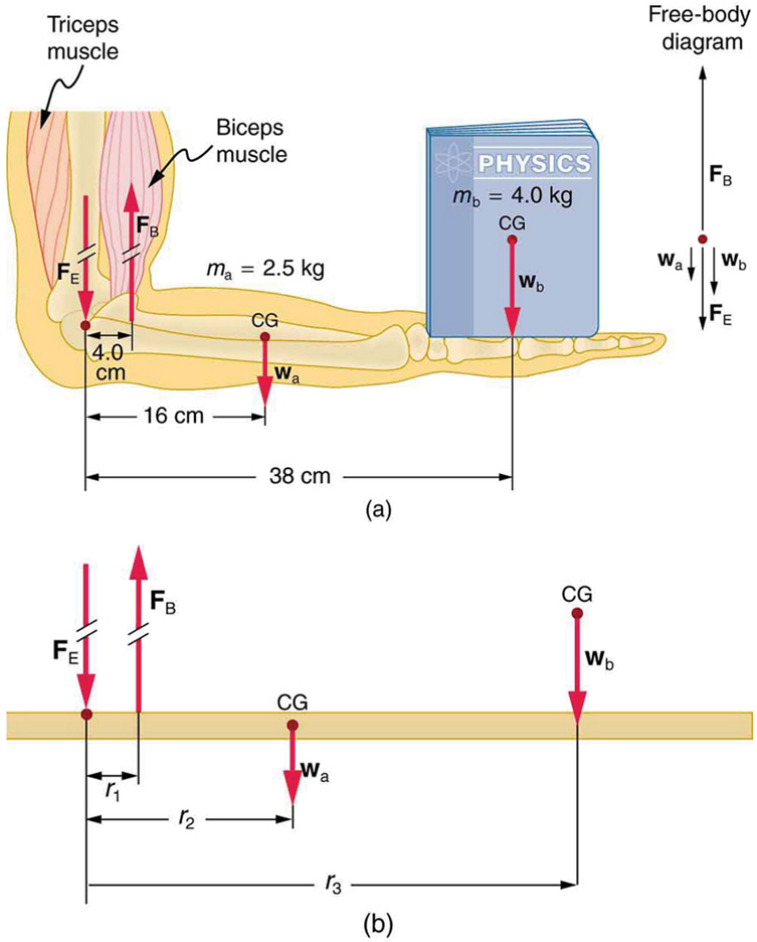



***Figure 1.*** *(a) The figure shows the forearm of a person holding a book. The biceps exert a force* $\mathbf{F_B}$ *to support the weight of the forearm and the book. The triceps are assumed to be relaxed. (b) Here, you can view an approximately equivalent mechanical system with the pivot at the elbow joint as seen in Example 1.*

## Example 1: Muscles Exert Bigger Forces Than You Might Think

Calculate the force the biceps muscle must exert to hold the forearm and its load as shown in Figure 1, and compare this force with the weight of the forearm plus its load. You may take the data in the figure to be accurate to three significant figures.

**Strategy**

There are four forces acting on the forearm and its load (the system of interest). The magnitude of the force of the biceps is $\mathbf{F_B}$; that of the elbow joint is $\mathbf{F_E}$; that of the weights of the forearm is $\boldsymbol{w_a}$, and its load is $\boldsymbol{w_b}$.Two of these are unknown ($\mathbf{F_B}$ and $\mathbf{F_E}$), so that the first condition for equilibrium cannot by itself yield $\mathbf{F_B}$. But if we use the second condition and choose the pivot to be at the elbow, then the torque due to $\mathbf{F_E}$ is zero, and the only unknown becomes $\mathbf{F_B}$.

**Solution**

The torques created by the weights are clockwise relative to the pivot, while the torque created by the biceps is counterclockwise; thus, the second condition for equilibrium (**net $\boldsymbol{\tau}$ = 0**) becomes

$$\boldsymbol{r_2 w_a + r_3 w_b = r_1 F_B}.$$

Note that **sin $\boldsymbol{\theta}$=1** for all forces, since **$\boldsymbol{\theta}$=90°** for all

forces. This equation can easily be solved for $\mathbf{F_B}$ in terms of known quantities, yielding

$$F_{\mathrm{B}} = \frac{r_2 w_{\mathrm{a}} + r_3 w_{\mathrm{b}}}{r_1}.$$

Entering the known values gives

$$F_{\mathrm{B}} = \frac{(0.160\ \mathrm{m})(2.50\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) + (0.380\ \mathrm{m})(4.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{0.0400\ \mathrm{m}}$$

which yields

$$F_{\mathrm{B}} = 470\ \mathrm{N}.$$

Now, the combined weight of the arm and its load is $(6.50\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = 63.7\ \mathrm{N}$, so that the ratio of the force exerted by the biceps to the total weight is

$$\frac{F_{\mathrm{B}}}{w_{\mathrm{a}} + w_{\mathrm{b}}} = \frac{470}{63.7} = 7.38.$$

**Discussion**

This means that the biceps muscle is exerting a force 7.38 times the weight supported.

In the above example of the biceps muscle, the angle between the forearm and upper arm is 90°. If this angle changes, the force exerted by the biceps muscle also changes. In addition, the length of the biceps muscle changes. The force the biceps muscle can exert depends upon its length; it is smaller when it is shorter than when it is stretched.

Very large forces are also created in the joints. In the previous example, the downward force $\mathbf{F_E}$ exerted by the humerus at the elbow joint equals 407 N, or 6.38 times the total weight supported. (The calculation of $\mathbf{F_E}$ is straightforward and is left as an end-of-chapter problem.) Because muscles can contract, but not expand

beyond their resting length, joints and muscles often exert forces that act in opposite directions and thus subtract. (In the above example, the upward force of the muscle minus the downward force of the joint equals the weight supported—that is, **470 N-407 N=63 N**, approximately equal to the weight supported.) Forces in muscles and joints are largest when their load is a long distance from the joint, as the book is in the previous example.

In racquet sports such as tennis the constant extension of the arm during game play creates large forces in this way. The mass times the lever arm of a tennis racquet is an important factor, and many players use the heaviest racquet they can handle. It is no wonder that joint deterioration and damage to the tendons in the elbow, such as "tennis elbow," can result from repetitive motion, undue torques, and possibly poor racquet selection in such sports. Various tried techniques for holding and using a racquet or bat or stick not only increases sporting prowess but can minimize fatigue and long-term damage to the body. For example, tennis balls correctly hit at the "sweet spot" on the racquet will result in little vibration or impact force being felt in the racquet and the body. Twisting the hand to provide top spin on the ball or using an extended rigid elbow in a backhand stroke can also aggravate the tendons in the elbow.

Training coaches and physical therapists use the knowledge of relationships between forces and torques in the treatment of muscles and joints. In physical therapy, an exercise routine can apply a particular force and torque which can, over a period of time, revive muscles and joints. Some exercises are designed to be carried out under water, because this requires greater forces to be exerted, further strengthening muscles. However, connecting tissues in the limbs, such as tendons and cartilage as well as joints are sometimes damaged by the large forces they carry. Often, this is due to accidents, but heavily muscled athletes, such as weightlifters, can tear muscles and connecting tissue through effort alone.

The back is considerably more complicated than the arm or leg, with various muscles and joints between vertebrae, all having

mechanical advantages less than 1. Back muscles must, therefore, exert very large forces, which are borne by the spinal column. Discs crushed by mere exertion are very common. The jaw is somewhat exceptional—the masseter muscles that close the jaw have a mechanical advantage greater than 1 for the back teeth, allowing us to exert very large forces with them. A cause of stress headaches is persistent clenching of teeth where the sustained large force translates into fatigue in muscles around the skull.

Figure 2 shows how bad posture causes back strain. In part (a), we see a person with good posture. Note that her upper body's cg is directly above the pivot point in the hips, which in turn is directly above the base of support at her feet. Because of this, her upper body's weight exerts no torque about the hips. The only force needed is a vertical force at the hips equal to the weight supported. No muscle action is required, since the bones are rigid and transmit this force from the floor. This is a position of unstable equilibrium, but only small forces are needed to bring the upper body back to vertical if it is slightly displaced. Bad posture is shown in part (b); we see that the upper body's cg is in front of the pivot in the hips. This creates a clockwise torque around the hips that is counteracted by muscles in the lower back. These muscles must exert large forces, since they have typically small mechanical advantages. (In other words, the perpendicular lever arm for the muscles is much smaller than for the cg.) Poor posture can also cause muscle strain for people sitting at their desks using computers. Special chairs are available that allow the body's CG to be more easily situated above the seat, to reduce back pain. Prolonged muscle action produces muscle strain. Note that the cg of the entire body is still directly above the base of support in part (b) of Figure 2. This is compulsory; otherwise the person would not be in equilibrium. We lean forward for the same reason when carrying a load on our backs, to the side when carrying a load in one arm, and backward when carrying a load in front of us, as seen in Figure 3.

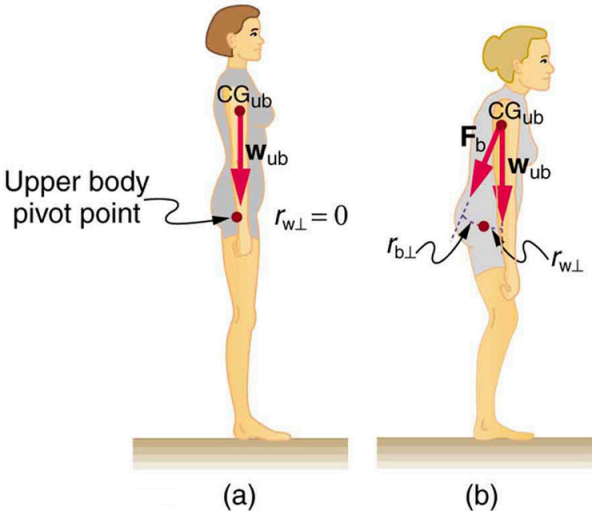



***Figure 2.*** *(a) Good posture places the upper body's cg over the pivots in the hips, eliminating the need for muscle action to balance the body. (b) Poor posture requires exertion by the back muscles to counteract the clockwise torque produced around the pivot by the upper body's weight. The back muscles have a small effective perpendicular lever arm, $\mathbf{r_{b\perp}}$, and must therefore exert a large force $\mathbf{F_b}$. Note that the legs lean backward to keep the cg of the entire body above the base of support in the feet.*

You have probably been warned against lifting objects with your back. This action, even more than bad posture, can cause muscle strain and damage discs and vertebrae, since abnormally large forces are created in the back muscles and spine.

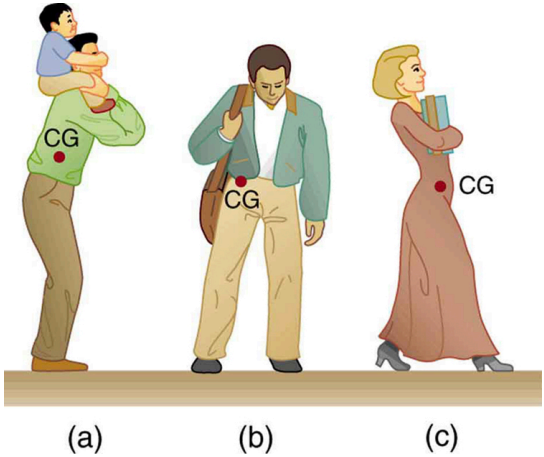



***Figure 3.*** *People adjust their stance to maintain balance. (a) A father carrying his son piggyback leans forward to position their overall cg above the base of support at his feet. (b) A student carrying a shoulder bag leans to the side to keep the overall cg over his feet. (c) Another student carrying a load of books in her arms leans backward for the same reason.*

## Example 2: Do Not Lift with Your Back

Consider the person lifting a heavy box with his back, shown in Figure 4. (a) Calculate the magnitude of the force $\mathbf{F_B}$– in the back muscles that is needed to support the upper body plus the box and compare this with his weight. The mass of the upper body is 55.0 kg and the mass of the box is 30.0 kg. (b) Calculate the magnitude

and direction of the force $\mathbf{F_V}$– exerted by the vertebrae on the spine at the indicated pivot point. Again, data in the figure may be taken to be accurate to three significant figures.

**Strategy**

By now, we sense that the second condition for equilibrium is a good place to start, and inspection of the known values confirms that it can be used to solve for $\mathbf{F_B}$– if the pivot is chosen to be at the hips. The torques created by $\boldsymbol{w_{ub}}$ and $\boldsymbol{w_{box}}$– are clockwise, while that created by $\vec{\mathbf{F}}_{\mathbf{B}}$ – is counterclockwise.

**Solution for (a)**

Using the perpendicular lever arms given in the figure, the second condition for equilibrium (**net $\boldsymbol{\tau}$=0**) becomes

$$(0.350\ \text{m})(55.0\ \text{kg})(9.80\ \text{m/s}^2) + (0.500\ \text{m})(30.0\ \text{kg})(9.80\ \text{m/s}^2) = (0.0800\ \text{m})F_{\text{B}}.$$

Solving for $\mathbf{F_B}$ yields

$$\boldsymbol{F_{\text{B}} = 4.20 \times 10^3\ \text{N}.}$$

The ratio of the force the back muscles exert to the weight of the upper body plus its load is

$$\frac{\boldsymbol{F_{\text{B}}}}{\boldsymbol{w_{\text{ub}} + w_{\text{box}}}} = \frac{\mathbf{4200\ N}}{\mathbf{833\ N}} = \mathbf{5.04.}$$

This force is considerably larger than it would be if the load were not present.

**Solution for (b)**

More important in terms of its damage potential is the force on the vertebrae $\mathbf{F_V}$. The first condition for

equilibrium (**net F=0**) can be used to find its magnitude and direction. Using **y** for vertical and **x** for horizontal, the condition for the net external forces along those axes to be zero

$$\textbf{net } F_y = 0 \textbf{ and net } F_x = 0.$$

Starting with the vertical (**y**) components, this yields

$$F_{\text{Vy}} - w_{\text{ub}} - w_{\text{box}} - F_{\text{B}} \sin 29.0^0 = 0.$$

Thus,

$$\begin{aligned} F_{\text{Vy}} &= w_{\text{ub}} + w_{\text{box}} + F_{\text{B}} \sin 29.0^0 \\ &= 833 \text{ N} + (4200 \text{ N}) \sin 29.0^0 \end{aligned}$$

yielding

$$F_{\text{Vy}} = 2.87 \times 10^3 \text{ N}.$$

Similarly, for the horizontal (**x**) components,

$$F_{\text{Vx}} - F_{\text{B}} \cos 29.0^0 = 0$$

yielding

$$F_{\text{Vx}} = 3.67 \times 10^3 \text{ N}.$$

The magnitude of $\vec{\mathbf{F}}_{\mathbf{V}}$ is given by the Pythagorean theorem:

$$F_{\text{V}} = \sqrt{F_{\text{Vx}}^2 + F_{\text{Vy}}^2} = 4.66 \times 10^3 \text{ N}.$$

The direction of $\vec{\mathbf{F}}_{\mathbf{V}}$ is

$$\theta = \tan^{-1}\left(\frac{F_{\text{Vy}}}{F_{\text{Vx}}}\right) = 38.0^0.$$

Note that the ratio of $F_{\text{V}}$ to the weight supported is

$$\frac{F_{\text{V}}}{w_{\text{ub}} + w_{\text{box}}} = \frac{4660 \text{ N}}{833 \text{ N}} = 5.59.$$

**Discussion**

This force is about 5.6 times greater than it would be if the person were standing erect. The trouble with the back is not so much that the forces are large—because similar forces are created in our hips, knees, and ankles—but that our spines are relatively weak. Proper lifting, performed with the back erect and using the legs to raise the body and load, creates much smaller forces in the back—in this case, about 5.6 times smaller.

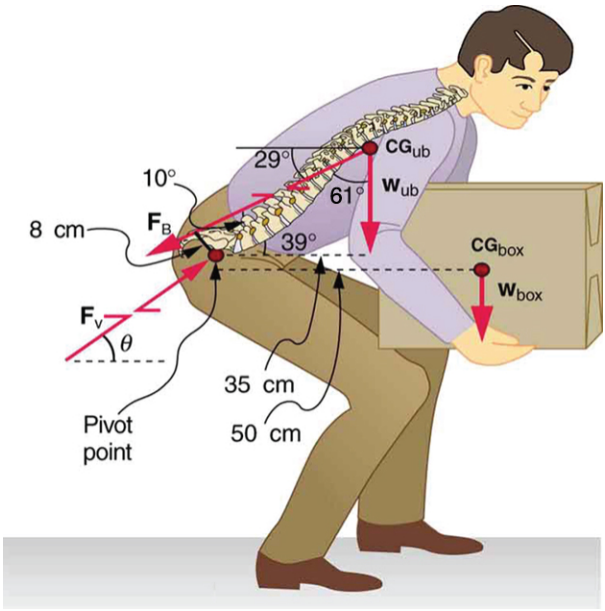



***Figure 4.*** *This figure shows that large forces are exerted by the back muscles and experienced in the vertebrae when a person lifts with their back, since these muscles have small effective perpendicular lever arms. The data shown here are analyzed in the preceding example, Example 2.*

What are the benefits of having most skeletal muscles attached so close to joints? One advantage is speed because small muscle contractions can produce large movements of limbs in a short period of time. Other advantages are flexibility and agility, made possible by the large numbers of joints and the ranges over which they function. For example, it is difficult to imagine a system with biceps muscles attached at the wrist that would be capable of the broad range of movement we vertebrates possess.

There are some interesting complexities in real systems of muscles, bones, and joints. For instance, the pivot point in many joints changes location as the joint is flexed, so that the perpendicular lever arms and the mechanical advantage of the

system change, too. Thus the force the biceps muscle must exert to hold up a book varies as the forearm is flexed. Similar mechanisms operate in the legs, which explain, for example, why there is less leg strain when a bicycle seat is set at the proper height. The methods employed in this section give a reasonable description of real systems provided enough is known about the dimensions of the system. There are many other interesting examples of force and torque in the body—a few of these are the subject of end-of-chapter problems.

## Section Summary

- Statics plays an important part in understanding everyday strains in our muscles and bones.
- Many lever systems in the body have a mechanical advantage of significantly less than one, as many of our muscles are attached close to joints.
- Someone with good posture stands or sits in such as way that their centre of gravity lies directly above the pivot point in their hips, thereby avoiding back strain and damage to disks.

### *Conceptual Questions*

**1:** Why are the forces exerted on the outside world by the limbs of our bodies usually much smaller than the forces exerted by muscles inside the body?

**2:** Explain why the forces in our joints are several times larger than the forces we exert on the outside world with

our limbs. Can these forces be even greater than muscle forces?

**3:** Certain types of dinosaurs were bipedal (walked on two legs). What is a good reason that these creatures invariably had long tails if they had long necks?

**4:** Swimmers and athletes during competition need to go through certain postures at the beginning of the race. Consider the balance of the person and why start-offs are so important for races.

**5:** If the maximum force the biceps muscle can exert is 1000 N, can we pick up an object that weighs 1000 N? Explain your answer.

**6:** Suppose the biceps muscle was attached through tendons to the upper arm close to the elbow and the forearm near the wrist. What would be the advantages and disadvantages of this type of construction for the motion of the arm?

**7:** Explain one of the reasons why pregnant women often suffer from back strain late in their pregnancy.

## *Problems & Exercises*

**1:** Verify that the force in the elbow joint in Example 1 is 407 N, as stated in the text.

**2:** Two muscles in the back of the leg pull on the Achilles tendon as shown in Figure 5. What total force do they exert?

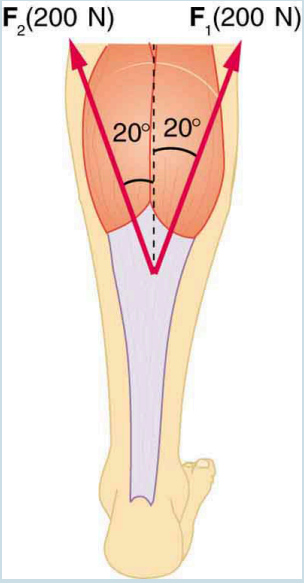



***Figure 5.*** *The Achilles tendon of the posterior leg serves to attach plantaris, gastrocnemius, and soleus muscles to calcaneus bone.*

**3:** The upper leg muscle (quadriceps) exerts a force of 1250 N, which is carried by a tendon over the kneecap (the patella) at the angles shown in Figure 6. Find the direction and magnitude of the force exerted by the kneecap on the upper leg bone (the femur).

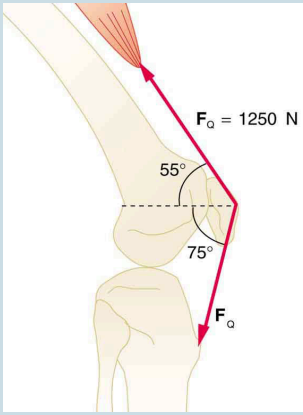



***Figure 6.*** *The knee joint works like a hinge to bend and straighten the lower leg. It permits a person to sit, stand, and pivot.*

**4:** A device for exercising the upper leg muscle is shown in Figure 7, together with a schematic representation of an equivalent lever system. Calculate the force exerted by the upper leg muscle to lift the mass at a constant speed. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium in Chapter 8.3 Applications of Statistics, Including Problem-Solving Strategies.

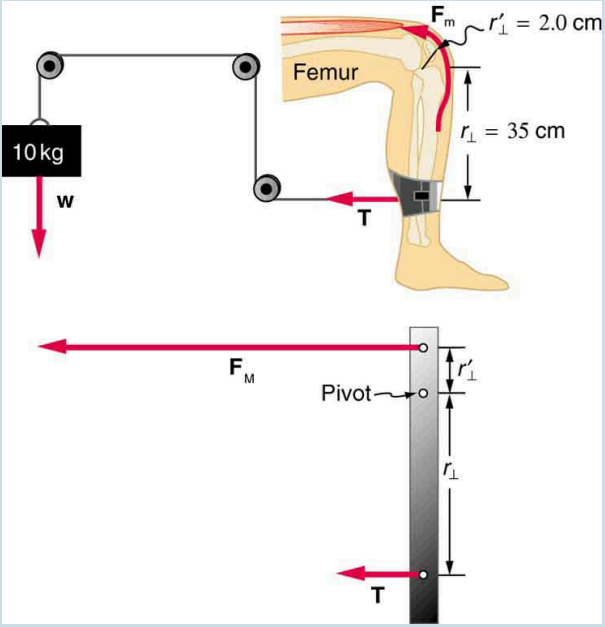



***Figure 7.*** *A mass is connected by pulleys and wires to the ankle in this exercise device.*

**5:** A person working at a drafting board may hold her head as shown in Figure 8, requiring muscle action to support the head. The three major acting forces are shown. Calculate the direction and magnitude of the force supplied by the upper vertebrae $F_V$ to hold the head stationary, assuming that this force acts along a line through the center of mass as do the weight and muscle force.

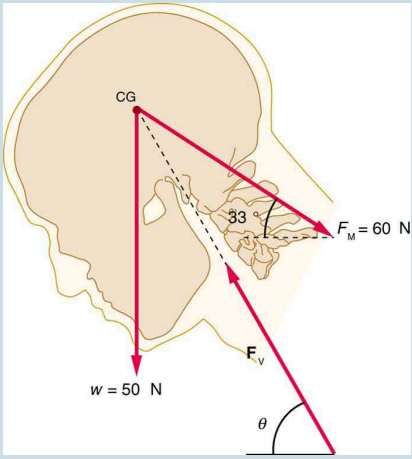



***Figure 8.***

**6:** We analyzed the biceps muscle example with the angle between forearm and upper arm set at 90°. Using the same numbers as in Example 1, find the force exerted by the biceps muscle when the angle is 120° and the forearm is in a downward position.

**7:** Even when the head is held erect, as in Figure 9, its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles at the back of the neck should therefore exert a force to keep the head erect. That is why your head falls forward when you fall asleep in the class. (a) Calculate the force exerted by these muscles using the information in the figure. (b) What is the force exerted by the pivot on the head?

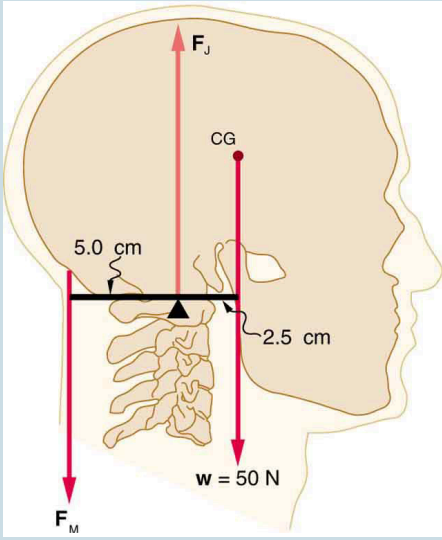



***Figure 9.*** *The center of mass of the head lies in front of its major point of support, requiring muscle action to hold the head erect. A simplified lever system is shown.*

**8:** A 75-kg man stands on his toes by exerting an upward force through the Achilles tendon, as in Figure 10. (a) What is the force in the Achilles tendon if he stands on one foot? (b) Calculate the force at the pivot of the simplified lever system shown—that force is representative of forces in the ankle joint.

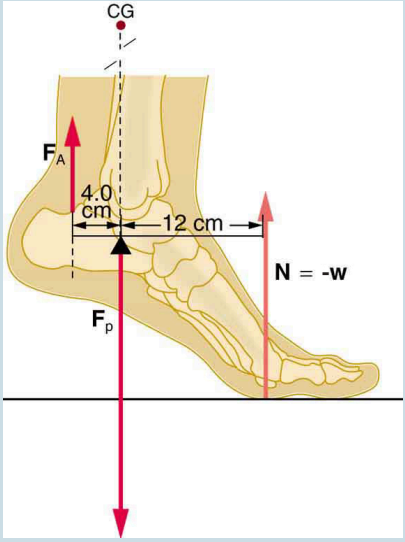



***Figure 10.*** *The muscles in the back of the leg pull the Achilles tendon when one stands on one's toes. A simplified lever system is shown.*

**9:** A father lifts his child as shown in Figure 11. What force should the upper leg muscle exert to lift the child at a constant speed?

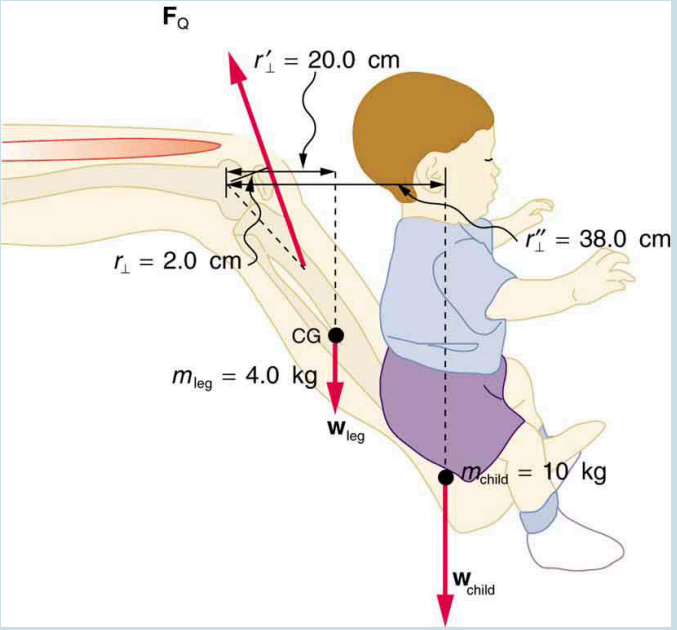



***Figure 11.*** *A child being lifted by a father's lower leg.*

**10:** Unlike most of the other muscles in our bodies, the masseter muscle in the jaw, as illustrated in Figure 12, is attached relatively far from the joint, enabling large forces to be exerted by the back teeth. (a) Using the information in the figure, calculate the force exerted by the lower teeth on the bullet. (b) Calculate the force on the joint.

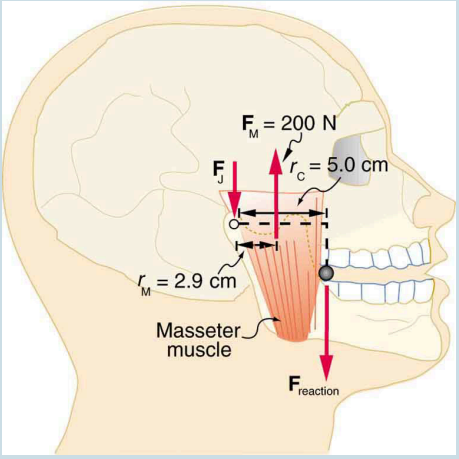



***Figure 12.*** *A person clenching a bullet between his teeth.*

**11: Integrated Concepts**

Suppose we replace the 4.0-kg book in Exercise 6 of the biceps muscle with an elastic exercise rope that obeys Hooke's Law. Assume its force constant $k$=600 N/m. (a) How much is the rope stretched (past equilibrium) to provide the same force $F_B$ as in this example? Assume the rope is held in the hand at the same location as the book. (b) What force is on the biceps muscle if the exercise rope is pulled straight up so that the forearm makes an angle of 25° with the horizontal? Assume the biceps muscle is still perpendicular to the forearm.

**12:** (a) What force should the woman in Figure 13 exert on the floor with each hand to do a push-up? Assume that she moves up at a constant speed. (b) The triceps muscle at the

back of her upper arm has an effective lever arm of 1.75 cm, and she exerts force on the floor at a horizontal distance of 20.0 cm from the elbow joint. Calculate the magnitude of the force in each triceps muscle, and compare it to her weight. (c) How much work does she do if her center of mass rises 0.240 m? (d) What is her useful power output if she does 25 pushups in one minute?

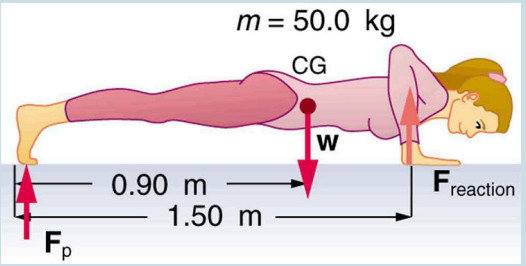



***Figure 13.*** *A woman doing pushups.*

**13:** You have just planted a sturdy 2-m-tall palm tree in your front lawn for your mother's birthday. Your brother kicks a 500 g ball, which hits the top of the tree at a speed of 5 m/s and stays in contact with it for 10 ms. The ball falls to the ground near the base of the tree and the recoil of the tree is minimal. (a) What is the force on the tree? (b) The length of the sturdy section of the root is only 20 cm. Furthermore, the soil around the roots is loose and we can assume that an effective force is applied at the tip of the 20 cm length. What is the effective force exerted by the end of the tip of the root to keep the tree from toppling? Assume the tree will be uprooted rather than bend. (c) What could

you have done to ensure that the tree does not uproot easily?

**14: Unreasonable Results**

Suppose two children are using a uniform seesaw that is 3.00 m long and has its centre of mass over the pivot. The first child has a mass of 30.0 kg and sits 1.40 m from the pivot. (a) Calculate where the second 18.0 kg child must sit to balance the seesaw. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

**15: Construct Your Own Problem**

Consider a method for measuring the mass of a person's arm in anatomical studies. The subject lies on her back, extends her relaxed arm to the side and two scales are placed below the arm. One is placed under the elbow and the other under the back of her hand. Construct a problem in which you calculate the mass of the arm and find its centre of mass based on the scale readings and the distances of the scales from the shoulder joint. You must include a free body diagram of the arm to direct the analysis. Consider changing the position of the scale under the hand to provide more information, if needed. You may wish to consult references to obtain reasonable mass values.

## *Solutions*

### Problems & Exercises

**1:**

$$\begin{aligned} F_{\mathrm{B}} &= 470\ \mathrm{N}; r_1 = 4.00\ \mathrm{cm}; w_{\mathrm{a}} = 2.50\ \mathrm{kg}; r_2 = 16.0\ \mathrm{cm}; w_{\mathrm{b}} = 4.00\ \mathrm{kg}; r_3 = 38.0\ \mathrm{cm} \\ F_{\mathrm{E}} &= w_{\mathrm{a}}\left(\tfrac{r_2}{r_1} - 1\right) + w_{\mathrm{b}}\left(\tfrac{r_3}{r_1} - 1\right) \\ &= (2.50\ \mathrm{kg})(9.80\ \mathrm{m/s^2})\left(\tfrac{16.0\ \mathrm{cm}}{4.0\ \mathrm{cm}} - 1\right) \\ &\quad + (4.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2})\left(\tfrac{38.0\ \mathrm{cm}}{4.00\ \mathrm{cm}} - 1\right) \\ &= 407\ \mathrm{N} \end{aligned}$$

**3:** $\mathbf{1.1 \times 10^3\ N,\ \theta = 190^0\ ccw\ from\ positive\ \mathit{x}\ axis}$

**5:** $\boldsymbol{F_{\mathrm{V}} = 97\ \mathrm{N}, \theta = 59^0}$

**7:** (a) 25 N downward (b) 75 N upward

**8:** (a) $\boldsymbol{F_{\mathrm{A}} = 2.21 \times 10^3\ \mathrm{N}}$ upward (b) $\boldsymbol{F_{\mathrm{B}} = 2.94 \times 10^3\ \mathrm{N}}$ downward

**10:** (a) $\boldsymbol{F_{\text{teeth on bullet}} = 1.2 \times 10^2\ \mathrm{N}}$ upward (b) $\boldsymbol{F_{\mathrm{J}} = 84\ \mathrm{N}}$ downward

**12:** (a) 147 N downward (b) 1690 N or 3.4 times her weight (c) 118 J (d) 49.0 W

**14:** (a) The second child would have to sit 2.33 metres from the pivot point (b) The seesaw is 3.0 m long, and hence, there is only 1.50 m of board on the other side of the pivot. The second child is off the board. (c) The position of the first child must be shortened, i.e. brought closer to the pivot.

# 48. 8.5 Optional: Simple Machines

*Summary*

- Describe different simple machines.
- Calculate the mechanical advantage.

Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we apply the force. The word for "machine" comes from the Greek word meaning "to help make things easier." Levers, gears, pulleys, wedges, and screws are some examples of machines. Energy is still conserved for these devices because a machine cannot do more work than the energy put into it. However, machines can reduce the input force that is needed to perform the job. The ratio of output to input force magnitudes for any simple machine is called its **mechanical advantage** (MA).

$$MA = (F_{output}) / (F_{input})$$

One of the simplest machines is the lever, which is a rigid bar pivoted at a fixed place called the fulcrum. Torques are involved in levers, since there is rotation about a pivot point. Distances from the physical pivot of the lever are crucial, and we can obtain a useful expression for the MA in terms of these distances.

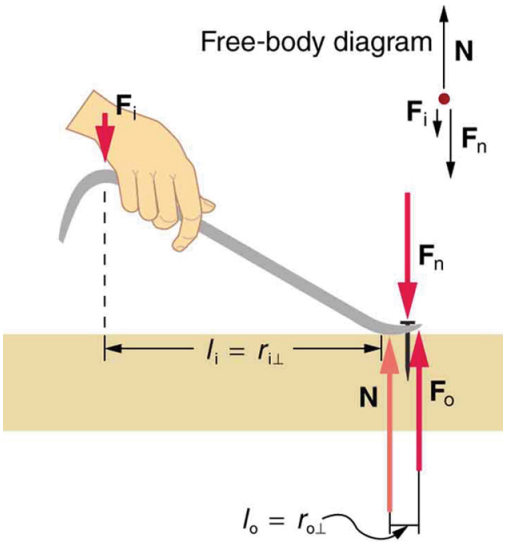



***Figure 1.*** *A nail puller is a lever with a large mechanical advantage. The external forces on the nail puller are represented by solid arrows. The force that the nail puller applies to the nail (**$F_o$**) is not a force on the nail puller. The reaction force the nail exerts back on the puller (**$F_n$**) is an external force and is equal and opposite to **$F_o$**. The perpendicular lever arms of the input and output forces are **$l_i$** and **$l_o$**.*

Figure 1 shows a lever type that is used as a nail puller. Crowbars, seesaws, and other such levers are all analogous to this one. $\vec{\mathbf{F}}_\mathbf{i}$ is the input force and $\vec{\mathbf{F}}_\mathbf{o}$ is the output force. There are three vertical forces acting on the nail puller (the system of interest) – these are $\vec{\mathbf{F}}_\mathbf{i}$, $\vec{\mathbf{F}}_\mathbf{o}$ and $\vec{\mathbf{N}}$. $\vec{\mathbf{F}}_\mathbf{n}$ is the reaction force back on the system, equal and opposite to $\vec{\mathbf{F}}_\mathbf{o}$. (Note that $\vec{\mathbf{F}}_\mathbf{o}$ is not a force on the system.) $\vec{\mathbf{N}}$ is the normal force upon the lever, and its torque is zero since it is exerted at the pivot. The torques due to $\vec{\mathbf{F}}_\mathbf{i}$ and $\vec{\mathbf{F}}_\mathbf{n}$ must be equal to each other if the nail is not moving, to satisfy the second

condition for equilibrium (**net τ = 0**). (In order for the nail to actually move, the torque due to $\vec{\mathbf{F}}_{\mathbf{i}}$ must be ever-so-slightly greater than torque due to $\vec{\mathbf{F}}_{\mathbf{n}}$.) Hence,

$$l_{\mathrm{i}} F_{\mathrm{i}} = l_{\mathrm{o}} F_{\mathrm{o}}$$

where $l_{\mathrm{i}}$ and $l_{\mathrm{o}}$ are the distances from where the input and output forces are applied to the pivot, as shown in the figure. Rearranging the last equation gives

$$\frac{F_{\mathrm{o}}}{F_{\mathrm{i}}} = \frac{l_{\mathrm{i}}}{l_{\mathrm{o}}}.$$

What interests us most here is that the magnitude of the force exerted by the nail puller, $F_{\mathrm{o}}$, is much greater than the magnitude of the input force applied to the puller at the other end, $F_{\mathrm{i}}$. For the nail puller,

$$\mathrm{MA} = \frac{F_{\mathrm{o}}}{F_{\mathrm{i}}} = \frac{l_{\mathrm{i}}}{l_{\mathrm{o}}}.$$

This equation is true for levers in general. For the nail puller, the MA is certainly greater than one. The longer the handle on the nail puller, the greater the force you can exert with it.

Two other types of levers that differ slightly from the nail puller are a wheelbarrow and a shovel, shown in Figure 2. All these lever types are similar in that only three forces are involved – the input force, the output force, and the force on the pivot – and thus their MAs are given by $\mathrm{MA} = \frac{F_{\mathrm{o}}}{F_{\mathrm{i}}}$ and $\mathrm{MA} = \frac{d_1}{d_2}$, with distances being measured relative to the physical pivot. The wheelbarrow and shovel differ from the nail puller because both the input and output forces are on the same side of the pivot.

In the case of the wheelbarrow, the output force or load is between the pivot (the wheel's axle) and the input or applied force. In the case of the shovel, the input force is between the pivot (at the

end of the handle) and the load, but the input lever arm is shorter than the output lever arm. In this case, the MA is less than one.

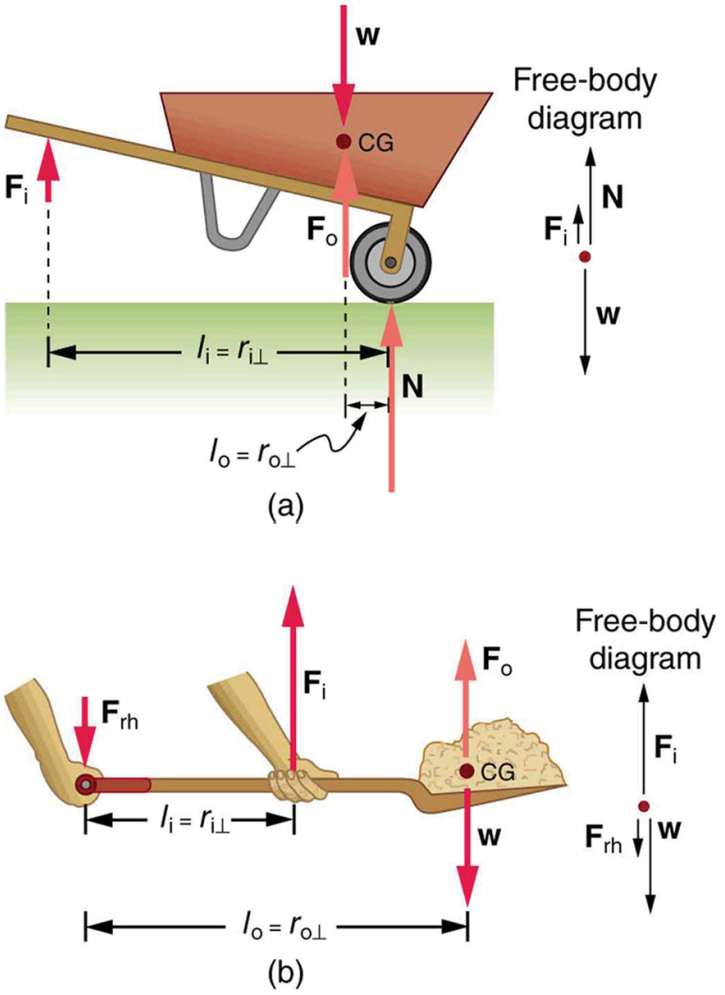



**Figure 2.** *(a) In the case of the wheelbarrow, the output force or load is between the pivot and the input force. The pivot is the wheel's axle. Here, the output force is greater than the input force. Thus, a wheelbarrow enables you to lift much heavier loads than you could with your body alone. (b) In the case of the shovel, the input force is between the pivot and the load, but the input lever arm is shorter than the output lever arm. The pivot is at the handle held by the right hand. Here, the output force (supporting the shovel's load) is less than the input force (from the hand nearest the load), because the input is exerted closer to the pivot than is the output.*

## Example 1: What is the Advantage for the Wheelbarrow?

In the wheelbarrow of Figure 2, the load has a perpendicular lever arm of 7.50 cm, while the hands have a perpendicular lever arm of 1.02 m. (a) What upward force must you exert to support the wheelbarrow and its load if their combined mass is 45.0 kg? (b) What force does the wheelbarrow exert on the ground?

**Strategy**

Here, we use the concept of mechanical advantage.

**Solution**

(a) In this case, $\frac{F_o}{F_i} = \frac{l_i}{l_o}$ becomes

$$F_i = F_o \frac{l_o}{l_i}.$$

Adding values into this equation yields

$$F_i = (45.0\ \text{kg})(9.80\ \text{m/s}^2)\frac{0.075\ \text{m}}{1.02\ \text{m}} = 32.4\ \text{N}.$$

The free-body diagram (see Figure 2) gives the following normal force: $F_i + N = W$. Therefore, $N = (45.0\ \text{kg})(9.80\ \text{m/s}^2) - 32.4\ \text{N} = 409\ \text{N}$. $N$ is the normal force

acting on the wheel; by Newton's third law, the force the wheel exerts on the ground is **409 N**.

**Discussion**

An even longer handle would reduce the force needed to lift the load. The MA here is **MA=1.02/0.0750=13.6**.

Another very simple machine is the inclined plane. Pushing a cart up a plane is easier than lifting the same cart straight up to the top using a ladder, because the applied force is less. However, the work done in both cases (assuming the work done by friction is negligible) is the same. Inclined lanes or ramps were probably used during the construction of the Egyptian pyramids to move large blocks of stone to the top.

A crank is a lever that can be rotated **360°** about its pivot, as shown in Figure 3. Such a machine may not look like a lever, but the physics of its actions remain the same. The MA for a crank is simply the ratio of the radii $\mathbf{r_i/r_0}$. Wheels and gears have this simple expression for their MAs too. The MA can be greater than 1, as it is for the crank, or less than 1, as it is for the simplified car axle driving the wheels, as shown. If the axle's radius is **2.0 cm** and the wheel's radius is **24.0 cm**, then **MA=2.0/24.0=0.083** and the axle would have to exert a force of **12,000 N** on the wheel to enable it to exert a force of **1000 N** on the ground.

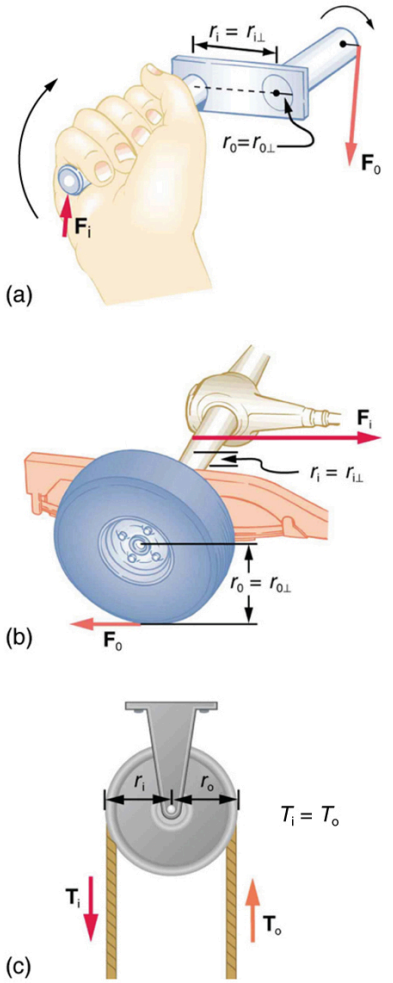



***Figure 3.*** *(a) A crank is a type of lever that can be rotated* **360°** *about its pivot. Cranks are usually designed to have a large MA. (b) A simplified automobile axle drives a wheel, which has a much larger diameter than the axle. The MA is less than 1. (c) An ordinary pulley is used to lift a heavy load. The pulley changes the direction of the force* **T** *exerted by the cord without changing its magnitude. Hence, this machine has an MA of 1.*

An ordinary pulley has an MA of 1; it only changes the direction of the force and not its magnitude. Combinations of pulleys, such as those illustrated in Figure 4, are used to multiply force. If the pulleys are friction-free, then the force output is approximately an integral multiple of the tension in the cable. The number of cables pulling directly upward on the system of interest, as illustrated in the figures given below, is approximately the MA of the pulley system. Since each attachment applies an external force in approximately the same direction as the others, they add, producing a total force that is nearly an integral multiple of the input force **T**.

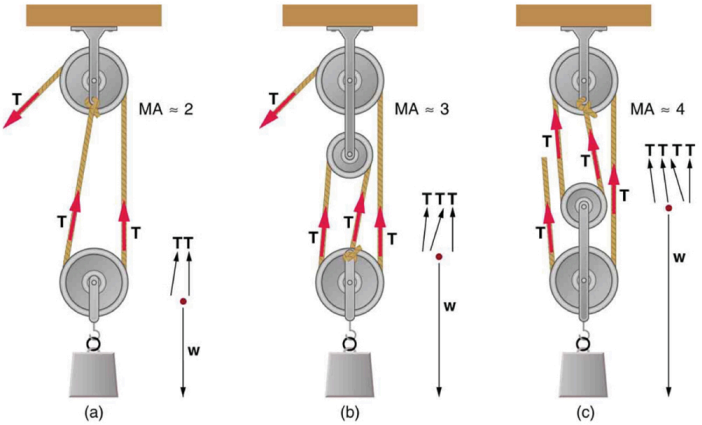



***Figure 4.*** *(a) The combination of pulleys is used to multiply force. The force is an integral multiple of tension if the pulleys are frictionless. This pulley system has two cables attached to its load, thus applying a force of approximately* **2T**. *This machine has* **MA≈2**. *(b) Three pulleys are used to lift a load in such a way that the mechanical advantage is about 3. Effectively, there are three cables attached to the load. (c) This pulley system applies a force of* **4T**, *so that it has* **MA≈4**. *Effectively, four cables are pulling on the system of interest.*

## Section Summary

- Simple machines are devices that can be used to multiply or

augment a force that we apply – often at the expense of a distance through which we have to apply the force.
- The ratio of output to input forces for any simple machine is called its mechanical advantage
- A few simple machines are the lever, nail puller, wheelbarrow, crank, etc.

## *Conceptual Questions*

**1:** Scissors are like a double-lever system. Which of the simple machines in Figure 1 and Figure 2 is most analogous to scissors?

**2:** Suppose you pull a nail at a constant rate using a nail puller as shown in Figure 1. Is the nail puller in equilibrium? What if you pull the nail with some acceleration – is the nail puller in equilibrium then? In which case is the force applied to the nail puller larger and why?

**3:** Why are the forces exerted on the outside world by the limbs of our bodies usually much smaller than the forces exerted by muscles inside the body?

**4:** Explain why the forces in our joints are several times larger than the forces we exert on the outside world with our limbs. Can these forces be even greater than muscle forces (see previous Question)?

## *Problems & Exercises*

**1:** What is the mechanical advantage of a nail puller—similar to the one shown in Figure 1 —where you exert a force 45 cm from the pivot and the nail is 1.8 cm on the other side? What minimum force must you exert to apply a force of 1250 N to the nail?

**2:** Suppose you needed to raise a 250-kg mower a distance of 6.0 cm above the ground to change a tire. If you had a 2.0-m long lever, where would you place the fulcrum if your force was limited to 300 N?

**3:** a) What is the mechanical advantage of a wheelbarrow, such as the one in Figure 2, if the center of gravity of the wheelbarrow and its load has a perpendicular lever arm of 5.50 cm, while the hands have a perpendicular lever arm of 1.02 m? (b) What upward force should you exert to support the wheelbarrow and its load if their combined mass is 55.0 kg? (c) What force does the wheel exert on the ground?

**4:** A typical car has an axle with 1.10 cm radius driving a tire with a radius of 27.5 cm. What is its mechanical advantage assuming the very simplified model in Figure 3(b)?

**5:** What force does the nail puller in Exercise 1 exert on the supporting surface? The nail puller has a mass of 2.10 kg.

**6:** If you used an ideal pulley of the type shown in Figure 4(a) to support a car engine of mass 115 kg, (a) What would

be the tension in the rope? (b) What force must the ceiling supply, assuming you pull straight down on the rope? Neglect the pulley system's mass.

**7:** Repeat Exercise 6 for the pulley shown in Figure 4(c), assuming you pull straight up on the rope. The pulley system's mass is 7.00 kg.

## Glossary

**mechanical advantage**
the ratio of output to input forces for any simple machine

### *Solutions*

**Problems & Exercises**

**1:** 25 and 50 N

**3:** (a) $\boldsymbol{MA = 18.5}$ (b) $\boldsymbol{F_{\text{i}} = 29.1\ \text{N}}$ (c) 510 N downward

**5:** $\boldsymbol{1.3 \times 10^3\ \text{N}}$

**7:** (a) $\boldsymbol{T = 299\ \text{N}}$ (b) 897 N upward

# 49. 8.6 Optional: Stability

*Summary*

- State the types of equilibrium.
- Describe stable and unstable equilibriums.
- Describe neutral equilibrium.

It is one thing to have a system in equilibrium; it is quite another for it to be stable. The toy doll perched on the man's hand in Figure 1, for example, is not in stable equilibrium. There are *three types of equilibrium*: *stable*, *unstable*, and *neutral*. Figures throughout this module illustrate various examples.

Figure 1 presents a balanced system, such as the toy doll on the man's hand, which has its center of gravity (cg) directly over the pivot, so that the torque of the total weight is zero. This is equivalent to having the torques of the individual parts balanced about the pivot point, in this case the hand. The cgs of the arms, legs, head, and torso are labeled with smaller type.

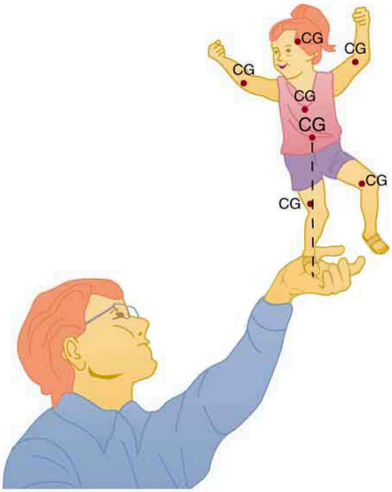



***Figure 1.*** *A man balances a toy doll on one hand.*

A system is said to be in **stable equilibrium** if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite to the direction of the displacement. For example, a marble at the bottom of a bowl will experience a *restoring* force when displaced from its equilibrium position. This force moves it back toward the equilibrium position. Most systems are in stable equilibrium, especially for small displacements. For another example of stable equilibrium, see the pencil in Figure 2.

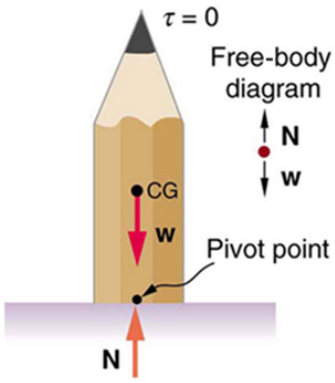



***Figure 2.*** *This pencil is in the condition of equilibrium. The net force on the pencil is zero and the total torque about any pivot is zero.*

A system is in **unstable equilibrium** if, when displaced, it experiences a net force or torque in the *same* direction as the displacement from equilibrium. A system in unstable equilibrium accelerates away from its equilibrium position if displaced even slightly. An obvious example is a ball resting on top of a hill. Once displaced, it accelerates away from the crest. See the next several figures for examples of unstable equilibrium.

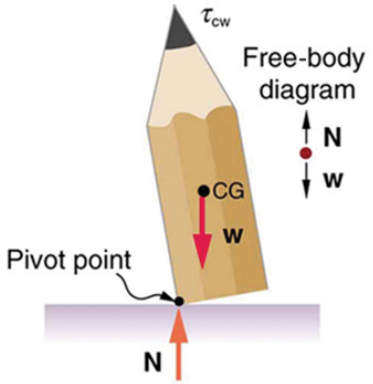



***Figure 3.*** *If the pencil is displaced slightly to the side (counterclockwise), it is no longer in equilibrium. Its weight produces a clockwise torque that returns the pencil to its equilibrium position.*

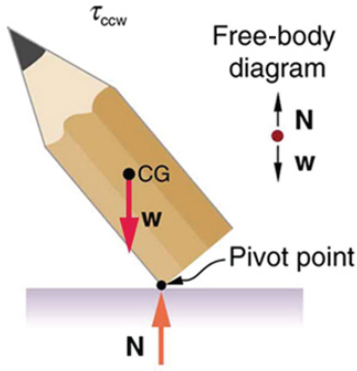



***Figure 4.*** *If the pencil is displaced too far, the torque caused by its weight changes direction to counterclockwise and causes the displacement to increase.*

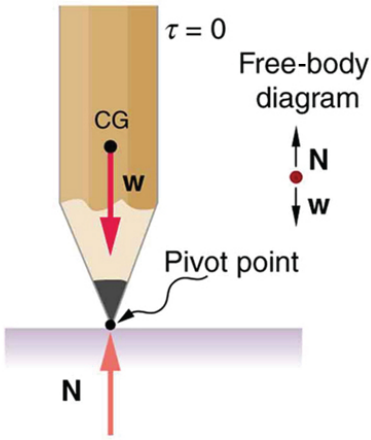



***Figure 5.*** *This figure shows unstable equilibrium, although both conditions for equilibrium are satisfied.*

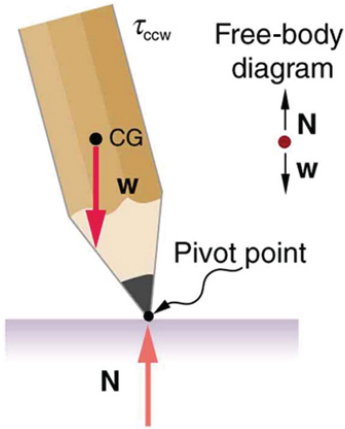



***Figure 6.*** *If the pencil is displaced even slightly, a torque is created by its weight that is in the same direction as the displacement, causing the displacement to increase.*

A system is in **neutral equilibrium** if its equilibrium is independent of displacements from its original position. A marble on a flat horizontal surface is an example. Combinations of these situations are possible. For example, a marble on a saddle is stable for displacements toward the front or back of the saddle and unstable for displacements to the side. Figure 7 shows another example of neutral equilibrium.

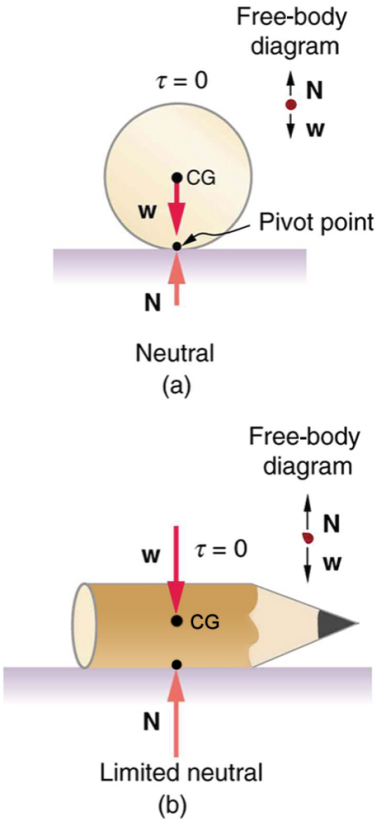



***Figure 7.*** *(a) Here we see neutral equilibrium. The cg of a sphere on a flat surface lies directly above the point of support, independent of the position on the surface. The sphere is therefore in equilibrium in any location, and if displaced, it will remain put. (b) Because it has a circular cross section, the pencil is in neutral equilibrium for displacements perpendicular to its length.*

When we consider how far a system in stable equilibrium can be displaced before it becomes unstable, we find that some systems

in stable equilibrium are more stable than others. The pencil in Figure 2 and the person in Figure 8(a) are in stable equilibrium, but become unstable for relatively small displacements to the side. The critical point is reached when the cg is no longer *above* the base of support. Additionally, since the cg of a person's body is above the pivots in the hips, displacements must be quickly controlled. This control is a central nervous system function that is developed when we learn to hold our bodies erect as infants. For increased stability while standing, the feet should be spread apart, giving a larger base of support. Stability is also increased by lowering one's center of gravity by bending the knees, as when a football player prepares to receive a ball or braces themselves for a tackle. A cane, a crutch, or a walker increases the stability of the user, even more as the base of support widens. Usually, the cg of a female is lower (closer to the ground) than a male. Young children have their center of gravity between their shoulders, which increases the challenge of learning to walk.

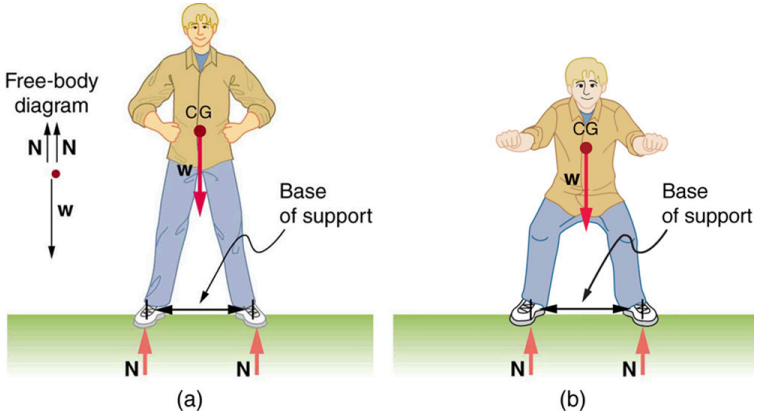



***Figure 8.*** *(a) The center of gravity of an adult is above the hip joints (one of the main pivots in the body) and lies between two narrowly-separated feet. Like a pencil standing on its eraser, this person is in stable equilibrium in relation to sideways displacements, but relatively small displacements take his cg outside the base of support and make him unstable. Humans are less stable relative to forward and backward displacements because the feet are not very long. Muscles are used extensively to balance the body in the front-to-back direction. (b) While bending in the manner shown, stability is increased by lowering the center of gravity. Stability is also increased if the base is expanded by placing the feet farther apart.*

Animals such as chickens have easier systems to control. Figure 9 shows that the cg of a chicken lies below its hip joints and between its widely separated and broad feet. Even relatively large displacements of the chicken's cg are stable and result in restoring forces and torques that return the cg to its equilibrium position with little effort on the chicken's part. Not all birds are like chickens, of course. Some birds, such as the flamingo, have balance systems that are almost as sophisticated as that of humans.

Figure 9 shows that the cg of a chicken is below the hip joints and lies above a broad base of support formed by widely-separated and large feet. Hence, the chicken is in very stable equilibrium, since a relatively large displacement is needed to render it unstable. The body of the chicken is supported from above by the hips and acts as a pendulum between the hips. Therefore, the chicken is

stable for front-to-back displacements as well as for side-to-side displacements.

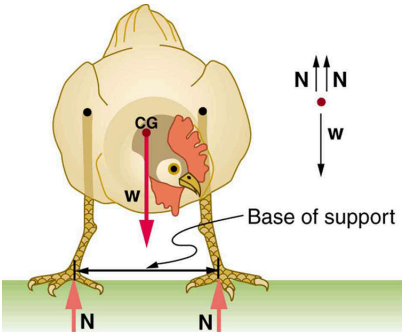



***Figure 9.*** *The center of gravity of a chicken is below the hip joints. The chicken is in stable equilibrium. The body of the chicken is supported from above by the hips and acts as a pendulum between them.*

Engineers and architects strive to achieve extremely stable equilibriums for buildings and other systems that must withstand wind, earthquakes, and other forces that displace them from equilibrium. Although the examples in this section emphasize gravitational forces, the basic conditions for equilibrium are the same for all types of forces. The net external force must be zero, and the net torque must also be zero.

### TAKE-HOME EXPERIMENT

Stand straight with your heels, back, and head against a wall. Bend forward from your waist, keeping your heels and bottom against the wall, to touch your toes. Can you do this without toppling over? Explain why and what you need to do to be able to touch your toes without losing your balance. Is it easier for a woman to do this?

## Section Summary

- A system is said to be in stable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite the direction of the displacement.
- A system is in unstable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in the same direction as the displacement from equilibrium.
- A system is in neutral equilibrium if its equilibrium is independent of displacements from its original position.

## *Conceptual Questions*

**1:** A round pencil lying on its side as in Figure 4 is in neutral equilibrium relative to displacements perpendicular to its length. What is its stability relative to displacements parallel to its length?

**2:** Explain the need for tall towers on a suspension bridge to ensure stable equilibrium.

## *Problems & Exercises*

**1:** Suppose a horse leans against a wall as in Figure 10. Calculate the force exerted on the wall assuming that force is horizontal while using the data in the schematic representation of the situation. Note that the force exerted on the wall is equal in magnitude and opposite in direction to the force exerted on the horse, keeping it in equilibrium. The total mass of the horse and rider is 500 kg. Take the data to be accurate to three digits.

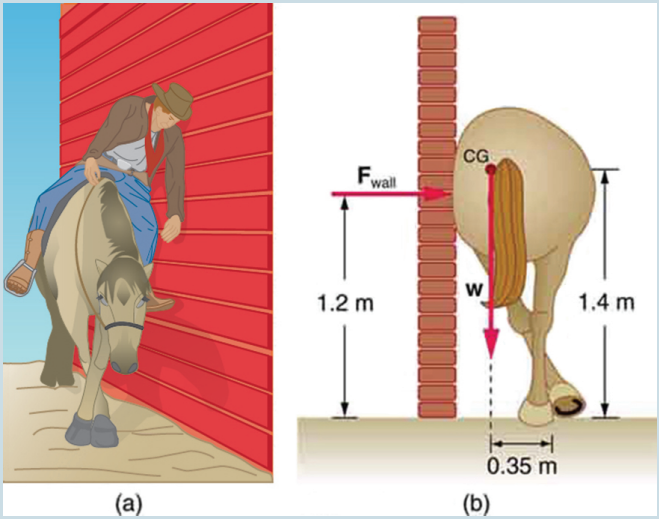



***Figure 10.***

**2:** Two children of mass 20.0 kg and 30.0 kg sit balanced on a seesaw with the pivot point located at the center of the seesaw. If the children are separated by a distance of 3.00 m, at what distance from the pivot point is the small child sitting in order to maintain the balance?

**3:** (a) Calculate the magnitude and direction of the force on each foot of the horse in Figure 10 (two are on the ground), assuming the center of mass of the horse is midway between the feet. The total mass of the horse and rider is 500kg. (b) What is the minimum coefficient of friction between the hooves and ground? Note that the force exerted by the wall is horizontal.

**4:** A person carries a plank of wood 2.00 m long with one

hand pushing down on it at one end with a force $F_1$ and the other hand holding it up at .500 m from the end of the plank with force $\mathbf{F_2}$. If the plank has a mass of 20.0 kg and its center of gravity is at the middle of the plank, what are the magnitudes of the forces $F_1$ and $F_2$?

**5:** A 17.0-m-high and 11.0-m-long wall under construction and its bracing are shown in Figure 11. The wall is in stable equilibrium without the bracing but can pivot at its base. Calculate the force exerted by each of the 10 braces if a strong wind exerts a horizontal force of 650 N on each square meter of the wall. Assume that the net force from the wind acts at a height halfway up the wall and that all braces exert equal forces parallel to their lengths. Neglect the thickness of the wall.

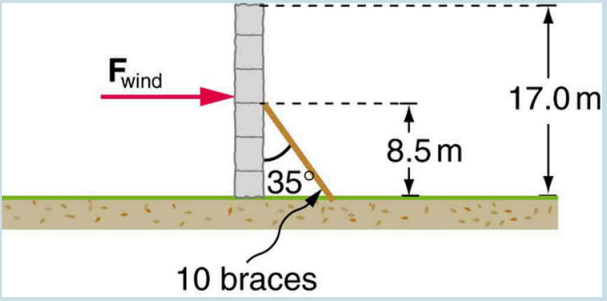



***Figure 11.***

**6:** (a) What force must be exerted by the wind to support a 2.50-kg chicken in the position shown in Figure 12? (b) What is the ratio of this force to the chicken's weight? (c) Does this support the contention that the chicken has a relatively stable construction?

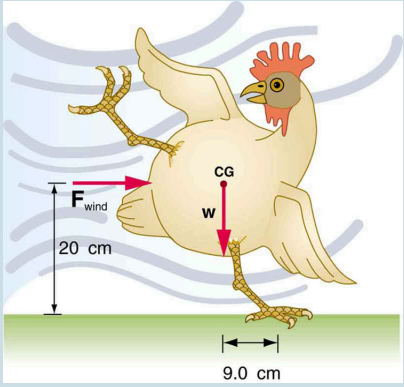



***Figure 12.***

**7:** Suppose the weight of the drawbridge in Figure 13 is supported entirely by its hinges and the opposite shore, so that its cables are slack. (a) What fraction of the weight is supported by the opposite shore if the point of support is directly beneath the cable attachments? (b) What is the direction and magnitude of the force the hinges exert on the bridge under these circumstances? The mass of the bridge is 2500 kg.

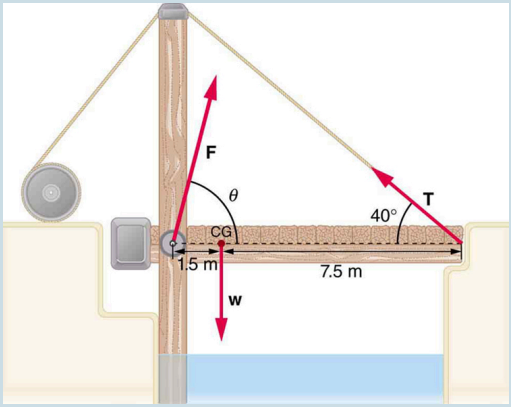



***Figure 13.*** *A small drawbridge, showing the forces on the hinges (**F**), its weight (**w**), and the tension in its wires (**T**).*

**8:** Suppose a 900-kg car is on the bridge in Figure 13 with its center of mass halfway between the hinges and the cable attachments. (The bridge is supported by the cables and hinges only.) (a) Find the force in the cables. (b) Find the direction and magnitude of the force exerted by the hinges on the bridge.

**9:** A sandwich board advertising sign is constructed as shown in Figure 14. The sign's mass is 8.00 kg. (a) Calculate the tension in the chain assuming no friction between the legs and the sidewalk. (b) What force is exerted by each side on the hinge?

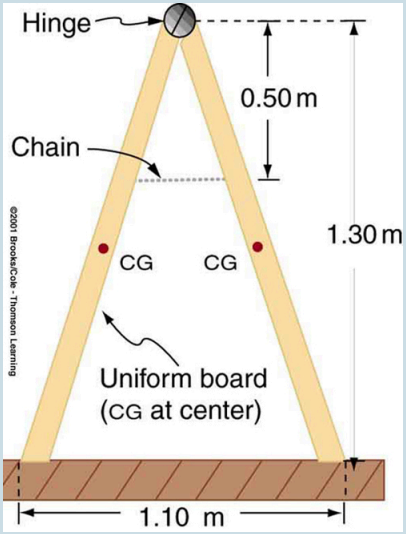



***Figure 14.*** *A sandwich board advertising sign demonstrates tension.*

**10:** (a) What minimum coefficient of friction is needed between the legs and the ground to keep the sign in Figure 14 in the position shown if the chain breaks? (b) What force is exerted by each side on the hinge?

**11:** A gymnast is attempting to perform splits. From the information given in Figure 15, calculate the magnitude and direction of the force exerted on each foot by the floor.

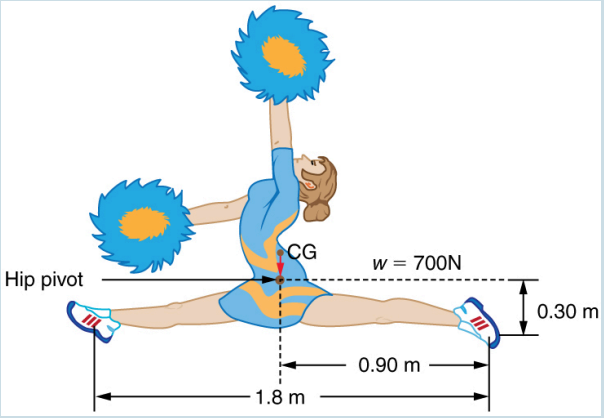



***Figure 15.*** *A gymnast performs full split. The center of gravity and the various distances from it are shown.*

## Glossary

**neutral equilibrium**
: a state of equilibrium that is independent of a system's displacements from its original position

**stable equilibrium**
: a system, when displaced, experiences a net force or torque in a direction opposite to the direction of the displacement

**unstable equilibrium**
: a system, when displaced, experiences a net force or torque in the same direction as the displacement from equilibrium

## *Solutions*

**Problems & Exercises**

**1:** $\boldsymbol{F_{\text{wall}} = 1.43 \times 10^3 \text{ N}}$

**3:** (a)

**$2.55 \times 10^3$ N, $16.3^0$ to the left of vertical (i.e., toward the wall)**

(b) 0.292

**5:** $\boldsymbol{F_{\text{B}} = 2.12 \times 10^4 \text{ N}}$

**7:** (a) 0.167, or about one-sixth of the weight is supported by the opposite shore. (b) $\boldsymbol{F = 2.0 \times 10^4 \text{ N}}$, straight up.

**9:** (a) 21.6 N (b) 21.6 N

**11:** 350 N directly upwards

PART X

# CHAPTER 9 FLUID STATICS (FLOATING AND SINKING)



***Figure 1.*** *The fluid essential to all life has a beauty of its own. It also helps support the weight of this swimmer. (credit: Terren, Wikimedia Commons)*

Much of what we value in life is fluid: a breath of fresh winter air; the hot blue flame in our gas cooker; the water we drink, swim in, and bathe in; the blood in our veins. What exactly is a fluid? Can we understand fluids with the laws already presented, or will new laws emerge from their study? The physical characteristics of static or stationary fluids and some of the laws that govern their behavior are the topics of this chapter. The original Open Stax book has a chapter on Fluid Dynamics and its Biology and Medial Application which explores aspects of fluid flow.

# 50. 9.1 What Is a Fluid?

*Summary*

- State the common phases of matter.
- Explain the physical characteristics of solids, liquids, and gases.
- Describe the arrangement of atoms in solids, liquids, and gases.

Matter most commonly exists as a solid, liquid, or gas; these states are known as the three common *phases of matter*. Solids have a definite shape and a specific volume, liquids have a definite volume but their shape changes depending on the container in which they are held, and gases have neither a definite shape nor a specific volume as their molecules move to fill the container in which they are held. (See Figure 1.) Liquids and gases are considered to be fluids because they yield to shearing forces, whereas solids resist them. Note that the extent to which fluids yield to shearing forces (and hence flow easily and quickly) depends on a quantity called the viscosity (not covered in this course, but details can be found in Chapter 12.4 Viscosity and Laminar Flow; Poiseuille's Law of the original OpenStax College Physics textbook). We can understand the phases of matter and what constitutes a fluid by considering the forces between atoms that make up matter in the three phases.

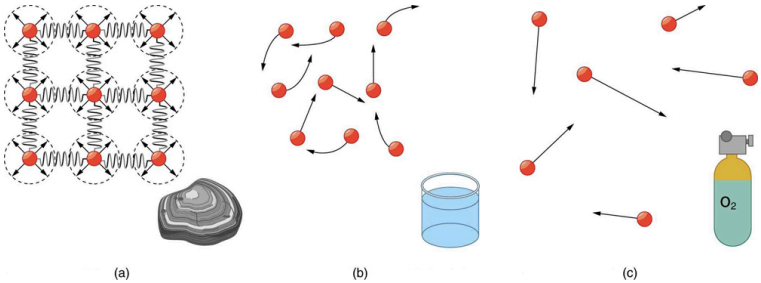



***Figure 1.*** *(a) Atoms in a solid always have the same neighbors, held near home by forces represented here by springs. These atoms are essentially in contact with one another. A rock is an example of a solid. This rock retains its shape because of the forces holding its atoms together. (b) Atoms in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. Water can flow, but it also remains in an open container because of the forces between its atoms. (c) Atoms in a gas are separated by distances that are considerably larger than the size of the atoms themselves, and they move about freely. A gas must be held in a closed container to prevent it from moving out freely.*

Atoms in *solids* are in close contact, with forces between them that allow the atoms to vibrate but not to change positions with neighboring atoms. (These forces can be thought of as springs that can be stretched or compressed, but not easily broken.) Thus a solid *resists* all types of stress. A solid cannot be easily deformed because the atoms that make up the solid are not able to move about freely. Solids also resist compression, because their atoms form part of a lattice structure in which the atoms are a relatively fixed distance apart. Under compression, the atoms would be forced into one another. Most of the examples we have studied so far have involved solid objects which deform very little when stressed.

## CONNECTIONS: SUBMICROSCOPIC EXPLANATION OF SOLIDS AND LIQUIDS

Atomic and molecular characteristics explain and underlie the macroscopic characteristics of solids and fluids. This submicroscopic explanation is one theme of this text and is highlighted in the Things Great and Small features in Chapter 8.3 Conservation of Momentum. See, for example, microscopic description of collisions and momentum or microscopic description of pressure in a gas. This present section is devoted entirely to the submicroscopic explanation of solids and liquids.

In contrast, *liquids* deform easily when stressed and do not spring back to their original shape once the force is removed because the atoms are free to slide about and change neighbors—that is, they *flow* (so they are a type of fluid), with the molecules held together by their mutual attraction. When a liquid is placed in a container with no lid on, it remains in the container (providing the container has no holes below the surface of the liquid!). Because the atoms are closely packed, liquids, like solids, resist compression.

Atoms in *gases* are separated by distances that are large compared with the size of the atoms. The forces between gas atoms are therefore very weak, except when the atoms collide with one another. Gases thus not only flow (and are therefore considered to be fluids) but they are relatively easy to compress because there is

much space and little force between atoms. When placed in an open container gases, unlike liquids, will escape. The major distinction is that gases are easily compressed, whereas liquids are not. We shall generally refer to both gases and liquids simply as **fluids**, and make a distinction between them only when they behave differently.

PHET EXPLORATIONS: STATES OF MATTER—BASICS

Heat, cool, and compress atoms and molecules and watch as they change between solid, liquid, and gas phases.



**PhET Interactive Simulation**

***Figure 2.*** *States of Matter: Basics*

## Section Summary

- A fluid is a state of matter that yields to sideways or shearing forces. Liquids and gases are both fluids. Fluid statics is the physics of stationary fluids.

## *Conceptual Questions*

**1:** What physical characteristic distinguishes a fluid from a solid?

**2:** Which of the following substances are fluids at room temperature: air, mercury, water, glass?

**3:** Why are gases easier to compress than liquids and solids?

**4:** How do gases differ from liquids?

## Glossary

**fluids**
liquids and gases; a fluid is a state of matter that yields to shearing forces

# 51. 9.2 Density

*Summary*

- Define density.
- Calculate the mass of a reservoir from its density.
- Compare and contrast the densities of various substances.

Which weighs more, a ton of feathers or a ton of bricks? This old riddle plays with the distinction between mass and density. A ton is a ton, of course; but bricks have much greater density than feathers, and so we are tempted to think of them as heavier.

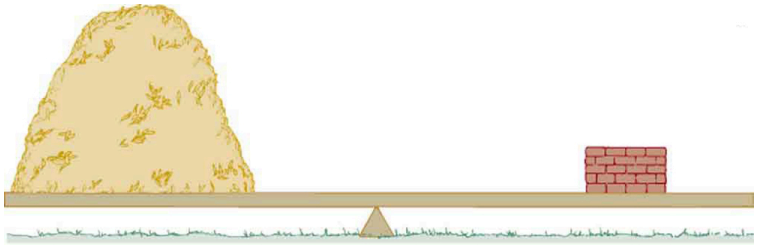


***Figure 1.*** *A ton of feathers and a ton of bricks have the same mass, but the feathers make a much bigger pile because they have a much lower density.*

**Density**, as you will see, is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid. Density is the mass per unit volume of a substance or object.

## DENSITY

Density is mass per unit volume.

In equation form, density is defined as **ρ = mass / Volume**

where the Greek letter ρ is the symbol for density, m is the mass, and V is the volume occupied by the substance.

In the riddle regarding the feathers and bricks, the masses are the same, but the volume occupied by the feathers is much greater, since their density is much lower. The SI unit of density is kg / $m^3$ representative values are given in Table 1. The metric system was originally devised so that water would have a density of 1 gram / cubic centimeter = 1 $g/cm^3$ equivalent to 1000 kg / $m^3$ . Thus the basic mass unit, the kilogram, was first devised to be the mass of 1000 mL of water, which has a volume of 1000 $cm^3$.

| Substance | ρ ($10^3$ kg/$m^3$ or g/mL) | Substance | ρ ($10^3$ kg/$m^3$ or g/mL) | Substance | ρ ($10^3$ kg/$m^3$ or g/mL) |
|---|---|---|---|---|---|
| **Solids** | | **Liquids** | | **Gases** | |
| Aluminum | 2.7 | Water (4°C) | 1.000 | Air | $1.29 \times 10^{-3}$ |
| Brass | 8.44 | Blood | 1.05 | Carbon dioxide | $1.29 \times 10^{-3}$ |
| Copper (average) | 8.8 | Sea water | 1.025 | Carbon monoxide | $1.25 \times 10^{-3}$ |
| | 19.32 | Mercury | 13.6 | Hydrogen | $0.090 \times 10^{-3}$ |
| Iron or steel | 7.8 | Ethyl alcohol | 0.79 | Helium | $0.18 \times 10^{-3}$ |
| Lead | 11.3 | Petrol | 0.68 | Methane | $0.72 \times 10^{-3}$ |
| Polystyrene | 0.10 | Glycerin | 1.26 | Nitrogen | $1.25 \times 10^{-3}$ |
| Tungsten | 19.30 | Olive oil | 0.92 | Nitrous oxide | $1.98 \times 10^{-3}$ |
| Uranium | 18.70 | | | Oxygen | $1.43 \times 10^{-3}$ |
| Concrete | 2.30–3.0 | | | Steam 100° C | $0.60 \times 10^{-3}$ |
| Cork | 0.24 | | | | |
| Glass, common (average) | 2.6 | | | | |
| Granite | 2.7 | | | | |
| Earth's crust | 3.3 | | | | |
| Wood | 0.3–0.9 | | | | |
| Ice (0°C) | 0.917 | | | | |
| Bone | 1.7–2.0 | | | | |

**Table 1.** Densities of Various Substances

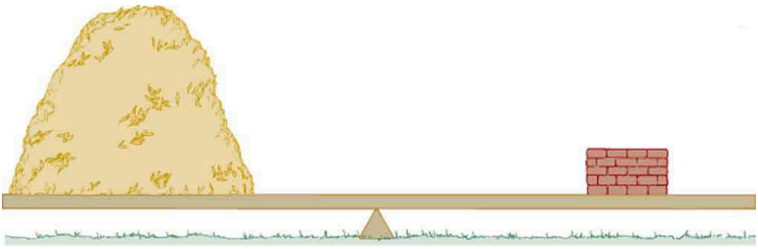


***Figure 1.*** *A ton of feathers and a ton of bricks have the same mass, but the feathers make a much bigger pile because they have a much lower density.*

As you can see by examining the tables of density, the density of an object may help identify its composition. The density of gold, for example, is about 2.5 times the density of iron, which is about 2.5 times the density of aluminum. Density also reveals something about the phase of the matter and its substructure. Notice that the densities of liquids and solids are roughly comparable, consistent with the fact that their atoms are in close contact. The densities of gases are much less than those of liquids and solids, because the atoms in gases are separated by large amounts of empty space.

## TAKE-HOME EXPERIMENT: SUGAR AND SALT

A pile of sugar and a pile of salt look pretty similar, but which weighs more? If the volumes of both piles are the same, any difference in mass is due to their different densities (including the air space between crystals). Which do you think has the greater density? What

values did you find? What method did you use to determine these values?

## Example 1: Calculating the Mass of a Reservoir From Its Volume

A reservoir has a surface area of 50.0 $km^2$ and an average depth of 40.0 m. What mass of water is held behind the dam? (See the figure below, or Google search it, for a view of a large reservoir – the Three Gorges Dam site on the Yangtze River in central China.)

**Strategy**

We can calculate the volume **V** of the reservoir from its dimensions, and find the density of water **ρ** from the table. Remember that by definition 1 gram = 1 millilitre or 1 kg = 1 litre or 1000 kg = 1 $m^3$ of water. Then the mass can be found from the definition of density

**ρ = mass / Volume gives mass = volume ρ**

**Solution**

Solving equation

The volume of the reservoir **V** is its surface area **A** times its average depth **h**.

Volume = 50 $km^2$ x 40 m = 50 x $(1000)^2$ $m^2$ x 40 m

$= 2.00 \times 10^9 \text{ m}^3$

\The density of water **$\rho$ water = 1000 kg/m$^3$ gives mass = volume $\rho$**

$$\text{mass} = (2.00 \times 10^9 \text{ m}^3) \times (1000 \text{ kg/m}^3) = 2.00 \times 10^{12} \text{ kg}$$

**Discussion**

A large reservoir contains a very large mass of water. In this example, the weight of the water in the reservoir is $1.96 \times 10^{13}$ N , where g is the acceleration due to the Earth's gravity (about 9.80 m/s$^2$ . It is reasonable to ask whether the dam must supply a force equal to this tremendous weight. The answer is no. As we shall see in the following sections, the force the dam must supply can be much smaller than the weight of the water it holds back.



***Figure 2.*** *Three Gorges Dam in central China. When completed in 2008, this became the world's largest hydroelectric plant, generating power equivalent to that generated by 22 average-sized nuclear power plants. The concrete dam is 181 m high and 2.3 km across. The reservoir made by this dam is 660 km long. Over 1 million people were displaced by the creation of the reservoir. (credit: Le Grand Portage)*

## Section Summary

- Density is the mass per unit volume of a substance or object. In equation form, density is defined as **ρ = mass / Volume**
- The SI unit of density is **$kg/m^3$**

### *Conceptual Questions*

**1:** Approximately how does the density of air vary with altitude?

**2:** Give an example in which density is used to identify the substance composing an object. Would information in addition to average density be needed to identify the substances in an object composed of more than one material?

**3:** Figure 3 shows a glass of ice water filled to the brim. Will the water overflow when the ice melts? Explain your answer.



***Figure 3.***

## *Problems & Exercises*

**1:** Gold is sold by the troy ounce (31.103 g). What is the volume of 1 troy ounce of pure gold?

**2:** Mercury is commonly supplied in flasks containing 34.5 kg (about 76 lb). What is the volume in liters of this much mercury?

**3:** (a) What is the mass of a deep breath of air having a volume of 2.00 L? (b) Discuss the effect taking such a breath has on your body's volume and density.

**4:** A straightforward method of finding the density of an object is to measure its mass and then measure its volume by submerging it in a graduated cylinder. What is the density of a 240-g rock that displaces 89.0 $cm^3$ of water? (Note that the accuracy and practical applications of this technique are more limited than a variety of others that are based on Archimedes' principle.)

**5:** Suppose you have a coffee mug with a circular cross section and vertical sides (uniform radius). What is its inside radius if it holds 375 g of coffee when filled to a depth of 7.50 cm? Assume coffee has the same density as water.

**6:** (a) A rectangular gasoline tank can hold 50.0 kg of gasoline when full. What is the depth of the tank if it is 0.500-m wide by 0.900-m long? (b) Discuss whether this gas tank has a reasonable volume for a passenger car.

**7:** A trash compactor can reduce the volume of its contents to 0.350 their original value. Neglecting the mass of air expelled, by what factor is the density of the rubbish increased?

**8:** A 2.50-kg steel gasoline can holds 20.0 L of gasoline when full. What is the average density of the full gas can,

taking into account the volume occupied by steel as well as by gasoline?

**9:** What is the density of 18.0-karat gold that is a mixture of 18 parts gold, 5 parts silver, and 1 part copper? (These values are parts by mass, not volume.) Assume that this is a simple mixture having an average density equal to the weighted densities of its constituents.

**10:** There is relatively little empty space between atoms in solids and liquids, so that the average density of an atom is about the same as matter on a macroscopic scale—approximately 1000 $kg/m^3$ . The nucleus of an atom has a radius about $1 \times 10^{-15}$ m and contains nearly all the mass of the entire atom. (a) What is the approximate density of a nucleus? (b) One remnant of a supernova, called a neutron star, can have the density of a nucleus. What would be the radius of a neutron star with a mass 10 times that of our Sun (the radius of the Sun is $7 \times 10^8$ m ?

## Glossary

**density**
the mass per unit volume of a substance or object

## *Solutions*

**Problems & Exercises**

**1:** 1.610 $cm^3$

**3:** (a) 2.58 g (b) The volume of your body increases by the volume of air you inhale. The average density of your body decreases when you take a deep breath, because the density of air is substantially smaller than the average density of the body before you took the deep breath.

**4:** 2.70 $g/cm^3$

**6:** (a) 0.163 m (b) Equivalent to 19.4 gallons, which is reasonable

**8:** 7.9 x $10^2$ $kg/m^3$

**9:** 15.6 $g/cm^3$

**10:** (a) 10^18 $kg/m^3$ (b) 2x $10^4$ m

# 52. 9.3 Pressure

*Summary*

- Define pressure.
- Explain the relationship between pressure and force.
- Calculate force given pressure and area.

You have no doubt heard the word **pressure** being used in relation to blood (high or low blood pressure) and in relation to the weather (high- and low-pressure weather systems). These are only two of many examples of pressures in fluids. Pressure $P$ is defined as

$$P = \frac{F}{A}$$

where $F$ is a force applied to an area $A$ that is perpendicular to the force.

PRESSURE

Pressure is defined as the force divided by the area

perpendicular to the force over which the force is applied, or

$$P = \frac{F}{A}.$$

A given force can have a significantly different effect depending on the area over which the force is exerted, as shown in Figure 1. The SI unit for pressure is the *pascal*, where

$$\mathbf{1\ Pa = 1\ N/m^2}.$$

In addition to the pascal, there are many other units for pressure that are in common use. In meteorology, atmospheric pressure is often described in units of millibar (mb), where

$$\mathbf{100\ mb = 1 \times 10^5\ Pa}.$$

Pounds per square inch $\mathbf{(lb/in^2\ or\ psi)}$ is still sometimes used as a measure of tire pressure, and millimeters of mercury (mm Hg) is still often used in the measurement of blood pressure. Pressure is defined for all states of matter but is particularly important when discussing fluids.

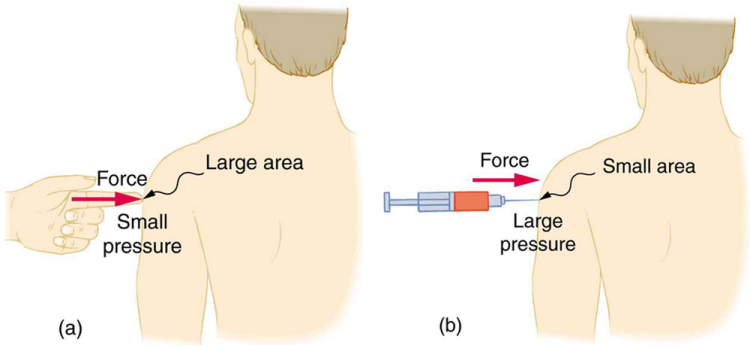



***Figure 1.*** *(a) While the person being poked with the finger might be irritated, the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is great enough to break the skin.*

## Example 1: Calculating Force Exerted by the Air: What Force Does a Pressure Exert?

An astronaut is working outside the International Space Station where the atmospheric pressure is essentially zero. The pressure gauge on her air tank reads $\mathbf{6.90 \times 10^6\ Pa}$. What force does the air inside the tank exert on the flat end of the cylindrical tank, a disk 0.150 m in diameter?

**Strategy**

We can find the force exerted from the definition of

pressure given in $P = \frac{F}{A}$, provided we can find the area $A$ acted upon.

**Solution**

By rearranging the definition of pressure to solve for force, we see that

$$F = PA.$$

Here, the pressure $P$ is given, as is the area of the end of the cylinder $A$, given by $A = \pi r^2$. Thus,

$$\begin{aligned} F &= (6.90 \times 10^6 \text{ N/m}^2)(3.14)(0.0750 \text{ m})^2 \\ &= 1.22 \times 10^5 \text{ N}. \end{aligned}$$

**Discussion**

Wow! No wonder the tank must be strong. Since we found $F = PA$, we see that the force exerted by a pressure is directly proportional to the area acted upon as well as the pressure itself.

The force exerted on the end of the tank is perpendicular to its inside surface. This direction is because the force is exerted by a static or stationary fluid. We have already seen that fluids cannot *withstand* shearing (sideways) forces; they cannot *exert* shearing forces, either. Fluid pressure has no direction, being a scalar quantity. The forces due to pressure have well-defined directions: they are always exerted perpendicular to any surface. (See the tire in Figure 2, for example.) Finally, note that pressure is exerted on all surfaces. Swimmers, as well as the tire, feel pressure on all sides. (See Figure 3.)

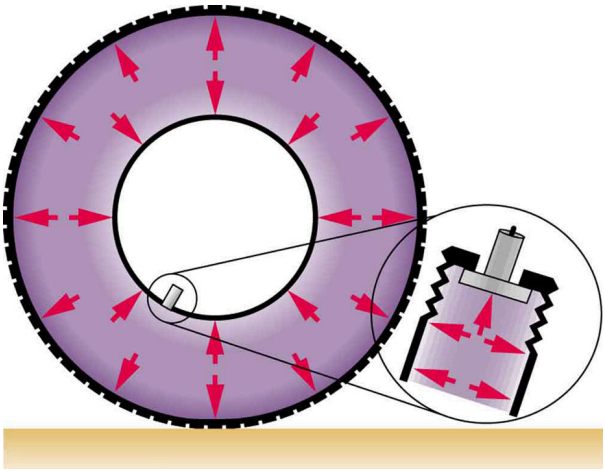


***Figure 2.*** *Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows give representative directions and magnitudes of the forces exerted at various points. Note that static fluids do not exert shearing forces.*

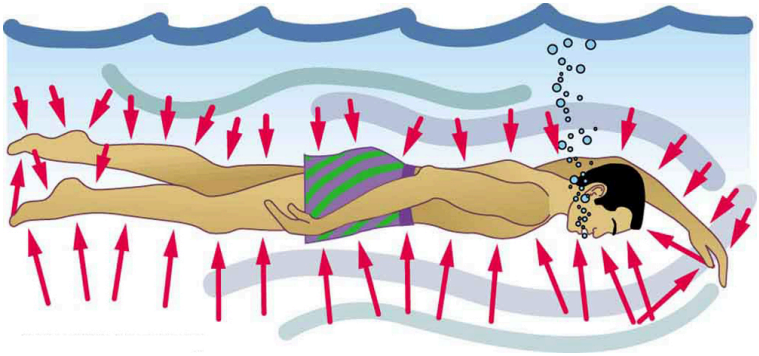


***Figure 3.*** *Pressure is exerted on all sides of this swimmer, since the water would flow into the space he occupies if he were not there. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth, giving a net upward or buoyant force that is balanced by the weight of the swimmer.*

## PHET EXPLORATIONS: GAS PROPERTIES

Pump gas molecules to a box and see what happens as you change the volume, add or remove heat, change gravity, and more. Measure the temperature and pressure, and discover how the properties of the gas vary in relation to each other.



PhET Interactive Simulation

***Figure 4.*** *Gas Properties*

## Section Summary

- Pressure is the force per unit perpendicular area over which the force is applied. In equation form, pressure is defined as

$$P = \frac{F}{A}.$$

- The SI unit of pressure is pascal and $1\ \text{Pa} = 1\ \text{N/m}^2$.

## *Conceptual Questions*

**1:** How is pressure related to the sharpness of a knife and its ability to cut?

**2:** Why does a dull hypodermic needle hurt more than a sharp one?

**3:** The outward force on one end of an air tank was calculated in Example 1. How is this force balanced? (The tank does not accelerate, so the force must be balanced.)

**4:** Why is force exerted by static fluids always perpendicular to a surface?

**5:** In a remote location near the North Pole, an iceberg floats in a lake. Next to the lake (assume it is not frozen) sits a comparably sized glacier sitting on land. If both chunks of ice should melt due to rising global temperatures (and the melted ice all goes into the lake), which ice chunk would give the greatest increase in the level of the lake water, if any?

**6:** How do jogging on soft ground and wearing padded shoes reduce the pressures to which the feet and legs are subjected?

**7:** Toe dancing (as in ballet) is much harder on toes than normal dancing or walking. Explain in terms of pressure.

**8:** How do you convert pressure units like millimeters of mercury, centimeters of water, and inches of mercury into

units like newtons per meter squared without resorting to a table of pressure conversion factors?

## *Problems & Exercises*

**1:** As a woman walks, her entire weight is momentarily placed on one heel of her high-heeled shoes. Calculate the pressure exerted on the floor by the heel if it has an area of $\mathbf{1.50\ cm^2}$ and the woman's mass is 55.0 kg. Express the pressure in Pa. (In the early days of commercial flight, women were not allowed to wear high-heeled shoes because aircraft floors were too thin to withstand such large pressures.)

**2:** The pressure exerted by a phonograph needle on a record is surprisingly large. If the equivalent of 1.00 g is supported by a needle, the tip of which is a circle 0.200 mm in radius, what pressure is exerted on the record in $\mathbf{N/m^2}$?

**3:** Nail tips exert tremendous pressures when they are hit by hammers because they exert a large force over a small area. What force must be exerted on a nail with a circular tip of 1.00 mm diameter to create a pressure of $\mathbf{3.00 \times 10^9\ N/m^2}$**?** (This high pressure is possible because the hammer striking the nail is brought to rest in such a short distance.)

## Glossary

**pressure**
the force per unit area perpendicular to the force, over which the force acts

### *Solutions*

**Problems & Exercises**

**1:** 3.59 x$10^6$ Pa or 521 lb/$in^2$

**3:** 2.36 x $10^3$ N

# 53. 9.4 Variation of Pressure with Depth in a Fluid

*Summary*

- Define pressure in terms of weight.
- Explain the variation of pressure with depth in a fluid.
- Calculate density given pressure and altitude.

If your ears have ever popped on a plane flight or ached during a deep dive in a swimming pool, you have experienced the effect of depth on pressure in a fluid. At the Earth's surface, the air pressure exerted on you is a result of the weight of air above you. This pressure is reduced as you climb up in altitude and the weight of air above you decreases. Under water, the pressure exerted on you increases with increasing depth. In this case, the pressure being exerted upon you is a result of both the weight of water above you *and* that of the atmosphere above you. You may notice an air pressure change on an elevator ride that transports you many stories, but you need only dive a meter or so below the surface of a pool to feel a pressure increase. The difference is that water is much denser than air, about 775 times as dense.

Consider the container in Figure 1. Its bottom supports the weight of the fluid in it. Let us calculate the pressure exerted on the bottom by the weight of the fluid. That **pressure** is the weight of the fluid $mg$ divided by the area $A$ supporting it (the area of the bottom of the container):

$$P = \frac{mg}{A}.$$

We can find the mass of the fluid from its volume and density:

$$m = \rho V.$$

The volume of the fluid $V$ is related to the dimensions of the container. It is

$$V = Ah,$$

where $A$ is the cross-sectional area and $h$ is the depth. Combining the last two equations gives

$$m = \rho Ah.$$

If we enter this into the expression for pressure, we obtain

$$P = \frac{(\rho Ah)g}{A}.$$

The area cancels, and rearranging the variables yields

$$P = h\rho g.$$

This value is the *pressure due to the weight of a fluid*. The equation has general validity beyond the special conditions under which it is derived here. Even if the container were not there, the surrounding fluid would still exert this pressure, keeping the fluid static. Thus the equation $P = h\rho g$ represents the pressure due to the weight of any fluid of *average density* $\rho$ at any depth $h$ below its surface. For liquids, which are nearly incompressible, this equation holds to great depths. For gases, which are quite compressible, one can apply this equation as long as the density changes are small over the depth considered. Example 2 illustrates this situation.

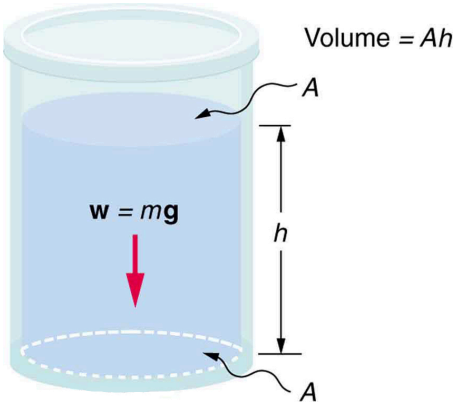



***Figure 1.*** *The bottom of this container supports the entire weight of the fluid in it. The vertical sides cannot exert an upward force on the fluid (since it cannot withstand a shearing force), and so the bottom must support it all.*

## Example 1: Calculating the Average Pressure and Force Exerted: What Force Must a Dam Withstand?

In Chapter 9.2 Example 1, we calculated the mass of water in a large reservoir. We will now consider the pressure and force acting on the dam retaining water. (See Figure 2.) The dam is 500 m wide, and the water is 80.0 m deep at the dam. (a) What is the average pressure on the dam due to the water? (b) Calculate the force

exerted against the dam and compare it with the weight of water in the dam (previously found to be $1.96 \times 10^{13}\ \text{N}$).

**Strategy for (a)**

The average pressure $\bar{P}$ due to the weight of the water is the pressure at the average depth $\bar{h}$ of 40.0 m, since pressure increases linearly with depth.

**Solution for (a)**

The average pressure due to the weight of a fluid is

$$\bar{P} = \bar{h}\rho g.$$

Entering the density of water from Table 1 and taking $\bar{h}$ to be the average depth of 40.0 m, we obtain

$$\begin{aligned} \bar{P} &= (40.0\ \text{m})(10^3\ \tfrac{\text{kg}}{\text{m}^3})(9.80\ \tfrac{\text{m}}{\text{s}^2}) \\ &= 3.92 \times 10^5\ \tfrac{\text{N}}{\text{m}^2} = 392\ \text{kPa}. \end{aligned}$$

**Strategy for (b)**

The force exerted on the dam by the water is the average pressure times the area of contact:

$$F = \bar{P}A.$$

**Solution for (b)**

We have already found the value for $\bar{P}$. The area of the dam is

$$A = 80.0\ \text{m} \times 500\ \text{m} = 4.00 \times 10^4\ \text{m}^2,$$

so that

$$\begin{aligned} F &= (3.92 \times 10^5\ \text{N/m}^2)(4.00 \times 10^4\ \text{m}^2) \\ &= 1.57 \times 10^{10}\ \text{N}. \end{aligned}$$

**Discussion**

Although this force seems large, it is small compared with the $\mathbf{1.96 \times 10^{13}\ N}$ weight of the water in the reservoir—in fact, it is only $\mathbf{0.0800\%}$ of the weight. Note that the pressure found in part (a) is completely independent of the width and length of the lake—it depends only on its average depth at the dam. Thus the force depends only on the water's average depth and the dimensions of the dam, *not* on the horizontal extent of the reservoir. In the diagram, the thickness of the dam increases with depth to balance the increasing force due to the increasing pressure.

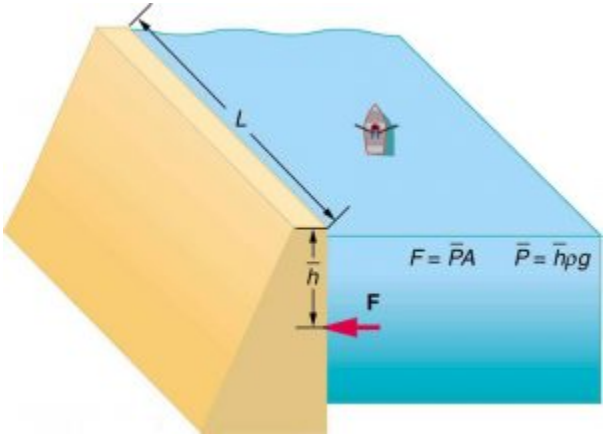



***Figure 2.*** *The dam must withstand the force exerted against it by the water it retains. This force is small compared with the weight of the water behind the dam.*

*Atmospheric pressure* is another example of pressure due to the

weight of a fluid, in this case due to the weight of *air* above a given height. The atmospheric pressure at the Earth's surface varies a little due to the large-scale flow of the atmosphere induced by the Earth's rotation (this creates weather "highs" and "lows"). However, the average pressure at sea level is given by the *standard atmospheric pressure* $P_{\mathbf{atm}}$, measured to be

$$1\ \textbf{atmosphere (atm)} = P_{\text{atm}} = 1.01 \times 10^5\ \text{N/m}^2 = 101\ \text{kPa}.$$

This relationship means that, on average, at sea level, a column of air above $1.00\ \mathbf{m}^2$ of the Earth's surface has a weight of $\mathbf{1.01 \times 10^5\ N}$, equivalent to $\mathbf{1\ atm}$ (see Figure 3).

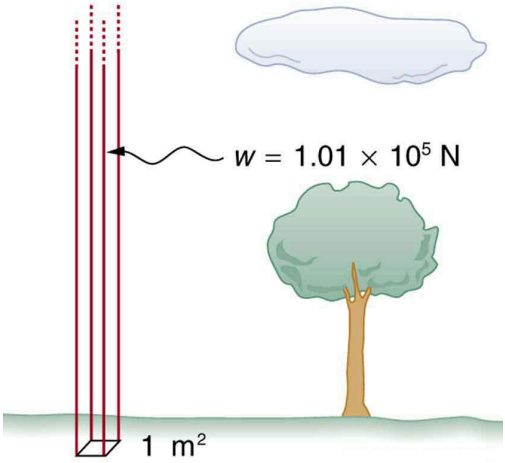



***Figure 3.*** *Atmospheric pressure at sea level averages* **1.01×10$^5$ Pa** *(equivalent to 1 atm), since the column of air over this* **1 m$^2$**, *extending to the top of the atmosphere, weighs* **1.01×10$^5$ N**.

## Example 2: Calculating Average Density: How Dense Is the Air?

Calculate the average density of the atmosphere, given that it extends to an altitude of 120 km. Compare this density with that of air listed in Table 1.

**Strategy**

If we solve $P = h\rho g$ for density, we see that

$$\bar{\rho} = \frac{P}{hg}.$$

We then take $P$ to be atmospheric pressure, $h$ is given, and $g$ is known, and so we can use this to calculate $\bar{\rho}$.

**Solution**

Entering known values into the expression for $\bar{\rho}$ yields

$$\bar{\rho} = \frac{1.01 \times 10^5 \text{ N/m}^2}{(120 \times 10^3 \text{ m})(9.80 \text{ m/s}^2)} = 8.59 \times 10^{-2} \text{ kg/m}^3.$$

**Discussion**

This result is the average density of air between the Earth's surface and the top of the Earth's atmosphere, which essentially ends at 120 km. The density of air at sea level is given in Table 1 as $1.29 \text{ kg/m}^3$—about 15 times its average value. Because air is so

compressible, its density has its highest value near the Earth's surface and declines rapidly with altitude.

## Example 3: Calculating Depth Below the Surface of Water: What Depth of Water Creates the Same Pressure as the Entire Atmosphere?

Calculate the depth below the surface of water at which the pressure due to the weight of the water equals 1.00 atm.

**Strategy**

We begin by solving the equation $P = h\rho g$ for depth $h$:

$$h = \frac{P}{\rho g}.$$

Then we take $P$ to be 1.00 atm and $\rho$ to be the density of the water that creates the pressure.

**Solution**

Entering the known values into the expression for $h$ gives

$$h =$$

$$\frac{1.01 \times 10^5 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 10.3 \text{ m}.$$

**Discussion**

Just 10.3 m of water creates the same pressure as 120 km of air. Since water is nearly incompressible, we can neglect any change in its density over this depth.

What do you suppose is the *total* pressure at a depth of 10.3 m in a swimming pool? Does the atmospheric pressure on the water's surface affect the pressure below? The answer is yes. This seems only logical, since both the water's weight and the atmosphere's weight must be supported. So the *total* pressure at a depth of 10.3 m is 2 atm—half from the water above and half from the air above. We shall see in Chapter 9.5 Pascal's Principle that fluid pressures always add in this way.

## Section Summary

- Pressure is the weight of the fluid $mg$ divided by the area $A$ supporting it (the area of the bottom of the container):
$$P = \frac{mg}{A}.$$
- Pressure due to the weight of a liquid is given by
$$P = h\rho g,$$
where $P$ is the pressure, $h$ is the height of the liquid, $\rho$ is the density of the liquid, and $g$ is the acceleration due to gravity.

## *Conceptual Questions*

**1:** Atmospheric pressure exerts a large force (equal to the weight of the atmosphere above your body—about 10 tons) on the top of your body when you are lying on the beach sunbathing. Why are you able to get up?

**2:** Why does atmospheric pressure decrease more rapidly than linearly with altitude?

**3:** What are two reasons why mercury rather than water is used in barometers?

**4:** Figure 4 shows how sandbags placed around a leak outside a river levee can effectively stop the flow of water under the levee. Explain how the small amount of water inside the column formed by the sandbags is able to balance the much larger body of water behind the levee.

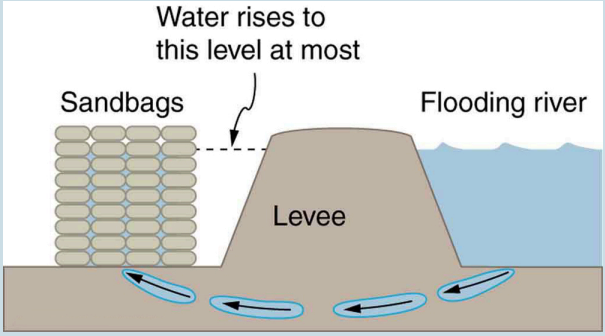



***Figure 4.*** *Because the river level is very high, it has started to leak under the levee. Sandbags are placed around the leak, and the water held by them rises until it is the same level as the river, at which point the water there stops rising.*

**5:** Why is it difficult to swim under water in the Great Salt Lake?

**6:** Is there a net force on a dam due to atmospheric pressure? Explain your answer.

**7:** Does atmospheric pressure add to the gas pressure in a rigid tank? In a toy balloon? When, in general, does atmospheric pressure *not* affect the total pressure in a fluid?

**8:** You can break a strong wine bottle by pounding a cork into it with your fist, but the cork must press directly against the liquid filling the bottle—there can be no air between the cork and liquid. Explain why the bottle breaks, and why it will not if there is air between the cork and liquid.

## *Problems & Exercises*

**1:** What depth of mercury creates a pressure of 1.00 atm?

**2:** The greatest ocean depths on the Earth are found in the Marianas Trench near the Philippines. Calculate the pressure due to the ocean at the bottom of this trench, given its depth is 11.0 km and assuming the density of seawater is constant all the way down.

**3:** Verify that the SI unit of $h\rho g$ is $\mathbf{N/m^2}$.

**4:** Water towers store water above the level of consumers for times of heavy use, eliminating the need for high-speed pumps. How high above a user must the water level be to create a gauge pressure of $\mathbf{3.00 \times 10^5\ N/m^2}$?

**5:** The aqueous humor in a person's eye is exerting a force of 0.300 N on the $\mathbf{1.10 - cm^2}$ area of the cornea. (a) What pressure is this in mm Hg? (b) Is this value within the normal range for pressures in the eye?

**6:** How much force is exerted on one side of an 8.50 cm by 11.0 cm sheet of paper by the atmosphere? How can the paper withstand such a force?

**7:** What pressure is exerted on the bottom of a 0.500-m-wide by 0.900-m-long gas tank that can hold 50.0 kg of gasoline by the weight of the gasoline in it when it is full?

**8:** Calculate the average pressure exerted on the palm of a shot-putter's hand by the shot if the area of contact is

$\mathbf{50.0\ cm^2}$ and he exerts a force of 800 N on it. Express the pressure in $\mathbf{N/m^2}$ and compare it with the $\mathbf{1.00 \times 10^6\ Pa}$ pressures sometimes encountered in the skeletal system.

**9:** The left side of the heart creates a pressure of 120 mm Hg by exerting a force directly on the blood over an effective area of $\mathbf{15.0\ cm^2}$. What force does it exert to accomplish this?

**10:** Show that the total force on a rectangular dam due to the water behind it increases with the *square* of the water depth. In particular, show that this force is given by $\boldsymbol{F = \rho g h^2\ L/2}$, where $\boldsymbol{\rho}$ is the density of water, $\boldsymbol{h}$ is its depth at the dam, and $\boldsymbol{L}$ is the length of the dam. You may assume the face of the dam is vertical. (Hint: Calculate the average pressure exerted and multiply this by the area in contact with the water. (See Figure 5.)

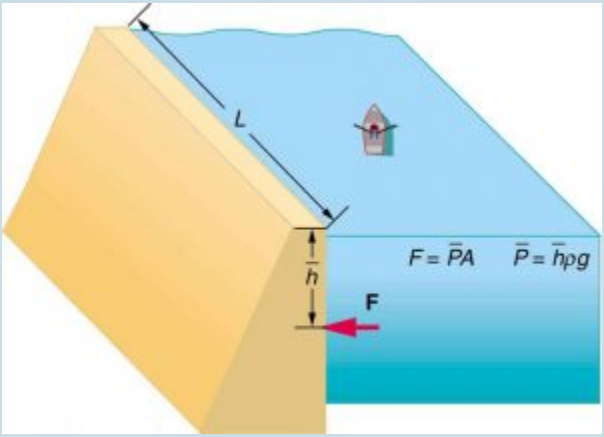



***Figure 5.***

## Glossary

**pressure**
the weight of the fluid divided by the area supporting it

## *Solutions*

**Problems & Exercises**

**1:** 0.760 m

**3:**

$$\begin{aligned}(h\rho g)_{\text{units}} &= (\text{m})(\text{kg/m}^3)(\text{m/s}^2) = (\text{kg}\cdot\text{m}^2)/(\text{m}^3\cdot\text{s}^2) \\ &= (\text{kg}\cdot\text{m/s}^2)(1/\text{m}^2) \\ &= \text{N/m}^2\end{aligned}$$

**5:** (a) 20.5 mm Hg

(b) The range of pressures in the eye is 12–24 mm Hg, so the result in part (a) is within that range

**7:** $\mathbf{1.09 \times 10^3\ N/m^2}$

**9:** 24.0 N

# 54. 9.5 Pascal's Principle

*Summary*

- Define pressure.
- State Pascal's principle.
- Apply of Pascal's principle.
- Derive relationships between forces in a hydraulic system.

**Pressure** is defined as force per unit area. Can pressure be increased in a fluid by pushing directly on the fluid? Yes, but it is much easier if the fluid is enclosed. The heart, for example, increases blood pressure by pushing directly on the blood in an enclosed system (valves closed in a chamber). If you try to push on a fluid in an open system, such as a river, the fluid flows away. An enclosed fluid cannot flow away, and so pressure is more easily increased by an applied force.

What happens to a pressure in an enclosed fluid? Since atoms in a fluid are free to move about, they transmit the pressure to all parts of the fluid and to the walls of the container. Remarkably, the pressure is transmitted *undiminished*. This phenomenon is called **Pascal's principle**, because it was first clearly stated by the French philosopher and scientist Blaise Pascal (1623–1662): A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

## PASCAL'S PRINCIPLE

A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

Pascal's principle, an experimentally verified fact, is what makes pressure so important in fluids. Since a change in pressure is transmitted undiminished in an enclosed fluid, we often know more about pressure than other physical quantities in fluids. Moreover, Pascal's principle implies that *the total pressure in a fluid is the sum of the pressures from different sources*. We shall find this fact—that pressures add—very useful.

Blaise Pascal had an interesting life in that he was home-schooled by his father who removed all of the mathematics textbooks from his house and forbade him to study mathematics until the age of 15. This, of course, raised the boy's curiosity, and by the age of 12, he started to teach himself geometry. Despite this early deprivation, Pascal went on to make major contributions in the mathematical fields of probability theory, number theory, and geometry. He is also well known for being the inventor of the first mechanical digital calculator, in addition to his contributions in the field of fluid statics.

# Application of Pascal's Principle

One of the most important technological applications of Pascal's principle is found in a *hydraulic system*, which is an enclosed fluid system used to exert forces. The most common hydraulic systems are those that operate car brakes. Let us first consider the simple hydraulic system shown in Figure 1.

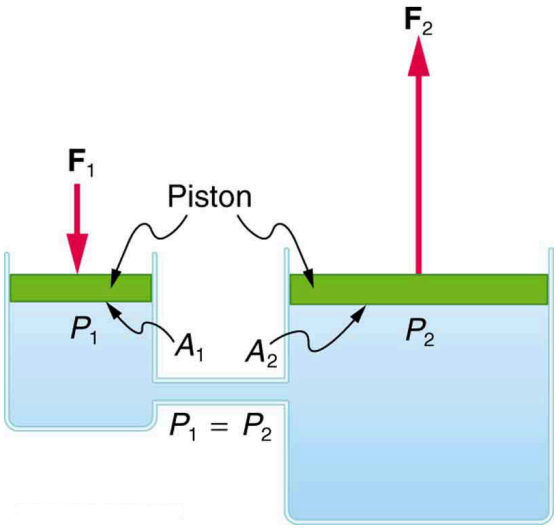



***Figure 1.*** *A typical hydraulic system with two fluid-filled cylinders, capped with pistons and connected by a tube called a hydraulic line. A downward force* $\mathbf{F_1}$ *on the left piston creates a pressure that is transmitted undiminished to all parts of the enclosed fluid. This results in an upward force* $\mathbf{F_2}$ *on the right piston that is larger than* $\mathbf{F_1}$ *because the right piston has a larger area.*

## Relationship Between Forces in a Hydraulic System

We can derive a relationship between the forces in the simple hydraulic system shown in Figure 1 by applying Pascal's principle. Note first that the two pistons in the system are at the same height, and so there will be no difference in pressure due to a difference in depth. Now the pressure due to $F_1$ acting on area $A_1$ is simply $P_1 = \frac{F_1}{A_1}$, as defined by $P = \frac{F}{A}$. According to Pascal's principle, this pressure is transmitted undiminished throughout the fluid and to all walls of the container. Thus, a pressure $P_2$ is felt at the other piston that is equal to $P_1$. That is $P_1 = P_2$.

But since $P_2 = \frac{F_2}{A_2}$, we see that $\frac{F_1}{A_1} = \frac{F_2}{A_2}$.

This equation relates the ratios of force to area in any hydraulic system, providing the pistons are at the same vertical height and that friction in the system is negligible. Hydraulic systems can increase or decrease the force applied to them. To make the force larger, the pressure is applied to a larger area. For example, if a 100-N force is applied to the left cylinder in Figure 1 and the right one has an area five times greater, then the force out is 500 N. Hydraulic systems are analogous to simple levers, but they have the advantage that pressure can be sent through tortuously curved lines to several places at once.

## Example 1: Calculating Force of Slave Cylinders: Pascal Puts on the Brakes

Consider the automobile hydraulic system shown in Figure 2.

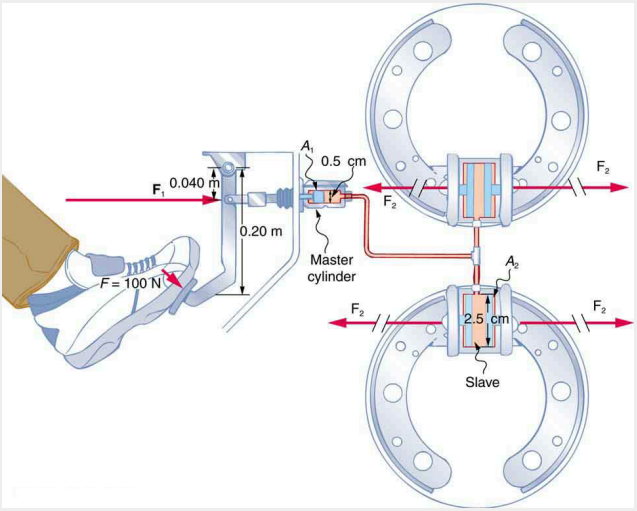



***Figure 2.*** *Hydraulic brakes use Pascal's principle. The driver exerts a force of 100 N on the brake pedal. This force is increased by the simple lever and again by the hydraulic system. Each of the identical slave cylinders receives the same pressure and, therefore, creates the same force output* $\mathbf{F_2}$*. The circular cross-sectional areas of the master and slave cylinders are represented by* $\mathbf{A_1}$ *and* $\mathbf{A_2}$*, respectively.*

A force of 100 N is applied to the brake pedal, which

acts on the cylinder—called the master—through a lever. A force of 500 N is exerted on the master cylinder. (The reader can verify that the force is 500 N using techniques of statics from Chapter 8.3 Applications of Statics, Including Problem-Solving Strategies.) Pressure created in the master cylinder is transmitted to four so-called slave cylinders. The master cylinder has a diameter of 0.500 cm, and each slave cylinder has a diameter of 2.50 cm. Calculate the force $\mathbf{F_2}$ created at each of the slave cylinders.

**Strategy**

We are given the force **F1** that is applied to the master cylinder. The cross-sectional areas $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$ can be calculated from their given diameters. Then

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

can be used to find the force $\mathbf{F_2}$. Manipulate this algebraically to get $\mathbf{F_2}$ on one side and substitute known values:

**Solution**

Pascal's principle applied to hydraulic systems is given by $\boldsymbol{\frac{F_1}{A_1} = \frac{F_2}{A_2}}$:

$$\boldsymbol{F_2 = \frac{A_2}{A_1} F_1 = \frac{\pi r_2^2}{\pi r_1^2} F_1 = \frac{(1.25\ \text{cm})^2}{(0.250\ \text{cm})^2}}$$

$$\times 500\ \text{N} = 1.25 \times 10^4\ \text{N}.$$

**Discussion**

This value is the force exerted by each of the four slave cylinders. Note that we can add as many slave cylinders as we wish. If each has a 2.50-cm diameter, each will exert 1.25 x 10 $^4$ N.

A simple hydraulic system, such as a simple machine, can increase force but cannot do more work than done on it. Work is force times distance moved, and the slave cylinder moves through a smaller distance than the master cylinder. Furthermore, the more slaves added, the smaller the distance each moves. Many hydraulic systems—such as power brakes and those in bulldozers—have a motorized pump that actually does most of the work in the system. The movement of the legs of a spider is achieved partly by hydraulics. Using hydraulics, a jumping spider can create a force that makes it capable of jumping 25 times its length!

## MAKING CONNECTIONS: CONSERVATION OF ENERGY

Conservation of energy applied to a hydraulic system tells us that the system cannot do more work than is done on it. Work transfers energy, and so the work output cannot exceed the work input. Power brakes and

other similar hydraulic systems use pumps to supply extra energy when needed.

## Section Summary

- Pressure is force per unit area.
- A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.
- A hydraulic system is an enclosed fluid system used to exert forces.

### *Conceptual Questions*

**1:** Suppose the master cylinder in a hydraulic system is at a greater height than the slave cylinder. Explain how this will affect the force produced at the slave cylinder.

### *Problems & Exercises*

**1:** How much pressure is transmitted in the hydraulic

system considered in Example 1? Express your answer in pascals and in atmospheres.

**2:** What force must be exerted on the master cylinder of a hydraulic lift to support the weight of a 2000-kg car (a large car) resting on the slave cylinder? The master cylinder has a 2.00-cm diameter and the slave has a 24.0-cm diameter.

**3:** A crass host pours the remnants of several bottles of wine into a jug after a party. He then inserts a cork with a 2.00-cm diameter into the bottle, placing it in direct contact with the wine. He is amazed when he pounds the cork into place and the bottom of the jug (with a 14.0-cm diameter) breaks away. Calculate the extra force exerted against the bottom if he pounded the cork with a 120-N force.

**4:** A certain hydraulic system is designed to exert a force 100 times as large as the one put into it. (a) What must be the ratio of the area of the slave cylinder to the area of the master cylinder? (b) What must be the ratio of their diameters? (c) By what factor is the distance through which the output force moves reduced relative to the distance through which the input force moves? Assume no losses to friction.

**5:** (a) Verify that work input equals work output for a hydraulic system assuming no losses to friction. Do this by showing that the distance the output force moves is reduced by the same factor that the output force is increased. Assume the volume of the fluid is constant. (b) What effect would friction within the fluid and between

components in the system have on the output force? How would this depend on whether or not the fluid is moving?

## Glossary

**Pascal's Principle**
a change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container

## *Solutions*

**Problems & Exercises**

**1:** 2.55 x $10^7$ Pa; or 251 atm

**3:** 5.76×$10^3$ N extra force

**5:** (a

$$V = d_i A_i = d_o A_o \Rightarrow d_o = d_i\left(\frac{A_i}{A_o}\right).$$

Now, using equation:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_o = F_i\left(\frac{A_o}{A_i}\right).$$

Finally,

$$W_o = F_o d_o = \left(\frac{F_i A_o}{A_i}\right)\left(\frac{d_i A_i}{A_o}\right)$$
$$= F_i d_i = W_i.$$

In other words, the work output equals the work input.

(b) If the system is not moving, friction would not play a role. With friction, we know there are losses, so that $W_{out} = W_{in} - W_f$; therefore, the work output is less than the work input. In other words, with friction, you need to push harder on the input piston than was calculated for the non-friction case.

# 55. 9.6 Archimedes' Principle

## *Summary*

- Define buoyant force.
- State Archimedes' principle.
- Explain why objects float or sink.
- Explain the relationship between density and Archimedes' principle.

When you rise from lounging in a warm bath, your arms feel strangely heavy. This is because you no longer have the buoyant support of the water. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected? (See Figure 1.)

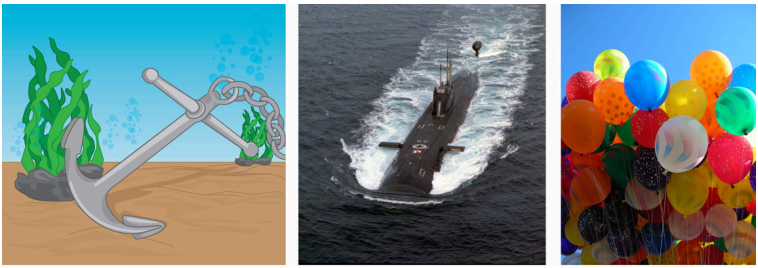



***Figure 1.*** *(a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (credit: Allied Navy) (c) Helium-filled balloons tug upward on their strings, demonstrating air's buoyant effect. (credit: Crystl)*

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on the top of the object. There is a net upward, or **buoyant force** on any object in any fluid. (See Figure 2.) If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.

### BUOYANT FORCE

The buoyant force is the net upward force on any object in any fluid.

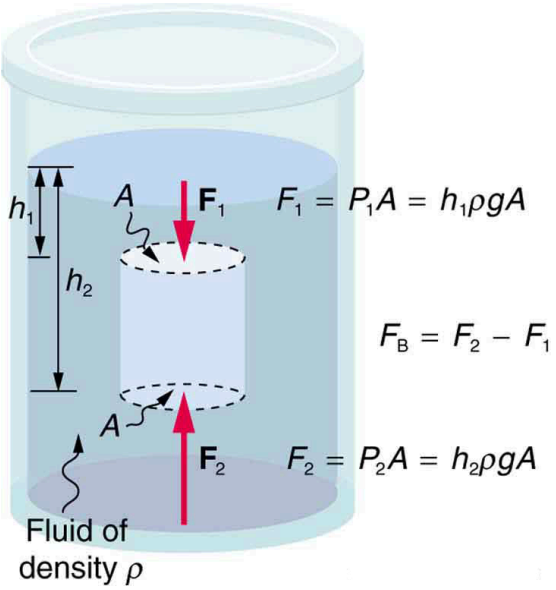



***Figure 2.*** *Pressure due to the weight of a fluid increases with depth since* $\mathbf{P=h\rho g}$*. This pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. Their difference is the buoyant force* $\mathbf{F_B}$*. (Horizontal forces cancel.)*

Just how great is this buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in Figure 3.

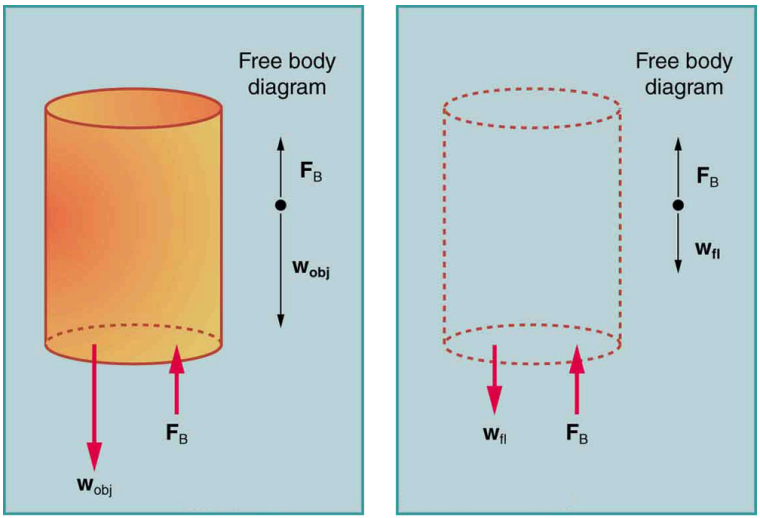



***Figure 3.*** *(a) An object submerged in a fluid experiences a buoyant force* $\mathbf{F_B}$*. If* $\mathbf{F_B}$ *is greater than the weight of the object, the object will rise. If* $\mathbf{F_B}$ *is less than the weight of the object, the object will sink. (b) If the object is removed, it is replaced by fluid having weight* $\boldsymbol{w_{fl}}$*. Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced. That is,* $\mathbf{F_B = w_{fl}}$*, a statement of Archimedes' principle.*

The space it occupied is filled by fluid having a weight $\boldsymbol{w_{fl}}$. This weight is supported by the surrounding fluid, and so the buoyant force must equal $\boldsymbol{w_{fl}}$, the weight of the fluid displaced by the object. It is a tribute to the genius of the Greek mathematician and inventor Archimedes (ca. 287–212 B.C.) that he stated this principle long before concepts of force were well established. Stated in words, Archimedes' principle is as follows: The buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$\boldsymbol{F_B = w_{fl}},$$

where $\boldsymbol{F_B}$ is the buoyant force and $\boldsymbol{w_{fl}}$ is the weight of the fluid

displaced by the object. Archimedes' principle is valid in general, for any object in any fluid, whether partially or totally submerged.

## ARCHIMEDES' PRINCIPLE

According to this principle the buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_B = w_{fl},$$

where $F_B$ is the buoyant force and $w_{fl}$ is the weight of the fluid displaced by the object.

*Humm* ... High-tech body swimsuits were introduced in 2008 in preparation for the Beijing Olympics. One concern (and international rule) was that these suits should not provide any buoyancy advantage. How do you think that this rule could be verified?

## MAKING CONNECTIONS: TAKE-HOME INVESTIGATION

The density of aluminum foil is 2.7 times the density of

water. Take a piece of foil, roll it up into a ball and drop it into water. Does it sink? Why or why not? Can you make it sink?

## Floating and Sinking

Drop a lump of clay in water. It will sink. Then mold the lump of clay into the shape of a boat, and it will float. Because of its shape, the boat displaces more water than the lump and experiences a greater buoyant force. The same is true of steel ships.

### Example 1: Calculating buoyant force: dependency on shape

(a) Calculate the buoyant force on 10,000 metric tons $(1.00 \times 10^7\ \text{kg})$ of solid steel completely submerged in water, and compare this with the steel's weight. (b) What is the maximum buoyant force that water could exert on this same steel if it were shaped into a boat that could displace $1.00 \times 10^5\ \text{m}^3$ of water?

**Strategy for (a)**

To find the buoyant force, we must find the weight of water displaced. We can do this by using the densities of water and steel given in Table 1. We note that, since the steel is completely submerged, its volume and the water's volume are the same. Once we know the volume of water, we can find its mass and weight.

**Solution for (a)**

First, we use the definition of density $\rho = \frac{m}{V}$ to find the steel's volume, and then we substitute values for mass and density. This gives

$$V_{\text{st}} = \frac{m_{\text{st}}}{\rho_{\text{st}}} = \frac{1.00 \times 10^7 \text{ kg}}{7.8 \times 10^3 \text{ kg/m}^3} = 1.28 \times 10^3 \text{ m}^3.$$

Because the steel is completely submerged, this is also the volume of water displaced, $V_{\text{w}}$. We can now find the mass of water displaced from the relationship between its volume and density, both of which are known. This gives

$$\begin{aligned} m_{\text{w}} &= \rho_{\text{w}} V_{\text{w}} = (1.000 \times 10^3 \text{ kg/m}^3)(1.28 \times 10^3 \text{ m}^3) \\ &= 1.28 \times 10^6 \text{ kg}. \end{aligned}$$

By Archimedes' principle, the weight of water displaced is $m_{\text{w}} g$, so the buoyant force is

$$\begin{aligned} F_{\text{B}} &= w_{\text{w}} = m_{\text{w}} g = (1.28 \times 10^6 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 1.3 \times 10^7 \text{ N}. \end{aligned}$$

The steel's weight is $m_{\text{w}} g = 9.80 \times 10^7 \text{ N}$, which is much greater than the buoyant force, so the steel will remain submerged. Note that the buoyant

force is rounded to two digits because the density of steel is given to only two digits.

**Strategy for (b)**

Here we are given the maximum volume of water the steel boat can displace. The buoyant force is the weight of this volume of water.

**Solution for (b)**

The mass of water displaced is found from its relationship to density and volume, both of which are known. That is,

$$\begin{aligned} m_{\text{w}} &= \rho_{\text{w}} V_{\text{w}} = (1.000 \times 10^3 \text{ kg/m}^3)(1.00 \times 10^5 \text{ m}^3) \\ &= 1.00 \times 10^8 \text{ kg}. \end{aligned}$$

The maximum buoyant force is the weight of this much water, or

$$\begin{aligned} F_{\text{B}} &= w_{\text{w}} = m_{\text{w}} g = (1.00 \times 10^8 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 9.80 \times 10^8 \text{ N}. \end{aligned}$$

**Discussion**

The maximum buoyant force is ten times the weight of the steel, meaning the ship can carry a load nine times its own weight without sinking.

## MAKING CONNECTIONS:

### TAKE-HOME INVESTIGATION

A piece of household aluminum foil is 0.016 mm thick. Use a piece of foil that measures 10 cm by 15 cm. (a) What is the mass of this amount of foil? (b) If the foil is folded to give it four sides, and paper clips or washers are added to this "boat," what shape of the boat would allow it to hold the most "cargo" when placed in water? Test your prediction.

## Density and Archimedes' Principle

Density plays a crucial role in Archimedes' principle. The average density of an object is what ultimately determines whether it floats. If its average density is less than that of the surrounding fluid, it will float. This is because the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object's density is related to that of the fluid. In Figure 4, for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship loaded. We can derive a quantitative expression for the fraction submerged by considering

density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or

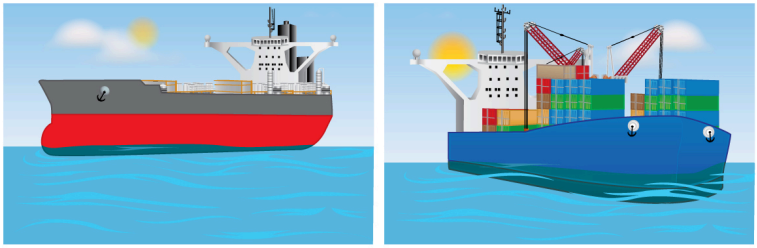
$$\textbf{fraction submerged} = \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{V_{\text{fl}}}{V_{\text{obj}}}.$$

The volume submerged equals the volume of fluid displaced, which we call $V_{\text{fl}}$. Now we can obtain the relationship between the densities by substituting $\rho = \frac{m}{V}$ into the expression. This gives

$$\frac{V_{\text{fl}}}{V_{\text{obj}}} = \frac{m_{\text{fl}}/\rho_{\text{fl}}}{m_{\text{obj}}/\bar{\rho}_{\text{obj}}},$$

where $\bar{\rho}_{\text{obj}}$ is the average density of the object and $\rho_{\text{fl}}$ is the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, and so they cancel from the equation, leaving

$$\textbf{fraction submerged} = \frac{\bar{\rho}_{\text{obj}}}{\rho_{\text{fl}}}.$$



(a) (b)

***Figure 4.*** *An unloaded ship (a) floats higher in the water than a loaded ship (b).*

We use this last relationship to measure densities. This is done by measuring the fraction of a floating object that is submerged–for example, with a hydrometer. It is useful to define the ratio of the density of an object to a fluid (usually water) as **specific gravity**:

$$\textbf{specific gravity} = \frac{\bar{\rho}}{\rho_{\text{w}}},$$

where $\bar{\rho}$ is the average density of the object or substance and $\rho_{\text{w}}$ is the density of water at 4.00°C. Specific gravity is dimensionless, independent of whatever units are used for $\rho$. If an object floats, its specific gravity is less than one. If it sinks, its specific gravity is greater than one. Moreover, the fraction of a floating object that is submerged equals its specific gravity. If an object's specific gravity is exactly 1, then it will remain suspended in the fluid, neither sinking nor floating. Scuba divers try to obtain this state so that they can hover in the water. We measure the specific gravity of fluids, such as battery acid, radiator fluid, and urine, as an indicator of their condition. One device for measuring specific gravity is shown in Figure 5.

SPECIFIC GRAVITY

Specific gravity is the ratio of the density of an object to a fluid (usually water).

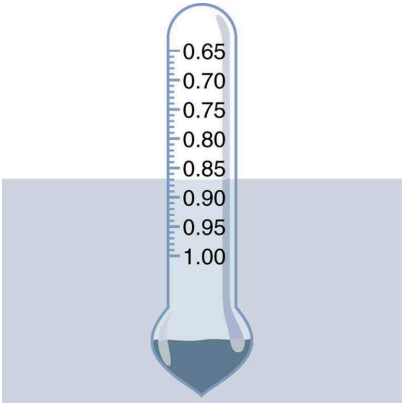



***Figure 5.*** *This hydrometer is floating in a fluid of specific gravity 0.87. The glass hydrometer is filled with air and weighted with lead at the bottom. It floats highest in the densest fluids and has been calibrated and labeled so that specific gravity can be read from it directly.*

## Example 2: Calculating Average Density: Floating Woman

Suppose a 60.0-kg woman floats in freshwater with **97.0%** of her volume submerged when her lungs are full of air. What is her average density?

**Strategy**

We can find the woman's density by solving the equation

$$\textbf{fraction submerged} = \frac{\bar{\rho}_{\textbf{obj}}}{\rho_{\textbf{fl}}}$$

for the density of the object. This yields

$$\bar{\rho}_{\textbf{obj}} = \bar{\rho}_{\textbf{person}} = (\textbf{fraction submerged}) \cdot \rho_{\textbf{fl}}.$$

We know both the fraction submerged and the density of water, and so we can calculate the woman's density.

**Solution**

Entering the known values into the expression for her density, we obtain

$$\begin{aligned} \bar{\rho}_{\textbf{person}} &= 0.970 \cdot \left(10^3 \frac{kg}{m^3}\right) \\ &= 970 \frac{\textbf{kg}}{m^3}. \end{aligned}$$

**Discussion**

Her density is less than the fluid density. We expect this because she floats. Body density is one indicator of a person's percent body fat, of interest in medical diagnostics and athletic training. (See Figure 6.)

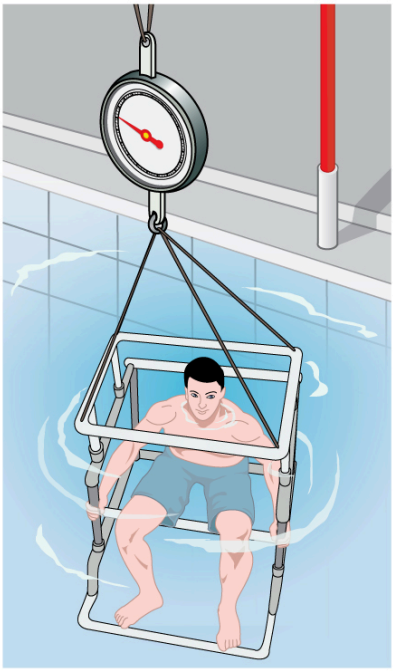


***Figure 6.*** *Subject in a "fat tank," where he is weighed while completely submerged as part of a body density determination. The subject must completely empty his lungs and hold a metal weight in order to sink. Corrections are made for the residual air in his lungs (measured separately) and the metal weight. His corrected submerged weight, his weight in air, and pinch tests of strategic fatty areas are used to calculate his percent body fat.*

There are many obvious examples of lower-density objects or substances floating in higher-density fluids—oil on water, a hot-air balloon, a bit of cork in wine, an iceberg, and hot wax in a "lava lamp," to name a few. Less obvious examples include lava rising in a volcano and mountain ranges floating on the higher-density crust

and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

## More Density Measurements

One of the most common techniques for determining density is shown in Figure 7.

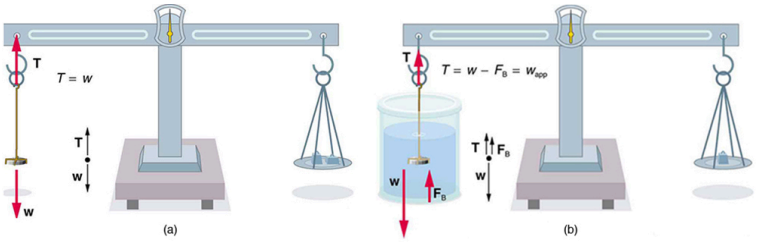



***Figure 7.*** *(a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin.*

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. This same technique can also be used to determine the density of the fluid if the density of the coin is known. All of these calculations are based on Archimedes' principle.

Archimedes' principle states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object *appears* to weigh less when submerged; we call this measurement the object's *apparent weight*. The object suffers an *apparent weight loss* equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an *apparent mass loss* equal to the mass of fluid displaced. That is

$$\textbf{apparent weight loss} = \textbf{weight of fluid displaced}$$

or

$$\textbf{apparent mass loss} = \textbf{mass of fluid displaced.}$$

The next example illustrates the use of this technique.

## Example 3: Calculating Density: Is the Coin Authentic?

The mass of an ancient Greek coin is determined in air to be 8.630 g. When the coin is submerged in water as shown in Figure 7, its apparent mass is 7.800 g. Calculate its density, given that water has a density of $\mathbf{1.000\ g/cm^3}$ and that effects caused by the wire suspending the coin are negligible.

**Strategy**

To calculate the coin's density, we need its mass (which is given) and its volume. The volume of the coin equals the volume of water displaced. The volume of water displaced $V_w$ can be found by solving the equation for density $\rho = \frac{m}{V}$ for $V$.

**Solution**

The volume of water is $V_w = \frac{m_w}{\rho_w}$ where $m_w$ is the mass of water displaced. As noted, the mass of the

water displaced equals the apparent mass loss, which is

$$m_{\mathrm{w}} = 8.630\ \mathrm{g} - 7.800\ \mathrm{g} = 0.830\ \mathrm{g}.$$

Thus the volume of water is

$$V_{\mathrm{w}} = \frac{0.830\ \mathrm{g}}{1.000\ \mathrm{g/cm^3}} = 0.830\ \mathrm{cm^3}.$$

This is also the volume of the coin, since it is completely submerged. We can now find the density of the coin using the definition of density:

$$\rho_{\mathrm{c}} = \frac{m_{\mathrm{c}}}{V_{\mathrm{c}}} = \frac{8.630\ \mathrm{g}}{0.830\ \mathrm{cm^3}} = 10.4\ \mathrm{g/cm^3}.$$

**Discussion**

You can see from Table 1 that this density is very close to that of pure silver, appropriate for this type of ancient coin. Most modern counterfeits are not pure silver.

This brings us back to Archimedes' principle and how it came into being. As the story goes, the king of Syracuse gave Archimedes the task of determining whether the royal crown maker was supplying a crown of pure gold. The purity of gold is difficult to determine by color (it can be diluted with other metals and still look as yellow as pure gold), and other analytical techniques had not yet been conceived. Even ancient peoples, however, realized that the density of gold was greater than that of any other then-known substance. Archimedes purportedly agonized over his task and had his inspiration one day while at the public baths, pondering the support the water gave his body. He came up with his now-famous principle, saw how to apply it to determine density, and ran naked down the streets of Syracuse crying "Eureka!" (Greek for "I have found it").

Similar behavior can be observed in contemporary physicists from time to time!

PHET EXPLORATIONS: BUOYANCY

When will objects float and when will they sink? Learn how buoyancy works with blocks. Arrows show the applied forces, and you can modify the properties of the blocks and the fluid.



PhET Interactive Simulation

***Figure 8.*** *Buoyancy*

## Section Summary

- Buoyant force is the net upward force on any object in any fluid. If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.

- Archimedes' principle states that the buoyant force on an object equals the weight of the fluid it displaces.
- Specific gravity is the ratio of the density of an object to a fluid (usually water).

## Conceptual Questions

### *Conceptual Questions*

**1:** More force is required to pull the plug in a full bathtub than when it is empty. Does this contradict Archimedes' principle? Explain your answer.

**2:** Do fluids exert buoyant forces in a "weightless" environment, such as in the space shuttle? Explain your answer.

**3:** Will the same ship float higher in salt water than in freshwater? Explain your answer.

**4:** Marbles dropped into a partially filled bathtub sink to the bottom. Part of their weight is supported by buoyant force, yet the downward force on the bottom of the tub increases by exactly the weight of the marbles. Explain why.

## *Problems & Exercises*

**1:** What fraction of ice is submerged when it floats in freshwater, given the density of ice is 917 $kg/m^3$ and the density of freshwater at 0°C is very close to 1000 $kg/m^3$?

**2:** Logs sometimes float vertically in a lake because one end has become water-logged and denser than the other. What is the average density of a uniform-diameter log that floats with 20.0% of its length above water?

**3:** Find the density of a fluid in which a hydrometer having a density of $\mathbf{0.750\ g/mL}$ floats with 92.0 % of its volume submerged.

**4:** If your body has a density of 995 $kg/m^3$, what fraction of you will be submerged when floating gently in: (a) Freshwater? (b) Salt water, which has a density of 1027 $kg/m^3$?

**5:** Bird bones have air pockets in them to reduce their weight—this also gives them an average density significantly less than that of the bones of other animals. Suppose an ornithologist weighs a bird bone in air and in water and finds its mass is $\mathbf{45.0\ g}$ and its apparent mass when submerged is $\mathbf{3.60\ g}$ (the bone is watertight). (a) What mass of water is displaced? (b) What is the volume of the bone? (c) What is its average density?

**6:** A rock with a mass of 540 g in air is found to have an apparent mass of 342 g when submerged in water. (a) What mass of water is displaced? (b) What is the volume of the

rock? (c) What is its average density? Is this consistent with the value for granite?

**7:** Archimedes' principle can be used to calculate the density of a fluid as well as that of a solid. Suppose a chunk of iron with a mass of 390.0 g in air is found to have an apparent mass of 350.5 g when completely submerged in an unknown liquid. (a) What mass of fluid does the iron displace? (b) What is the volume of iron, using its density as given in Table 1 (c) Calculate the fluid's density and identify it.

**8:** In an immersion measurement of a woman's density, she is found to have a mass of 62.0 kg in air and an apparent mass of 0.0850 kg when completely submerged with lungs empty. (a) What mass of water does she displace? (b) What is her volume? (c) Calculate her density. (d) If her lung capacity is 1.75 L, is she able to float without treading water with her lungs filled with air?

**9:** Some fish have a density slightly less than that of water and must exert a force (swim) to stay submerged. What force must an 85.0-kg grouper exert to stay submerged in salt water if its body density is $\mathbf{1015\ kg/m^3}$?

**10:** (a) Calculate the buoyant force on a 2.00-L helium balloon. (b) Given the mass of the rubber in the balloon is 1.50 g, what is the net vertical force on the balloon if it is let go? You can neglect the volume of the rubber.

**11:** (a) What is the density of a woman who floats in freshwater with $\mathbf{4.00\%}$ of her volume above the surface? This could be measured by placing her in a tank with marks

on the side to measure how much water she displaces when floating and when held under water (briefly). (b) What percent of her volume is above the surface when she floats in seawater?

**12:** A certain man has a mass of 80 kg and a density of $\mathbf{955\ kg/m^3}$ (excluding the air in his lungs). (a) Calculate his volume. (b) Find the buoyant force air exerts on him. (c) What is the ratio of the buoyant force to his weight?

**13:** A simple compass can be made by placing a small bar magnet on a cork floating in water. (a) What fraction of a plain cork will be submerged when floating in water? (b) If the cork has a mass of 10.0 g and a 20.0-g magnet is placed on it, what fraction of the cork will be submerged? (c) Will the bar magnet and cork float in ethyl alcohol?

**14:** What fraction of an iron anchor's weight will be supported by buoyant force when submerged in saltwater?

**15:** Scurrilous con artists have been known to represent gold-plated tungsten ingots as pure gold and sell them to the greedy at prices much below gold value but deservedly far above the cost of tungsten. With what accuracy must you be able to measure the mass of such an ingot in and out of water to tell that it is almost pure tungsten rather than pure gold?

**16:** A twin-sized air mattress used for camping has dimensions of 100 cm by 200 cm by 15 cm when blown up. The weight of the mattress is 2 kg. How heavy a person could the air mattress hold if it is placed in freshwater?

**17:** Referring to Figure 3, prove that the buoyant force on the cylinder is equal to the weight of the fluid displaced (Archimedes' principle). You may assume that the buoyant force is $F_2 - F_1$ and that the ends of the cylinder have equal areas $A$. Note that the volume of the cylinder (and that of the fluid it displaces) equals $(h_2 - h_1)A$.

**18:** (a) A 75.0-kg man floats in freshwater with $3.00\%$ of his volume above water when his lungs are empty, and $5.00\%$ of his volume above water when his lungs are full. Calculate the volume of air he inhales—called his lung capacity—in liters. (b) Does this lung volume seem reasonable?

## Glossary

**Archimedes' principle**
: the buoyant force on an object equals the weight of the fluid it displaces

**buoyant force**
: the net upward force on any object in any fluid

**specific gravity**
: the ratio of the density of an object to a fluid (usually water)

## *Solutions*

**Problems & Exercises**

**1:** $\mathbf{91.7\%}$

**3:** $\mathbf{815\ kg/m^3}$

**5:** (a) 41.4 g (b)$\mathbf{41.4\ cm^3}$ (c)$\mathbf{1.09\ g/cm^3}$

**7:** (a) 39.5 g

(b)$\mathbf{50\ cm^3}$

(c)$\mathbf{0.79\ g/cm^3}$

It is ethyl alcohol.

**9:** 8.21 N

**11:** (a)$\mathbf{960\ kg/m^3}$ (b)$\mathbf{6.34\%}$

She indeed floats more in seawater.

**13:** (a)$\mathbf{0.24}$

(b)$\mathbf{0.68}$

(c) Yes, the cork will float because $\rho_{\text{obj}} < \rho_{\text{ethyl alcohol}} (0.678\ \text{g/cm}^3 < 0.79\ \text{g/cm}^3)$

**15:**

The difference is$\mathbf{0.006\%}$.

**17:**

$$\begin{aligned} F_{\text{net}} &= F_2 - F_1 = P_2 A - P_1 A = (P_2 - P_1) A \\ &= (h_2 \rho_{\text{fl}} g - h_1 \rho_{\text{fl}} g) A \\ &= (h_2 - h_1) \rho_{\text{fl}} g A \end{aligned}$$

where $\rho_{\text{fl}}$ = density of fluid. Therefore,

$$F_{\text{net}} = (h_2 - h_1) A \rho_{\text{fl}} g = V_{\text{fl}} \rho_{\text{fl}} g = m_{\text{fl}} g = w_{\text{fl}}$$

where is $w_{\text{fl}}$ the weight of the fluid displaced.

# 56. 9.7 Pressures in the Body

*Summary*

- Explain the concept of pressure the in human body.
- Explain systolic and diastolic blood pressures.
- Describe pressures in the eye, lungs, spinal column, bladder, and skeletal system.

## Pressure in the Body

Next to taking a person's temperature and weight, measuring blood pressure is the most common of all medical examinations. Control of high blood pressure is largely responsible for the significant decreases in heart attack and stroke fatalities achieved in the last three decades. The pressures in various parts of the body can be measured and often provide valuable medical indicators. In this section, we consider a few examples together with some of the physics that accompanies them.

Table 5 lists some of the measured pressures in mm Hg, the units most commonly quoted.

| Body system | Gauge pressure in mm Hg |
|---|---|
| Blood pressures in large arteries (resting) | |
| *Maximum (systolic)* | 100–140 |
| *Minimum (diastolic)* | 60–90 |
| Blood pressure in large veins | 4–15 |
| Eye | 12–24 |
| Brain and spinal fluid (lying down) | 5–12 |
| Bladder | |
| *While filling* | 0–25 |
| *When full* | 100–150 |
| Chest cavity between lungs and ribs | −8 to −4 |
| Inside lungs | −2 to +3 |
| Digestive tract | |
| *Esophagus* | −2 |
| *Stomach* | 0–20 |
| *Intestines* | 10–20 |
| Middle ear | <1 |

**Table 5.** Typical Pressures in Humans

## Blood Pressure

Common arterial blood pressure measurements typically produce values of 120 mm Hg and 80 mm Hg, respectively, for systolic and diastolic pressures. Both pressures have health implications. When systolic pressure is chronically high, the risk of stroke and heart attack is increased. If, however, it is too low, fainting is a problem. **Systolic pressure** increases dramatically during exercise to increase blood flow and returns to normal afterward. This change produces no ill effects and, in fact, may be beneficial to the tone of the circulatory system. **Diastolic pressure** can be an indicator of fluid

balance. When low, it may indicate that a person is hemorrhaging internally and needs a transfusion. Conversely, high diastolic pressure indicates a ballooning of the blood vessels, which may be due to the transfusion of too much fluid into the circulatory system. High diastolic pressure is also an indication that blood vessels are not dilating properly to pass blood through. This can seriously strain the heart in its attempt to pump blood.

Blood leaves the heart at about 120 mm Hg but its pressure continues to decrease (to almost 0) as it goes from the aorta to smaller arteries to small veins (see Figure 1). The pressure differences in the circulation system are caused by blood flow through the system as well as the position of the person. For a person standing up, the pressure in the feet will be larger than at the heart due to the weight of the blood $(P = h\rho g)$. If we assume that the distance between the heart and the feet of a person in an upright position is 1.4 m, then the increase in pressure in the feet relative to that in the heart (for a static column of blood) is given by

$$\Delta P = \Delta h\rho g = (1.4\ \text{m})(1050\ \text{kg/m}^3)(9.80\ \text{m/s}^2) = 1.4 \times 10^4\ \text{Pa} = 108\ \text{mm Hg}.$$

INCREASE IN PRESSURE IN THE FEET OF A PERSON

$$\Delta P = \Delta h\rho g = (1.4\ \text{m})(1050\ \text{kg/m}^3)(9.80\ \text{m/s}^2) = 1.4 \times 10^4\ \text{Pa} = 108\ \text{mm Hg}.$$

Standing a long time can lead to an accumulation of blood in the legs and swelling. This is the reason why soldiers who are required to stand still for long periods of time have been known to faint. Elastic bandages around the calf can help prevent this accumulation and can also help provide increased pressure to enable the veins to send blood back up to the heart. For similar reasons, doctors recommend tight stockings for long-haul flights.

Blood pressure may also be measured in the major veins, the heart chambers, arteries to the brain, and the lungs. But these pressures are usually only monitored during surgery or for patients in intensive care since the measurements are invasive. To obtain these pressure measurements, qualified health care workers thread thin tubes, called catheters, into appropriate locations to transmit pressures to external measuring devices.

The heart consists of two pumps—the right side forcing blood through the lungs and the left causing blood to flow through the rest of the body (Figure 1). Right-heart failure, for example, results in a rise in the pressure in the vena cavae and a drop in pressure in the arteries to the lungs. Left-heart failure results in a rise in the pressure entering the left side of the heart and a drop in aortal pressure.

## TWO PUMPS OF THE HEART

The heart consists of two pumps—the right side forcing blood through the lungs and the left causing blood to flow through the rest of the body.

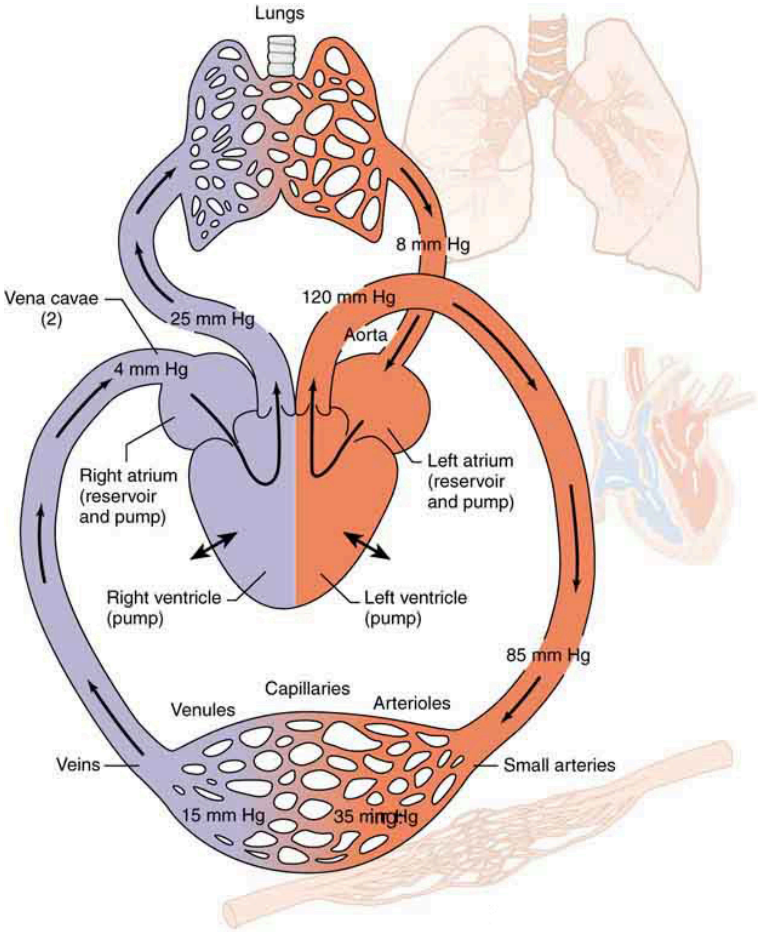



***Figure 1.*** *Schematic of the circulatory system showing typical pressures. The two pumps in the heart increase pressure and that pressure is reduced as the blood flows through the body.*

## Pressure in the Eye

The shape of the eye is maintained by fluid pressure, called **intraocular pressure**, which is normally in the range of 12.0 to 24.0

mm Hg. When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called **glaucoma**. The net pressure can become as great as 85.0 mm Hg, an abnormally large pressure that can permanently damage the optic nerve. To get an idea of the force involved, suppose the back of the eye has an area of $\mathbf{6.0\ cm^2}$, and the net pressure is 85.0 mm Hg. Force is given by $\boldsymbol{F = PA}$. To get $\boldsymbol{F}$ in newtons, we convert the area to $\mathbf{m^2\ (1\ m^2 = 10^4\ cm^2)}$. Then we calculate as follows:

$$\boldsymbol{F = h\rho gA = (85.0 \times 10^{-3}\ m)(13.6 \times 10^3\ kg/m^3)(9.80\ m/s^2)(6.0 \times 10^{-4}\ m^2) = 6.8\ N.}$$

## EYE PRESSURE

The shape of the eye is maintained by fluid pressure, called intraocular pressure. When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma. The force is calculated as

$$\boldsymbol{F = h\rho gA = (85.0 \times 10^{-3}\ m)(13.6 \times 10^3\ kg/m^3)(9.80\ m/s^2)(6.0 \times 10^{-4}\ m^2) = 6.8\ N.}$$

This force is the weight of about a 680-g mass. A mass of 680 g resting on the eye (imagine 1.5 lb resting on your eye) would be sufficient to cause it damage. (A normal force here would be the weight of about 120 g, less than one-quarter of our initial value.)

People over 40 years of age are at greatest risk of developing glaucoma and should have their intraocular pressure tested routinely. Most measurements involve exerting a force on the (anesthetized) eye over some area (a pressure) and observing the eye's response. A noncontact approach uses a puff of air and a measurement is made of the force needed to indent the eye (Figure 2). If the intraocular pressure is high, the eye will deform less and rebound more vigorously than normal. Excessive intraocular pressures can be detected reliably and sometimes controlled effectively.

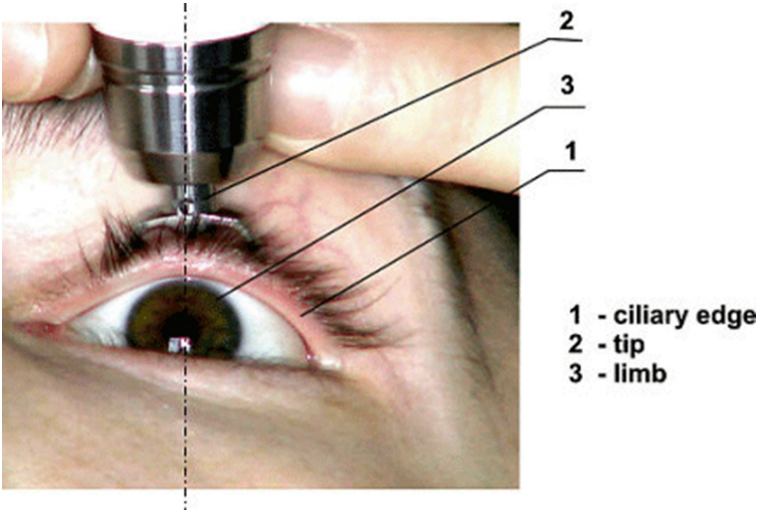



***Figure 2.*** *The intraocular eye pressure can be read with a tonometer. (credit: DevelopAll at the Wikipedia Project.)*

## Example 1: Calculating Gauge Pressure and Depth: Damage to the Eardrum

Suppose a 3.00-N force can rupture an eardrum. (a) If the eardrum has an area of $\mathbf{1.00\ cm^2}$, calculate the maximum tolerable gauge pressure on the eardrum in newtons per meter squared and convert it to millimeters of mercury. (b) At what depth in freshwater would this person's eardrum rupture, assuming the gauge pressure in the middle ear is zero?

**Strategy for (a)**

The pressure can be found directly from its definition since we know the force and area. We are looking for the gauge pressure.

**Solution for (a)**

$\boldsymbol{P_g = F/A = 3.00\ N/(1.00 \times 10^{-4}\ m^2) = 3.00 \times 10^4\ N/m^2}$. We now need to convert this to units of mm Hg:

$$\boldsymbol{P_g = 3.0 \times 10^4\ N/m^2 \left(\frac{1.0\ mm\ Hg}{133\ N/m^2}\right) = 226\ mm\ Hg.}$$

**Strategy for (b)**

Here we will use the fact that the water pressure varies linearly with depth $\boldsymbol{h}$ below the surface.

**Solution for (b)**

$P = h\rho g$ and therefore $h = P/\rho g$. Using the value above for $P$, we have

$$h = \frac{3.0 \times 10^4 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 3.06 \text{ m}.$$

**Discussion**

Similarly, increased pressure exerted upon the eardrum from the middle ear can arise when an infection causes a fluid buildup.

## Pressure Associated with the Lungs

The pressure inside the lungs increases and decreases with each breath. The pressure drops to below atmospheric pressure (negative gauge pressure) when you inhale, causing air to flow into the lungs. It increases above atmospheric pressure (positive gauge pressure) when you exhale, forcing air out.

Lung pressure is controlled by several mechanisms. Muscle action in the diaphragm and rib cage is necessary for inhalation; this muscle action increases the volume of the lungs thereby reducing the pressure within them Figure 3. Surface tension in the alveoli creates a positive pressure opposing inhalation. You can exhale without muscle action by letting surface tension in the alveoli create its own positive pressure. Muscle action can add to this positive pressure to produce forced exhalation, such as when you blow up a balloon, blow out a candle, or cough.

The lungs, in fact, would collapse due to the surface tension in the alveoli, if they were not attached to the inside of the chest wall by liquid adhesion. The gauge pressure in the liquid attaching the lungs to the inside of the chest wall is thus negative, ranging from $-4$ to $-8$ mm Hg during exhalation and inhalation, respectively. If air is allowed to enter the chest cavity, it breaks the attachment, and one or both lungs may collapse. Suction is applied to the chest cavity of surgery patients and trauma victims to reestablish negative pressure and inflate the lungs.

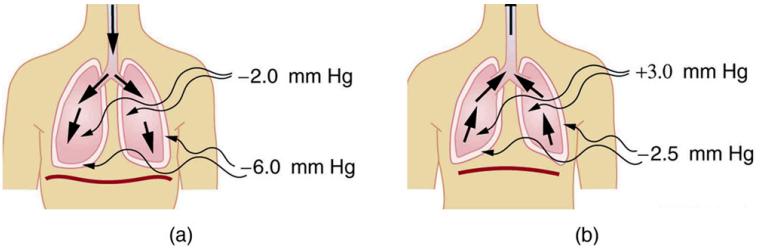



***Figure 3.*** *(a) During inhalation, muscles expand the chest, and the diaphragm moves downward, reducing pressure inside the lungs to less than atmospheric (negative gauge pressure). Pressure between the lungs and chest wall is even lower to overcome the positive pressure created by surface tension in the lungs. (b) During gentle exhalation, the muscles simply relax and surface tension in the alveoli creates a positive pressure inside the lungs, forcing air out. Pressure between the chest wall and lungs remains negative to keep them attached to the chest wall, but it is less negative than during inhalation.*

# Other Pressures in the Body

## Spinal Column and Skull

Normally, there is a 5- to12-mm Hg pressure in the fluid surrounding the brain and filling the spinal column. This cerebrospinal fluid serves many purposes, one of which is to supply flotation to the brain. The buoyant force supplied by the fluid nearly

equals the weight of the brain, since their densities are nearly equal. If there is a loss of fluid, the brain rests on the inside of the skull, causing severe headaches, constricted blood flow, and serious damage. Spinal fluid pressure is measured by means of a needle inserted between vertebrae that transmits the pressure to a suitable measuring device.

## Bladder Pressure

This bodily pressure is one of which we are often aware. In fact, there is a relationship between our awareness of this pressure and a subsequent increase in it. Bladder pressure climbs steadily from zero to about 25 mm Hg as the bladder fills to its normal capacity of $\mathbf{500\ cm^3}$. This pressure triggers the **micturition reflex**, which stimulates the feeling of needing to urinate. What is more, it also causes muscles around the bladder to contract, raising the pressure to over 100 mm Hg, accentuating the sensation. Coughing, straining, tensing in cold weather, wearing tight clothes, and experiencing simple nervous tension all can increase bladder pressure and trigger this reflex. So can the weight of a pregnant woman's fetus, especially if it is kicking vigorously or pushing down with its head! Bladder pressure can be measured by a catheter or by inserting a needle through the bladder wall and transmitting the pressure to an appropriate measuring device. One hazard of high bladder pressure (sometimes created by an obstruction), is that such pressure can force urine back into the kidneys, causing potentially severe damage.

## Pressures in the Skeletal System

These pressures are the largest in the body, due both to the high

values of initial force, and the small areas to which this force is applied, such as in the joints.. For example, when a person lifts an object improperly, a force of 5000 N may be created between vertebrae in the spine, and this may be applied to an area as small as $\mathbf{10\ cm^2}$. The pressure created is $\boldsymbol{P = F/A = (5000\ \mathrm{N})/(10^{-3}\ \mathrm{m^2}) = 5.0 \times 10^6\ \mathrm{N/m^2}}$ or about 50 atm! This pressure can damage both the spinal discs (the cartilage between vertebrae), as well as the bony vertebrae themselves. Even under normal circumstances, forces between vertebrae in the spine are large enough to create pressures of several atmospheres. Most causes of excessive pressure in the skeletal system can be avoided by lifting properly and avoiding extreme physical activity.

There are many other interesting and medically significant pressures in the body. For example, pressure caused by various muscle actions drives food and waste through the digestive system. Stomach pressure behaves much like bladder pressure and is tied to the sensation of hunger. Pressure in the relaxed esophagus is normally negative because pressure in the chest cavity is normally negative. Positive pressure in the stomach may thus force acid into the esophagus, causing "heartburn." Pressure in the middle ear can result in significant force on the eardrum if it differs greatly from atmospheric pressure, such as while scuba diving. The decrease in external pressure is also noticeable during plane flights (due to a decrease in the weight of air above relative to that at the Earth's surface). The Eustachian tubes connect the middle ear to the throat and allow us to equalize pressure in the middle ear to avoid an imbalance of force on the eardrum.

## Section Summary

- Measuring blood pressure is among the most common of all

medical examinations.

- The pressures in various parts of the body can be measured and often provide valuable medical indicators.
- The shape of the eye is maintained by fluid pressure, called intraocular pressure.
- When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma.
- Some of the other pressures in the body are spinal and skull pressures, bladder pressure, pressures in the skeletal system.

## *Problems & Exercises*

**1:** During forced exhalation, such as when blowing up a balloon, the diaphragm and chest muscles create a pressure of 60.0 mm Hg between the lungs and chest wall. What force in newtons does this pressure create on the 600 $cm^2$ surface area of the diaphragm?

**2:** You can chew through very tough objects with your incisors because they exert a large force on the small area of a pointed tooth. What pressure in pascals can you create by exerting a force of **500 N** with your tooth on an area of **1.00 $mm^2$**?

**3:** One way to force air into an unconscious person's lungs is to squeeze on a balloon appropriately connected to the subject. What force must you exert on the balloon with your hands to create a gauge pressure of 4.00 cm water, assuming you squeeze on an effective area of **50.0 $cm^2$**?

**4:** Heroes in movies hide beneath water and breathe

through a hollow reed (villains never catch on to this trick). In practice, you cannot inhale in this manner if your lungs are more than 60.0 cm below the surface. What is the maximum negative gauge pressure you can create in your lungs on dry land, assuming you can achieve $\mathbf{-3.00\ cm}$ water pressure with your lungs 60.0 cm below the surface?

**5:** Gauge pressure in the fluid surrounding an infant's brain may rise as high as 85.0 mm Hg (5 to 12 mm Hg is normal), creating an outward force large enough to make the skull grow abnormally large. (a) Calculate this outward force in newtons on each side of an infant's skull if the effective area of each side is $\mathbf{70.0\ cm^2}$. (b) What is the net force acting on the skull?

**6:** A full-term fetus typically has a mass of 3.50 kg. (a) What pressure does the weight of such a fetus create if it rests on the mother's bladder, supported on an area of $\mathbf{90.0\ cm^2}$? (b) Convert this pressure to millimeters of mercury and determine if it alone is great enough to trigger the micturition reflex (it will add to any pressure already existing in the bladder).

**7:** If the pressure in the esophagus is $\mathbf{-2.00\ mm\ Hg}$ while that in the stomach is $\mathbf{+20.0\ mm\ Hg}$, to what height could stomach fluid rise in the esophagus, assuming a density of 1.10 g/mL? (This movement will not occur if the muscle closing the lower end of the esophagus is working properly.)

**8:** Pressure in the spinal fluid is measured as shown in

Figure 4. If the pressure in the spinal fluid is 10.0 mm Hg: (a) What is the reading of the water manometer in cm water? (b) What is the reading if the person sits up, placing the top of the fluid 60 cm above the tap? The fluid density is 1.05 g/mL.

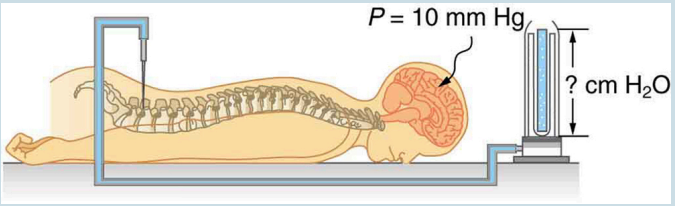



***Figure 4.*** *A water manometer used to measure pressure in the spinal fluid. The height of the fluid in the manometer is measured relative to the spinal column, and the manometer is open to the atmosphere. The measured pressure will be considerably greater if the person sits up.*

**9:** Calculate the maximum force in newtons exerted by the blood on an aneurysm, or ballooning, in a major artery, given the maximum blood pressure for this person is 150 mm Hg and the effective area of the aneurysm is $\mathbf{20.0\ cm^2}$. Note that this force is great enough to cause further enlargement and subsequently greater force on the ever-thinner vessel wall.

**10:** During heavy lifting, a disk between spinal vertebrae is subjected to a 5000-N compressional force. What pressure is created, assuming that the disk has a uniform circular cross section 2.00 cm in radius?

**11:** When a person sits erect, increasing the vertical position of their brain by 36.0 cm, the heart must continue

to pump blood to the brain at the same rate. (a) What is the gain in gravitational potential energy for 100 mL of blood raised 36.0 cm? (b) What is the drop in pressure, neglecting any losses due to friction? (c) Discuss how the gain in gravitational potential energy and the decrease in pressure are related.

**12:** (a) How high will water rise in a glass capillary tube with a 0.500-mm radius? (b) How much gravitational potential energy does the water gain? (c) Discuss possible sources of this energy.

**13:** A negative pressure of 25.0 atm can sometimes be achieved with the device in Figure 5 before the water separates. (a) To what height could such a negative gauge pressure raise water? (b) How much would a steel wire of the same diameter and length as this capillary stretch if suspended from above?

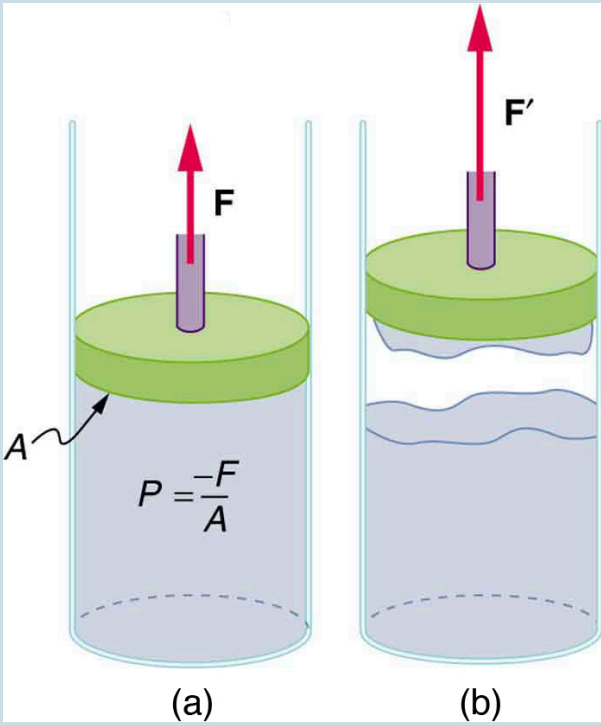



***Figure 5.*** *(a) When the piston is raised, it stretches the liquid slightly, putting it under tension and creating a negative absolute pressure* **P=−F/A** *(b) The liquid eventually separates, giving an experimental limit to negative pressure in this liquid.*

**14:** Suppose you hit a steel nail with a 0.500-kg hammer, initially moving at $\mathbf{15.0\ m/s}$ and brought to rest in 2.80 mm. (a) What average force is exerted on the nail? (b) How much is the nail compressed if it is 2.50 mm in diameter and 6.00-cm long? (c) What pressure is created on the 1.00-mm-diameter tip of the nail?

**15:** Calculate the pressure due to the ocean at the bottom of the Marianas Trench near the Philippines, given its depth is 11.0 km and assuming the density of sea water is constant all the way down. (b) Calculate the percent decrease in volume of sea water due to such a pressure, assuming its bulk modulus is the same as water and is constant. (c) What would be the percent increase in its density? Is the assumption of constant density valid? Will the actual pressure be greater or smaller than that calculated under this assumption?

**16:** The hydraulic system of a backhoe is used to lift a load as shown in Figure 6. (a) Calculate the force $F$ the slave cylinder must exert to support the 400-kg load and the 150-kg brace and shovel. (b) What is the pressure in the hydraulic fluid if the slave cylinder is 2.50 cm in diameter? (c) What force would you have to exert on a lever with a mechanical advantage of 5.00 acting on a master cylinder 0.800 cm in diameter to create this pressure?

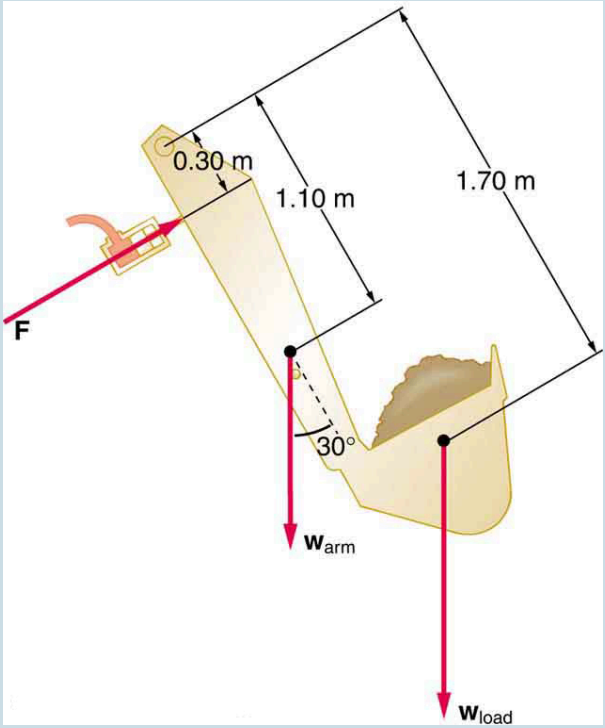



***Figure 6.*** *Hydraulic and mechanical lever systems are used in heavy machinery such as this back hoe.*

**17:** Some miners wish to remove water from a mine shaft. A pipe is lowered to the water 90 m below, and a negative pressure is applied to raise the water. (a) Calculate the pressure needed to raise the water. (b) What is unreasonable about this pressure? (c) What is unreasonable about the premise?

**18:** You are pumping up a bicycle tire with a hand pump, the piston of which has a 2.00-cm radius.

(a) What force in newtons must you exert to create a pressure of $\mathbf{6.90 \times 10^5\ Pa}$ (b) What is unreasonable about this (a) result? (c) Which premises are unreasonable or inconsistent?

**19:** Consider a group of people trying to stay afloat after their boat strikes a log in a lake. Construct a problem in which you calculate the number of people that can cling to the log and keep their heads out of the water. Among the variables to be considered are the size and density of the log, and what is needed to keep a person's head and arms above water without swimming or treading water.

## Glossary

**diastolic pressure**
: minimum arterial blood pressure; indicator for the fluid balance

**glaucoma**
: condition caused by the buildup of fluid pressure in the eye

**intraocular pressure**
: fluid pressure in the eye

**micturition reflex**
: stimulates the feeling of needing to urinate, triggered by bladder pressure

**systolic pressure**
: maximum arterial blood pressure; indicator for the blood flow

## *Solutions*

**Problems & Exercises**

**1:** 8000 Pa, pressure = force / area  force = 479 N

2: 5 x 10 $^{8}$ N/m  or Pa

**3:** 1.96 N

**4:** $\mathbf{-63.0\ cm\ H_2O}$

**6:** (a) $\mathbf{3.81 \times 10^3\ N/m^2}$ (b) $\mathbf{28.7\ mm\ Hg}$, which is sufficient to trigger micturition reflex

**8:** (a) 13.6 m water  (b) 76.5 cm water

**10:** $\mathbf{3.98 \times 10^6\ Pa}$

**12:** (a) 2.97 cm (b)$\mathbf{3.39 \times 10^{-6}\ J}$

(c) Work is done by the surface tension force through an effective distance $\boldsymbol{h/2}$ to raise the column of water.

**14:** (a) $\mathbf{2.01 \times 10^4\ N}$ (b) $\mathbf{1.17 \times 10^{-3}\ m}$

(c) $\mathbf{2.56 \times 10^{10}\ N/m^2}$

**16:** (a) $\mathbf{1.38 \times 10^4\ N}$ (b) $\mathbf{2.81 \times 10^7\ N/m^2}$ (c) 283 N

**18:** (a) 867 N (b) This is too much force to exert with a hand pump. (c) The assumed radius of the pump is too large; it would be nearly two inches in diameter—too large for a pump or even a master cylinder. The pressure is reasonable for bicycle tires.

PART XI

# CHAPTER 10 TEMPERATURE AND THERMAL EXPANSION



***Figure 1.*** *The welder's gloves and helmet protect him from the electric arc that transfers enough thermal energy to melt the rod, spray sparks, and burn the retina of an unprotected eye. The thermal energy can be felt on exposed skin a few meters away, and its light can be seen for kilometers. (credit: Kevin S. O'Brien/U.S. Navy)*

Heat is something familiar to each of us. We feel the warmth of the summer Sun, the chill of a clear summer night, the heat of coffee after a winter stroll, and the cooling effect of our sweat. Heat transfer is maintained by temperature differences. Manifestations of heat transfer—the movement of heat energy from one place or material to another—are apparent throughout the universe. Heat from beneath Earth's surface is brought to the surface in flows of incandescent lava. The Sun warms Earth's surface and is the source of much of the energy we find on it. Rising levels of atmospheric carbon dioxide threaten to trap more of the Sun's energy, perhaps

fundamentally altering the ecosphere. In space, supernovas explode, briefly radiating more heat than an entire galaxy does.

What is heat? How do we define it? How is it related to temperature? What are heat's effects? How is it related to other forms of energy and to work? We will find that, in spite of the richness of the phenomena, there is a small set of underlying physical principles that unite the subjects and tie them to other fields.

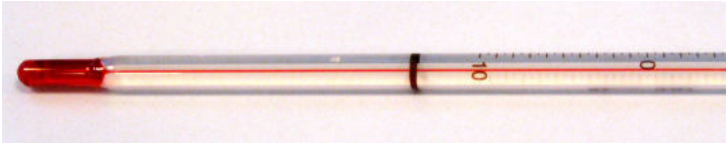



***Figure 2.*** *In a typical thermometer like this one, the alcohol, with a red dye, expands more rapidly than the glass containing it. When the thermometer's temperature increases, the liquid from the bulb is forced into the narrow tube, producing a large change in the length of the column for a small change in temperature. (credit: Chemical Engineer, Wikimedia Commons)*

# 57. 10.1 Temperature

*Summary*

- Define temperature.
- Convert temperatures between the Celsius, Fahrenheit, and Kelvin scales.
- Define thermal equilibrium.
- State the zeroth law of thermodynamics.

The concept of temperature has evolved from the common concepts of hot and cold. Human perception of what feels hot or cold is a relative one. For example, if you place one hand in hot water and the other in cold water, and then place both hands in tepid water, the tepid water will feel cool to the hand that was in hot water, and warm to the one that was in cold water. The scientific definition of temperature is less ambiguous than your senses of hot and cold. **Temperature** is operationally defined to be what we measure with a thermometer. (Many physical quantities are defined solely in terms of how they are measured. We shall see later how temperature is related to the kinetic energies of atoms and molecules, a more physical explanation.) Two accurate thermometers, one placed in hot water and the other in cold water, will show the hot water to have a higher temperature. If they are then placed in the tepid water, both will give identical readings (within measurement uncertainties). In this section, we discuss temperature, its measurement by thermometers, and its relationship to thermal equilibrium. Again, temperature is the quantity measured by a thermometer.

## MISCONCEPTION ALERT: HUMAN PERCEPTION VS. REALITY

On a cold winter morning, the wood on a porch feels warmer than the metal of your bike. The wood and bicycle are in thermal equilibrium with the outside air, and are thus the same temperature. They *feel* different because of the difference in the way that they conduct heat away from your skin. The metal conducts heat away from your body faster than the wood does (see more about conductivity in Chapter 11.5 Conduction). This is just one example demonstrating that the human sense of hot and cold is not determined by temperature alone.

Another factor that affects our perception of temperature is humidity. Most people feel much hotter on hot, humid days than on hot, dry days. This is because on humid days, sweat does not evaporate from the skin as efficiently as it does on dry days. It is the evaporation of sweat (or water from a sprinkler or pool) that cools us off.

Any physical property that depends on temperature, and whose response to temperature is reproducible, can be used as the basis of a thermometer. Because many physical properties depend on temperature, the variety of thermometers is remarkable. For example, volume increases with temperature for most substances. This property is the basis for the common alcohol thermometer, the old mercury thermometer, and the bimetallic strip (Figure 1).

Other properties used to measure temperature include electrical resistance and color, as shown in Figure 2, and the emission of infrared radiation, as shown in Figure 3.

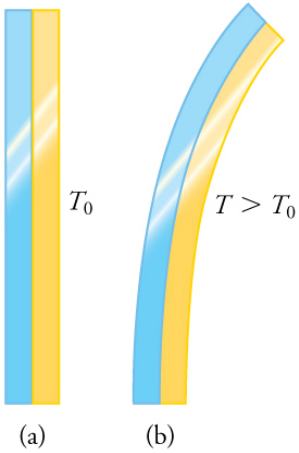



***Figure 1.*** *The curvature of a bimetallic strip depends on temperature. (a) The strip is straight at the starting temperature, where its two components have the same length. (b) At a higher temperature, this strip bends to the right, because the metal on the left has expanded more than the metal on the right.*




***Figure 2.*** *Each of the six squares on this plastic (liquid crystal) thermometer contains a film of a different heat-sensitive liquid crystal material. Below* **95°F**, *all six squares are black. When the plastic thermometer is exposed to temperature that increases to* **95°F**, *the first liquid crystal square changes color. When the temperature increases above* **96.8°F** *the second liquid crystal square also changes color, and so forth. (credit: Arkrishna, Wikimedia Commons)*



***Figure 3.*** *Fireman Jason Ormand uses a pyrometer to check the temperature of an aircraft carrier's ventilation system. Infrared radiation (whose emission varies with temperature) from the vent is measured and a temperature readout is quickly produced. Infrared measurements are also frequently used as a measure of body temperature. These modern thermometers, placed in the ear canal, are more accurate than alcohol thermometers placed under the tongue or in the armpit. (credit: Lamel J. Hinton/U.S. Navy)*

## Temperature Scales

Thermometers are used to measure temperature according to well-defined scales of measurement, which use pre-defined reference points to help compare quantities. The three most common

temperature scales are the Fahrenheit, Celsius, and Kelvin scales. A temperature scale can be created by identifying two easily reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used.

The **Celsius** scale (which replaced the slightly different *centigrade* scale) has the freezing point of water at $0^\circ\text{C}$ and the boiling point at $100^\circ\text{C}$. Its unit is the **degree Celsius** $(^\circ\text{C})$. On the **Fahrenheit** scale (still the most frequently used in the United States), the freezing point of water is at $32^\circ\text{F}$ and the boiling point is at $212^\circ\text{F}$. The unit of temperature on this scale is the **degree Fahrenheit** $(^\circ\text{F})$. Note that a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Only 100 Celsius degrees span the same range as 180 Fahrenheit degrees, thus one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale $180/100 = 9/5$.

The **Kelvin** scale is the temperature scale that is commonly used in science. It is an *absolute temperature* scale defined to have 0 K at the lowest possible temperature, called **absolute zero**. The official temperature unit on this scale is the *kelvin*, which is abbreviated K, and is not accompanied by a degree sign. The freezing and boiling points of water are 273.15 K and 373.15 K, respectively. Thus, the magnitude of temperature differences is the same in units of kelvins and degrees Celsius. Unlike other temperature scales, the Kelvin scale is an absolute scale. It is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature. The kelvin is the SI unit used in scientific work.

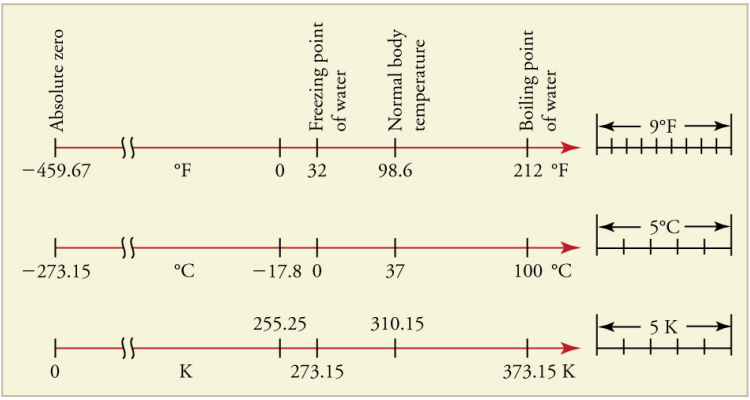



***Figure 4.*** *Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.*

The relationships between the three common temperature scales is shown in Figure 4. Temperatures on these scales can be converted using the equations in Table 1.

| To convert from . . . | Use this equation . . . | Also written as |
|---|---|---|
| Celsius to Fahrenheit | $T(^\circ\mathrm{F}) = \frac{9}{5}T(^\circ\mathrm{C}) + 32$ | $T_{^\circ\mathrm{F}} = \frac{9}{5}T$ |
| Fahrenheit to Celsius | $T(^\circ\mathrm{C}) = \frac{5}{9}T(^\circ\mathrm{F}) - 32$ | $T_{^\circ\mathrm{C}} = \frac{5}{9}($ |
| Celsius to Kelvin | $T(\mathrm{K}) = T(^\circ\mathrm{C}) + 273.15$ | $T_{\mathrm{K}} = T_{^\circ\mathrm{C}}$ |
| Kelvin to Celsius | $T(^\circ\mathrm{C}) = T(\mathrm{K}) - 273.15$ | $T_{^\circ\mathrm{C}} = T_{\mathrm{K}}$ |
| Fahrenheit to Kelvin | $T(\mathrm{K}) = \frac{5}{9}(T(^\circ\mathrm{F}) - 32) + 273.15$ | $T_{\mathrm{K}} = \frac{5}{9}($ |
| Kelvin to Fahrenheit | $T(^\circ\mathrm{F}) = \frac{9}{5}T(\mathrm{K}) - 273.15) + 32$ | $T_{\mathrm{F}} = \frac{9}{5}($ |

**Table 1.** Temperature Conversions

Notice that the conversions between Fahrenheit and Kelvin look quite complicated. In fact, they are simple combinations of the conversions between Fahrenheit and Celsius, and the conversions between Celsius and Kelvin.

### Example 1: Converting between Temperature Scales: Room Temperature

"Room temperature" is generally defined to be

$25^\circ\text{C}$. (a) What is room temperature in $^\circ\text{F}$? (b) What is it in K?

**Strategy**

To answer these questions, all we need to do is choose the correct conversion equations and plug in the known values.

**Solution for (a)**

1. Choose the right equation. To convert from $^\circ\text{C}$ to $^\circ\text{F}$, use the equation

$$T_{^\circ\text{F}} = \frac{9}{5}T_{^\circ\text{C}} + 32.$$

2. Plug the known value into the equation and solve:

$$T_{^\circ\text{F}} = \frac{9}{5}25^\circ\text{C} + 32 = 77^\circ\text{F}.$$

**Solution for (b)**

1. Choose the right equation. To convert from $^\circ\text{C}$ to K, use the equation

$$T_\text{K} = T^{^\circ\text{C}} + 273.15.$$

2. Plug the known value into the equation and solve:

$$T_\text{K} = 25^\circ\text{C} + 273.15 = 298\text{K}.$$

## Example 2: Converting between Temperature Scales: the Reaumur Scale

The Reaumur scale is a temperature scale that was used widely in Europe in the 18th and 19th centuries. On the Reaumur temperature scale, the freezing point of water is $0^\circ\text{R}$ and the boiling temperature is $80^\circ\text{R}$. If "room temperature" is $25^\circ\text{C}$ on the Celsius scale, what is it on the Reaumur scale?

**Strategy**

To answer this question, we must compare the Reaumur scale to the Celsius scale. The difference between the freezing point and boiling point of water on the Reaumur scale is $80^\circ\text{R}$. On the Celsius scale it is $100^\circ\text{C}$. Therefore $100^\circ\text{C} = 80^\circ\text{R}$. Both scales start at $0^\circ$ for freezing, so we can derive a simple formula to convert between temperatures on the two scales.

**Solution**

1. Derive a formula to convert from one scale to the other:

$$T_{^\circ\text{R}} = \frac{0.8^\circ\text{R}}{^\circ\text{C}} \times T_{^\circ\text{C}}.$$

2. Plug the known value into the equation and solve:

$$T_{^\circ\text{R}} = \frac{0.8^\circ\text{R}}{^\circ\text{C}} \times 25^\circ\text{C} = 20^\circ\text{R}.$$

## Temperature Ranges in the Universe

Figure 6 shows the wide range of temperatures found in the universe. Human beings have been known to survive with body temperatures within a small range, from $24^\circ C$ to $44^\circ C$ ($75^\circ F$ to $111^\circ F$). The average normal body temperature is usually given as $37.0^\circ C$ ($98.6^\circ F$), and variations in this temperature can indicate a medical condition: a fever, an infection, a tumor, or circulatory problems (see Figure 5).

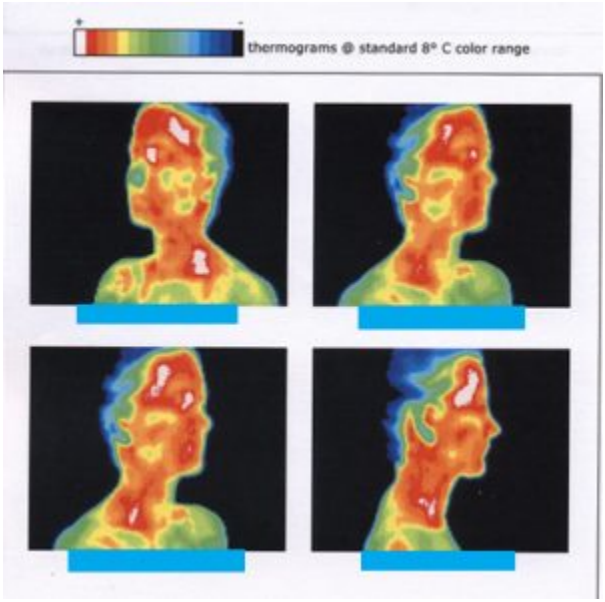



***Figure 5.*** *This image of radiation from a person's body (an infrared thermograph) shows the location of temperature abnormalities in the upper body. Dark blue corresponds to cold areas and red to white corresponds to hot areas. An elevated temperature might be an indication of malignant tissue (a cancerous tumor in the breast, for example), while a depressed temperature might be due to a decline in blood flow from a clot. In this case, the abnormalities are caused by a condition called hyperhidrosis. (credit: Porcelina81, Wikimedia Commons)*

The lowest temperatures ever recorded have been measured during laboratory experiments: $\mathbf{4.5 \times 10^{-10}\ K}$ at the Massachusetts Institute of Technology (USA), and $\mathbf{1.0 \times 10^{-10}\ K}$ at Helsinki University of Technology (Finland). In comparison, the coldest recorded place on Earth's surface is Vostok, Antarctica at 183 K $\mathbf{(-89^{\circ}C)}$, and the coldest place (outside the lab) known in the universe is the Boomerang Nebula, with a temperature of 1 K.

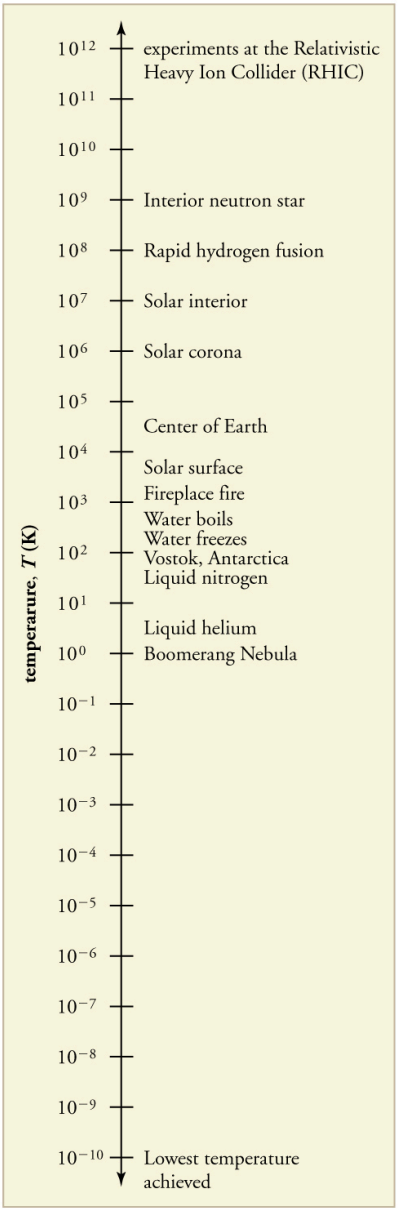



***Figure 6.*** *Each increment on this logarithmic scale indicates an increase by a factor of ten, and thus illustrates the*

*tremendous range of temperatures in nature. Note that zero on a logarithmic scale would occur off the bottom of the page at infinity.*

## MAKING CONNECTIONS: ABSOLUTE ZERO

What is absolute zero? Absolute zero is the temperature at which all molecular motion has ceased. The concept of absolute zero arises from the behavior of gases. Figure 7 shows how the pressure of gases at a constant volume decreases as temperature decreases. Various scientists have noted that the pressures of gases extrapolate to zero at the same temperature, $\mathbf{-273.15^\circ C}$. This extrapolation implies that there is a lowest temperature. This temperature is called *absolute zero*. Today we know that most gases first liquefy and then freeze, and it is not actually possible to reach absolute zero. The numerical value of absolute zero temperature is $\mathbf{-273.15^\circ C}$ or 0 K.

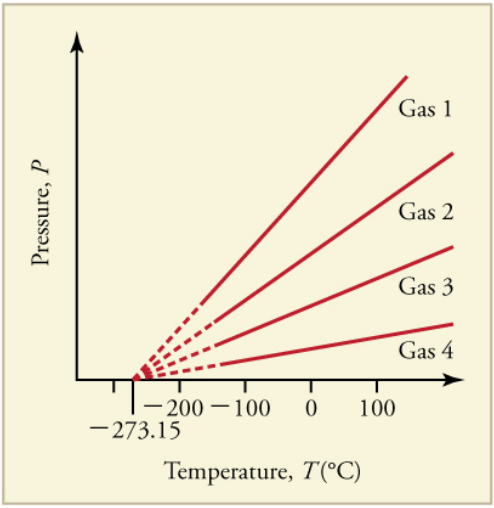



***Figure 7.*** *Graph of pressure versus temperature for various gases kept at a constant volume. Note that all of the graphs extrapolate to zero pressure at the same temperature.*

## Thermal Equilibrium and the Zeroth Law of Thermodynamics

Thermometers actually take their *own* temperature, not the temperature of the object they are measuring. This raises the question of how we can be certain that a thermometer measures the temperature of the object with which it is in contact. It is based on the fact that any two systems placed in *thermal contact* (meaning heat transfer can occur between them) will reach the same temperature. That is, heat will flow from the hotter object to the cooler one until they have exactly the same temperature. The objects are then in **thermal equilibrium**, and no further changes will occur. The systems interact and change because their temperatures differ, and the changes stop once their temperatures

are the same. Thus, if enough time is allowed for this transfer of heat to run its course, the temperature a thermometer registers *does* represent the system with which it is in thermal equilibrium. Thermal equilibrium is established when two bodies are in contact with each other and can freely exchange energy.

Furthermore, experimentation has shown that if two systems, A and B, are in thermal equilibrium with each another, and B is in thermal equilibrium with a third system C, then A is also in thermal equilibrium with C. This conclusion may seem obvious, because all three have the same temperature, but it is basic to thermodynamics. It is called the **zeroth law of thermodynamics**.

### THE ZEROTH LAW OF THERMODYNAMICS

If two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

This law was postulated in the 1930s, after the first and second laws of thermodynamics had been developed and named. It is called the *zeroth law* because it comes logically before the first and second laws. An example of this law in action is seen in babies in incubators: babies in incubators normally have very few clothes on, so to an observer they look as if they may not be warm enough. However, the temperature of the air, the cot, and the baby is the same, because they are in thermal equilibrium, which is

accomplished by maintaining air temperature to keep the baby comfortable.

### *Check Your Understanding*

**1:** Does the temperature of a body depend on its size?

## Section Summary

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.
- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.
- Temperatures on one scale can be converted to temperatures on another scale using the following equations:

$$T_{^\circ\mathrm{F}} = \frac{9}{5}T_{^\circ\mathrm{C}} + 32$$
$$T_{^\circ\mathrm{C}} = \frac{5}{9}(T_{^\circ\mathrm{F}} - 32)$$
$$T_{\mathrm{K}} = T_{^\circ\mathrm{C}} + 273.15$$
$$T_{^\circ\mathrm{C}} = T_{\mathrm{K}} - 273.15$$

- Systems are in thermal equilibrium when they have the same temperature.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy.
- The zeroth law of thermodynamics states that when two

systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

## *Conceptual Questions*

**1:** What does it mean to say that two systems are in thermal equilibrium?

**2:** Give an example of a physical property that varies with temperature and describe how it is used to measure temperature.

**3:** When a cold alcohol thermometer is placed in a hot liquid, the column of alcohol goes *down* slightly before going up. Explain why.

**4:** If you add boiling water to a cup at room temperature, what would you expect the final equilibrium temperature of the unit to be? You will need to include the surroundings as part of the system. Consider the zeroth law of thermodynamics.

## *Problems & Exercises*

**1:** What is the Fahrenheit temperature of a person with a $\mathbf{39.0^\circ C}$ fever?

**2:** Frost damage to most plants occurs at temperatures of $\mathbf{28.0^\circ F}$ or lower. What is this temperature on the Kelvin scale?

**3:** To conserve energy, room temperatures are kept at $\mathbf{68.0^\circ F}$ in the winter and $\mathbf{78.0^\circ F}$ in the summer. What are these temperatures on the Celsius scale?

**4:** A tungsten light bulb filament may operate at 2900 K. What is its Fahrenheit temperature? What is this on the Celsius scale?

**5:** The surface temperature of the Sun is about 5750 K. What is this temperature on the Fahrenheit scale?

**6:** One of the hottest temperatures ever recorded on the surface of Earth wa s $\mathbf{134^\circ F}$ in Death Valley, CA. What is this temperature in Celsius degrees? What is this temperature in Kelvin?

**7:** (a) Suppose a cold front blows into your locale and drops the temperature by 40.0 Fahrenheit degrees. How many degrees Celsius does the temperature decrease when there is a $\mathbf{40.0^\circ F}$ decrease in temperature? (b) Show that any change in temperature in Fahrenheit degrees is nine-fifths the change in Celsius degrees.

**8:** (a) At what temperature do the Fahrenheit and Celsius scales have the same numerical value? (b) At what temperature do the Fahrenheit and Kelvin scales have the same numerical value?

## Glossary

**temperature**
: the quantity measured by a thermometer

**Celsius scale**
: temperature scale in which the freezing point of water is $0^\circ\text{C}$ and the boiling point of water is $100^\circ\text{C}$

**degree Celsius**
: unit on the Celsius temperature scale

**Fahrenheit scale**
: temperature scale in which the freezing point of water is $32^\circ\text{F}$ and the boiling point of water is $212^\circ\text{F}$

**degree Fahrenheit**
: unit on the Fahrenheit temperature scale

**Kelvin scale**
: temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

**absolute zero**
: the lowest possible temperature; the temperature at which all molecular motion ceases

**thermal equilibrium**
: the condition in which heat no longer flows between two objects that are in contact; the two objects have the same temperature

**zeroth law of thermodynamics**
: law that states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object

## *Solutions*

**Check Your Understanding**

**1:** No, the system can be divided into smaller parts each of which is at the same temperature. We say that the temperature is an *intensive* quantity. Intensive quantities are independent of size.

**Problems & Exercises**

**1:** 102 degrees F

**3:** 20.0 degrees C and 25.6 degrees C

**5:** 9890 degrees F

7: (a) 22.2 degrees C

(b)

$$\begin{aligned} \mathbf{\Delta}T(^\circ\mathbf{F}) &= T_2(^\circ\mathbf{F}) - T_1(^\circ\mathbf{F}) \\ &= \tfrac{9}{5}T_2(^\circ\mathbf{C}) + \mathbf{32.0}^\circ - (\tfrac{9}{5}T_1(^\circ\mathbf{C}) + \mathbf{32.0}^\circ) \\ &= \tfrac{9}{5}T_2(^\circ\mathbf{C}) - T_1(^\circ\mathbf{C}) = \tfrac{9}{5}\mathbf{\Delta}T(^\circ\mathbf{C}) \end{aligned}$$

# 58. 10.2 Thermal Expansion of Solids and Liquids

*Summary*

- Define and describe thermal expansion.
- Calculate the linear expansion of an object given its initial length, change in temperature, and coefficient of linear expansion.
- Calculate the volume expansion of an object given its initial volume, change in temperature, and coefficient of volume expansion.
- Calculate thermal stress on an object given its original volume, temperature change, volume change, and bulk modulus.



***Figure 1.*** *Thermal expansion joints like these in the Auckland Harbour Bridge in New Zealand allow bridges to change length without buckling. (credit: Ingolfson, Wikimedia Commons)*

The expansion of alcohol in a thermometer is one of many commonly encountered examples of **thermal expansion**, the change in size or volume of a given mass with temperature. Hot air rises because its volume increases, which causes the hot air's density to be smaller than the density of surrounding air, causing a buoyant (upward) force on the hot air. The same happens in all liquids and gases, driving natural heat transfer upwards in homes, oceans, and weather systems. Solids also undergo thermal

expansion. Railroad tracks and bridges, for example, have expansion joints to allow them to freely expand and contract with temperature changes.

What are the basic properties of thermal expansion? First, thermal expansion is clearly related to temperature change. The greater the temperature change, the more a bimetallic strip will bend. Second, it depends on the material. In a thermometer, for example, the expansion of alcohol is much greater than the expansion of the glass containing it.

What is the underlying cause of thermal expansion? As is discussed earlier an increase in temperature implies an increase in the kinetic energy of the individual atoms. In a solid, unlike in a gas, the atoms or molecules are closely packed together, but their kinetic energy (in the form of small, rapid vibrations) pushes neighbouring atoms or molecules apart from each other. This neighbour-to-neighbour pushing results in a slightly greater distance, on average, between neighbours, and adds up to a larger size for the whole body. For most substances under ordinary conditions, there is no preferred direction, and an increase in temperature will increase the solid's size by a certain fraction in each dimension.

## LINEAR THERMAL EXPANSION—THERMAL EXPANSION IN ONE DIMENSION

The change in length $\Delta L$ is proportional to length $L$. The dependence of thermal expansion on

temperature, substance, and length is summarized in the equation

$$\Delta L = \alpha L \Delta T,$$

where $\Delta L$ is the change in length $L$, $\Delta T$ is the change in temperature, and $\alpha$ is the coefficient of linear expansion, which varies slightly with temperature.

Table 2 lists representative values of the coefficient of linear expansion, which may have units of $1/^{\circ}\text{C}$ or 1/K. Because the size of a kelvin and a degree Celsius are the same, both $\alpha$ and $\Delta T$ can be expressed in units of kelvins or degrees Celsius. The equation $\Delta L = \alpha L \Delta T$ is accurate for small changes in temperature and can be used for large changes in temperature if an average value of $\alpha$ is used.

| Material | Coefficient of linear expansion α(1/°C) | Coefficient of volume expansion β(1/°C) |
|---|---|---|
| **Solids** | | |
| Aluminum | $25 \times 10^{-6}$ | $75 \times 10^{-6}$ |
| Brass | $19 \times 10^{-6}$ | $56 \times 10^{-6}$ |
| Copper | $17 \times 10^{-6}$ | $51 \times 10^{-6}$ |
| Gold | $14 \times 10^{-6}$ | $42 \times 10^{-6}$ |
| Iron or Steel | $12 \times 10^{-6}$ | $35 \times 10^{-6}$ |
| Invar (Nickel-iron alloy) | $0.9 \times 10^{-6}$ | $2.7 \times 10^{-6}$ |
| Lead | $29 \times 10^{-6}$ | $87 \times 10^{-6}$ |
| Silver | $18 \times 10^{--6}$ | $54 \times 10^{-6}$ |
| Glass (ordinary) | $9 \times 10^{-6}$ | $27 \times 10^{-6}$ |
| Glass (Pyrex®) | $3 \times 10{-}6$ | $9 \times 10^{-6}$ |
| Quartz | $0.4 \times 10^{-6}$ | $1 \times 10^{-6}$ |
| Concrete, Brick | $\sim 12 \times 10^{-6}$ | $\sim 36 \times 10^{-6}$ |
| Marble (average) | $7 \times 10^{-6}$ | $2.1 \times 10^{-5}$ |
| **Liquids** | | |
| Ether | | $1650 \times 10^{-6}$ |
| Ethyl alcohol | | $1100 \times 10^{-6}$ |
| Petrol | | $950 \times 10^{-6}$ |
| Glycerin | | $500 \times 10^{-6}$ |
| Mercury | | $180 \times 10^{-6}$ |

| Material | Coefficient of linear expansion α(1/°C) | Coefficient of volume expansion β(1/°C) |
|---|---|---|
| Water | | $210 \times 10^{-6}$ |
| **Gases** | | |
| Air and most other gases at atmospheric pressure | | $3400 \times 10^{-6}$ |

**Table 2.** Thermal Expansion Coefficients at 20°C[1]

## Example 1: Calculating Linear Thermal Expansion: The Golden Gate Bridge

The main span of San Francisco's Golden Gate Bridge is 1275 m long at its coldest. The bridge is exposed to temperatures ranging from $-15^\circ\text{C}$ to $40^\circ\text{C}$. What is its change in length between these temperatures? Assume that the bridge is made entirely of steel.

**Strategy**

Use the equation for linear thermal expansion $\Delta L = \alpha L \Delta T$ to calculate the change in length, $\Delta L$. Use the coefficient of linear expansion, $\alpha$, for steel from Table 2, and note that the change in temperature, $\Delta T$, is $55^\circ\text{C}$.

**Solution**

Plug all of the known values into the equation to solve for $\Delta L$.

$$\Delta L = \alpha L \Delta T = \left( \frac{12 \times 10^{-6}}{^\circ\text{C}} \right) (1275 \text{ m})(55^\circ\text{C}) = 0.84 \text{ m}.$$

**Discussion**

Although not large compared with the length of the bridge, this change in length is observable. It is generally spread over many expansion joints so that the expansion at each joint is small.

## Thermal Expansion in Two and Three Dimensions

Objects expand in all dimensions, as illustrated in Figure 2. That is, their areas and volumes, as well as their lengths, increase with temperature. Holes also get larger with temperature. If you cut a hole in a metal plate, the remaining material will expand exactly as it would if the plug was still in place. The plug would get bigger, and so the hole must get bigger too. (Think of the ring of neighboring atoms or molecules on the wall of the hole as pushing each other farther apart as temperature increases. Obviously, the ring of neighbors must get slightly larger, so the hole gets slightly larger).

## THERMAL EXPANSION IN TWO DIMENSIONS

For small temperature changes, the change in area $\Delta A$ is given by

$$\Delta A = 2\alpha A \Delta T,$$

where $\Delta A$ is the change in area $A$, $\Delta T$ is the change in temperature, and $\alpha$ is the coefficient of linear expansion, which varies slightly with temperature.

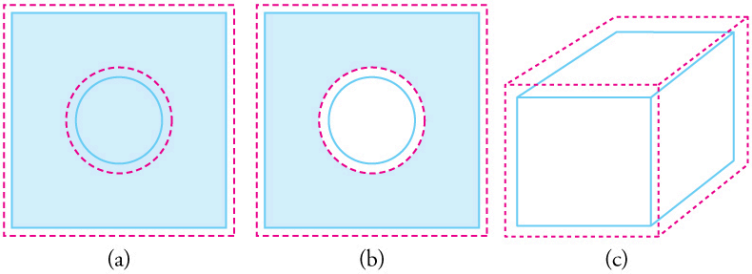



***Figure 2.*** *In general, objects expand in all directions as temperature increases. In these drawings, the original boundaries of the objects are shown with solid lines, and the expanded boundaries with dashed lines. (a) Area increases because both length and width increase. The area of a circular plug also increases. (b) If the plug is removed, the hole it leaves becomes larger with increasing temperature, just as if the expanding plug were still in place. (c) Volume also increases, because all three dimensions increase.*

## THERMAL EXPANSION IN THREE DIMENSIONS

The change in volume $\Delta V$ is very nearly $\Delta V = 3\alpha V \Delta T$. This equation is usually written as

$$\Delta V = \beta V \Delta T,$$

where $\beta$ is the coefficient of volume expansion and $\beta \approx 3\alpha$. Note that the values of $\beta$ in Table 2 are almost exactly equal to $3\alpha$.

In general, objects will expand with increasing temperature. Water is the most important exception to this rule. Water expands with increasing temperature (its density *decreases*) when it is at temperatures greater than $4^\circ\text{C}\ (40^\circ\text{F})$. However, it expands with *decreasing* temperature when it is between $+4^\circ\text{C}$ and $0^\circ\text{C}\ (40^\circ\text{F}$ to $32^\circ\text{F})$. Water is densest at $+4^\circ\text{C}$. (See Figure 3.) Perhaps the most striking effect of this phenomenon is the freezing of water in a pond. When water near the surface cools down to $4^\circ\text{C}$ it is denser than the remaining water and thus will sink to the bottom. This "turnover" results in a layer of warmer water near the surface, which is then cooled. Eventually the pond has a uniform temperature of $4^\circ\text{C}$. If the temperature in the surface layer drops below $4^\circ\text{C}$, the water is less dense than the water below, and thus stays near the top. As a result, the pond surface can completely freeze over. The ice on top of liquid water provides an insulating layer from winter's harsh exterior air

temperatures. Fish and other aquatic life can survive in $4^\circ C$ water beneath ice, due to this unusual characteristic of water. It also produces circulation of water in the pond that is necessary for a healthy ecosystem of the body of water.

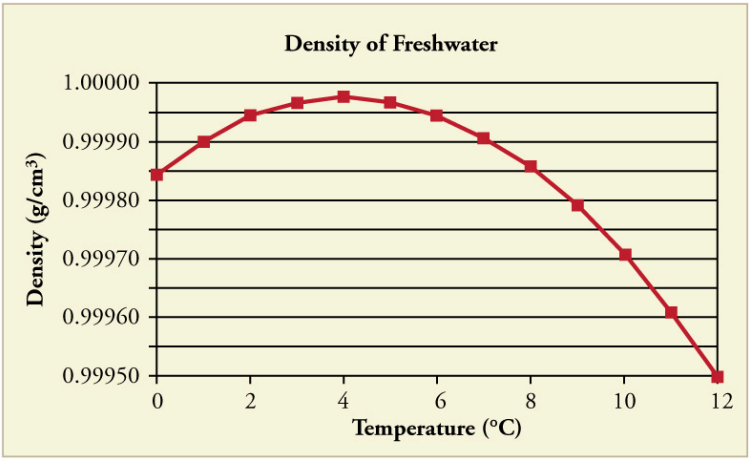



***Figure 3.*** *The density of water as a function of temperature. Note that the thermal expansion is actually very small. The maximum density at* **+4$^0$C** *is only 0.0075% greater than the density at* **2°C**, *and 0.012% greater than that at* **0°C**.

## MAKING CONNECTIONS: REAL-WORD CONNECTIONS—FILLING THE TANK

Differences in the thermal expansion of materials can lead to interesting effects at the gas station. One example is the dripping of gasoline from a freshly filled tank on a hot day. Gasoline starts out at the temperature of the ground under the gas station, which is cooler than the air temperature above. The gasoline cools the steel tank when it is filled. Both gasoline and steel tank expand as they warm to air temperature, but gasoline expands much more than steel, and so it may overflow.

This difference in expansion can also cause problems when interpreting the gasoline gauge. The actual amount (mass) of gasoline left in the tank when the gauge hits "empty" is a lot less in the summer than in the winter. The gasoline has the same volume as it does in the winter when the "add fuel" light goes on, but because the gasoline has expanded, there is less mass. If you are used to getting another 40 miles on "empty" in the winter, beware—you will probably run out much more quickly in the summer.

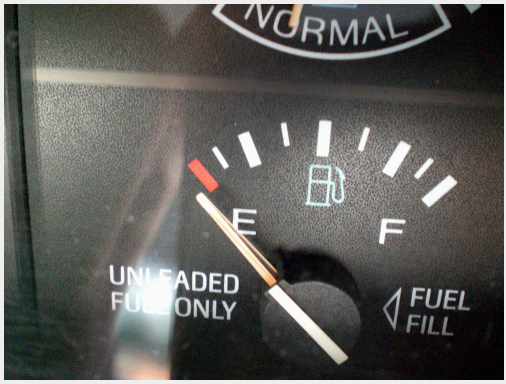



***Figure 4.*** *Because the gas expands more than the gas tank with increasing temperature, you can't drive as many miles on "empty" in the summer as you can in the winter. (credit: Hector Alejandro, Flickr)*

## Example 2: Calculating Thermal Expansion: Gas vs. Gas Tank

Suppose your 60.0-L (15.9-gal) steel gasoline tank is full of gas, so both the tank and the gasoline have a temperature of $\mathbf{15.0^\circ C}$. How much gasoline has spilled by the time they warm to $\mathbf{35.0^\circ C}$?

**Strategy**

The tank and gasoline increase in volume, but the gasoline increases more, so the amount spilled is the difference in their volume changes. (The gasoline tank can be treated as solid steel.) We can use the equation for volume expansion to calculate the change in volume of the gasoline and of the tank.

**Solution**

1. Use the equation for volume expansion to calculate the increase in volume of the steel tank:

$$\Delta V_{\text{s}} = \beta_{\text{s}} V_{\text{s}} \Delta T.$$

2. The increase in volume of the gasoline is given by this equation:

$$\Delta V_{\text{gas}} = \beta_{\text{gas}} V_{\text{gas}} \Delta T.$$

3. Find the difference in volume to determine the amount spilled as

$$V_{\text{spill}} = \Delta V_{\text{gas}} - \Delta V_{\text{s}}.$$

Alternatively, we can combine these three equations into a single equation. (Note that the original volumes are equal.)

$$\begin{aligned} V_{\text{spill}} &= (\beta_{\text{gas}} - \beta_{\text{s}}) V \Delta T \\ &= [(950 - 35) \times 10^{-6}/^\circ\text{C}](60.0\ \text{L})(20.0^\circ\text{C}) \\ &= 1.10\ \text{L}. \end{aligned}$$

**Discussion**

This amount is significant, particularly for a 60.0-L tank. The effect is so striking because the gasoline and steel expand quickly. The rate of change in thermal

properties is discussed in Chapter 14 Heat and Heat Transfer Methods.

If you try to cap the tank tightly to prevent overflow, you will find that it leaks anyway, either around the cap or by bursting the tank. Tightly constricting the expanding gas is equivalent to compressing it, and both liquids and solids resist being compressed with extremely large forces. To avoid rupturing rigid containers, these containers have air gaps, which allow them to expand and contract without stressing them.

## Thermal Stress

**Thermal stress** is created by thermal expansion or contraction. Thermal stress can be destructive, such as when expanding gasoline ruptures a tank. It can also be useful, for example, when two parts are joined together by heating one in manufacturing, then slipping it over the other and allowing the combination to cool. Thermal stress can explain many phenomena, such as the weathering of rocks and pavement by the expansion of ice when it freezes.

### Example 3: Calculating Thermal Stress:

## Gas Pressure

What pressure would be created in the gasoline tank considered in Example 2, if the gasoline increases in temperature from $15.0^\circ\,\mathrm{C}$ to $35.0^\circ\,\mathrm{C}$ without being allowed to expand? Assume that the bulk modulus $B$ for gasoline is $1.00 \times 10^9\ \mathrm{N/m^2}$.

**Strategy**

To solve this problem, we must use the following equation, which relates a change in volume $\Delta V$ to pressure:

$$\Delta V = \frac{1}{B}\frac{F}{A}V_0,$$

where $F/A$ is pressure, $V_0$ is the original volume, and $B$ is the bulk modulus of the material involved. We will use the amount spilled in Example 2 as the change in volume, $\Delta V$.

**Solution**

1. Rearrange the equation for calculating pressure:

$$P = \frac{F}{A} = \frac{\Delta V}{V_0}B.$$

2. Insert the known values. The bulk modulus for gasoline is $B = 1.00 \times 10^9\ \mathrm{N/m^2}$. In the previous example, the change in volume $\Delta V = 1.10\ \mathrm{L}$ is the amount that would spill.

Here, $V_0 = 60.0\ \text{L}$ is the original volume of the gasoline. Substituting these values into the equation, we obtain

$$P = \frac{1.10\ \text{L}}{60.0\ \text{L}}(1.00 \times 10^9\ \text{Pa}) = 1.83 \times 10^7\ \text{Pa}.$$

**Discussion**

This pressure is about $2500\ \text{lb/in}^2$, *much* more than a gasoline tank can handle.

Forces and pressures created by thermal stress are typically as great as that in the example above. Railroad tracks and roadways can buckle on hot days if they lack sufficient expansion joints. (See Figure 5.) Power lines sag more in the summer than in the winter, and will snap in cold weather if there is insufficient slack. Cracks open and close in plaster walls as a house warms and cools. Glass cooking pans will crack if cooled rapidly or unevenly, because of differential contraction and the stresses it creates. (Pyrex® is less susceptible because of its small coefficient of thermal expansion.) Nuclear reactor pressure vessels are threatened by overly rapid cooling, and although none have failed, several have been cooled faster than considered desirable. *Biological cells are ruptured when foods are frozen, detracting from their taste. Repeated thawing and freezing accentuate the damage. Even the oceans can be affected. A significant portion of the rise in sea level that is resulting from global warming is due to the thermal expansion of sea water.*



***Figure 5.*** *Thermal stress contributes to the formation of potholes. (credit: Editor5807, Wikimedia Commons)*

Metal is regularly used in the human body for hip and knee implants. Most implants need to be replaced over time because, among other things, metal does not bond with bone. Researchers are trying to find better metal coatings that would allow metal-to-bone bonding. One challenge is to find a coating that has an expansion coefficient similar to that of metal. If the expansion coefficients are too different, the thermal stresses during the manufacturing process lead to cracks at the coating-metal interface.

Another example of thermal stress is found in the mouth. Dental fillings can expand differently from tooth enamel. It can give pain when eating ice cream or having a hot drink. Cracks might occur in the filling. Metal fillings (gold, silver, etc.) are being replaced by composite fillings (porcelain), which have smaller coefficients of expansion, and are closer to those of teeth.

## *Check Your Understanding*

**1:** Two blocks, A and B, are made of the same material. Block A has dimensions $l \times w \times h = L \times 2L \times L$ and Block B has dimensions $2L \times 2L \times 2L$. If the temperature changes, what is (a) the change in the volume of the two blocks, (b) the change in the cross-sectional area $l \times w$, and (c) the change in the height $h$ of the two blocks?

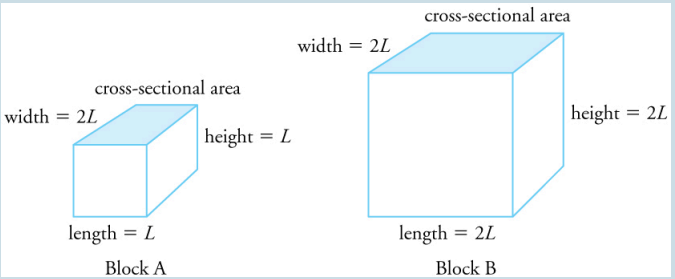



***Figure 6.***

## Section Summary

- Thermal expansion is the increase, or decrease, of the size (length, area, or volume) of a body due to a change in temperature.
- Thermal expansion is large for gases, and relatively small, but not negligible, for liquids and solids.

- Linear thermal expansion is
$$\Delta L = \alpha L \Delta T,$$
where $\Delta L$ is the change in length $L$, $\Delta T$ is the change in temperature, and $\alpha$ is the coefficient of linear expansion, which varies slightly with temperature.
- The change in area due to thermal expansion is
$$\Delta A = 2\alpha A \Delta T,$$
where $\Delta A$ is the change in area.
- The change in volume due to thermal expansion is
$$\Delta V = \beta V \Delta T,$$
where $\beta$ is the coefficient of volume expansion and $\beta \approx 3\alpha$. Thermal stress is created when thermal expansion is constrained.

## *Conceptual Questions*

**1:** Thermal stresses caused by uneven cooling can easily break glass cookware. Explain why Pyrex®, a glass with a small coefficient of linear expansion, is less susceptible.

**2:** Water expands significantly when it freezes: a volume increase of about 9% occurs. As a result of this expansion and because of the formation and growth of crystals as water freezes, anywhere from 10% to 30% of biological cells are burst when animal or plant material is frozen. Discuss the implications of this cell damage for the prospect of preserving human bodies by freezing so that they can be thawed at some future date when it is hoped that all diseases are curable.

**3:** One method of getting a tight fit, say of a metal peg in a hole in a metal block, is to manufacture the peg slightly larger than the hole. The peg is then inserted when at a different temperature than the block. Should the block be hotter or colder than the peg during insertion? Explain your answer.

**4:** Does it really help to run hot water over a tight metal lid on a glass jar before trying to open it? Explain your answer.

**5:** Liquids and solids expand with increasing temperature, because the kinetic energy of a body's atoms and molecules increases. Explain why some materials *shrink* with increasing temperature.

## *Problems & Exercises*

**1:** The height of the Washington Monument is measured to be 170 m on a day when the temperature is $35.0^\circ\text{C}$. What will its height be on a day when the temperature falls to $-10.0^\circ\text{C}$? Although the monument is made of limestone, assume that its thermal coefficient of expansion is the same as marble's.

**2:** How much taller does the Eiffel Tower become at the end of a day when the temperature has increased by 15 degrees Celsius? Its original height is 321 m and you can

assume it is made of steel with a coefficient of linear expansion of 12 x $10^{-6}$ per degree Celsius.

**3:** What is the change in length of a 3.00-cm-long column of mercury if its temperature changes from $37.0^\circ\text{C}$ to $40.0^\circ\text{C}$, assuming the mercury is unconstrained?

**4:** How large an expansion gap should be left between steel railroad rails if they may reach a maximum temperature $35.0^\circ\text{C}$ greater than when they were laid? Their original length is 10.0 m.

**5:** You are looking to purchase a small piece of land in Hong Kong. The price is "only" $60,000 per square meter! The land title says the dimensions are $20\text{ m} \times 30\text{ m}$. By how much would the total price change if you measured the parcel with a steel tape measure on a day when the temperature wa s $20^\circ\text{C}$ above normal?

**6:** Global warming will produce rising sea levels partly due to melting ice caps but also due to the expansion of water as average ocean temperatures rise. To get some idea of the size of this effect, calculate the change in length of a column of water 1.00 km high for a temperature increase of $1.00^\circ\text{C}$. Note that this calculation is only approximate because ocean warming is not uniform with depth.

**7:** Show that 60.0 L of gasoline originally a t $15.0^\circ\text{C}$ will expand to 61.1 L when it warms to $35.0^\circ\text{C}$, as claimed in Example 2.

**8:** (a) Suppose a meter stick made of steel and one made of invar (an alloy of iron and nickel) are the same length at

$0^\circ C$. What is their difference in length at $22.0^\circ C$? (b) Repeat the calculation for two 30.0-m-long surveyor's tapes.

**9:** (a) If a 500-mL glass beaker is filled to the brim with ethyl alcohol at a temperature of $5.00^\circ C$, how much will overflow when its temperature reaches $22.0^\circ C$? (b) How much less water would overflow under the same conditions?

**10:** Most automobiles have a coolant reservoir to catch radiator fluid that may overflow when the engine is hot. A radiator is made of copper and is filled to its 16.0-L capacity when at $10.0^\circ C$. What volume of radiator fluid will overflow when the radiator and fluid reach their $95.0^\circ C$ operating temperature, given that the fluid's volume coefficient of expansion is $\beta = 400 \times 10{-}6/^\circ C$? Note that this coefficient is approximate, because most car radiators have operating temperatures of greater than $95.0^\circ C$.

**11:** A physicist makes a cup of instant coffee and notices that, as the coffee cools, its level drops 3.00 mm in the glass cup. Show that this decrease cannot be due to thermal contraction by calculating the decrease in level if the $350\ cm^3$ of coffee is in a 7.00-cm-diameter cup and decreases in temperature from $95.0^\circ C$ to $45.0^\circ C$. (Most of the drop in level is actually due to escaping bubbles of air.)

**12:** (a) The density of water at $0^\circ C$ is very nearly

$1000\ \text{kg/m}^3$ (it is actually $999.84\ \text{kg/m}^3$), whereas the density of ice a t $0^\circ\text{C}$ is $917\ \text{kg/m}^3$. Calculate the pressure necessary to keep ice from expanding when it freezes, neglecting the effect such a large pressure would have on the freezing temperature. (This problem gives you only an indication of how large the forces associated with freezing water might be.) (b) What are the implications of this result for biological cells that are frozen?

## Footnotes

1. [1] Values for liquids and gases are approximate.

## Glossary

**thermal expansion**
: the change in size or volume of an object with change in temperature

**coefficient of linear expansion**
: $\alpha$, the change in length, per unit length, per $1^\circ\text{C}$ change in temperature; a constant used in the calculation of linear expansion; the coefficient of linear expansion depends on the material and to some degree on the temperature of the material

**coefficient of volume expansion**
: $\beta$, the change in volume, per unit volume, per $1^\circ\text{C}$ change in

temperature

**thermal stress**

stress caused by thermal expansion or contraction

## *Solutions*

**Check Your Understanding**

**1:** (a) The change in volume is proportional to the original volume. Block A has a volume of $L \times 2L \times L = 2L^3$. Block B has a volume of $2L \times 2L \times 2L = 8L^3$, which is 4 times that of Block A. Thus the change in volume of Block B should be 4 times the change in volume of Block A.

(b) The change in area is proportional to the area. The cross-sectional area of Block A is $L \times 2L = 2L^2$, while that of Block B is $2L \times 2L = 4L^2$. Because cross-sectional area of Block B is twice that of Block A, the change in the cross-sectional area of Block B is twice that of Block A.

(c) The change in height is proportional to the original height. Because the original height of Block B is twice that of A, the change in the height of Block B is twice that of Block A.

**Problems & Exercises**

**1:** 169.98 m

**2:** 0.058 m to two significant figures.

**3:** $5.4 \times 10^{-6}$ m

**5:** Because the area gets smaller, the price of the land DECREASES by 17,000

**7:**

$$\begin{aligned} V &= V_0 + \Delta V = V_0(1 + \beta \Delta T) \\ &= (60.00\ \text{L})[1 + (950 \times 10^{-6}/^\circ\text{C})(35.0^\circ\text{C} - 15.0^\circ\text{C})] \\ &= 61.1\ \text{L} \end{aligned}$$

**9:** (a) 9.35 mL (b) 7.56 mL

**11:** 0.832 mm

**13:** We know how the length changes with temperature: $\Delta L = \alpha L_0 \Delta T$. Also we know that the volume of a cube is related to its length by $V = L^3$, so the final volume is then $V = V_0 + \Delta V = L_0 + \Delta L^3$. Substituting for $\Delta L$ gives

$$V = (L_0 + \alpha L_0 \Delta T)^3 = L_0^3(1 + \alpha \Delta T)^3.$$

Now, because $\alpha \Delta T$ is small, we can use the binomial expansion:

$$V \approx L_0^3(1 + 3\alpha \Delta T) = L_0^3 + 3\alpha L_0^3 \Delta T.$$

So writing the length terms in terms of volumes gives $V = V_0 + \Delta V \approx V_0 + 3\alpha V_0 \Delta T$, and so

$$\Delta V = \beta V_0 \Delta T \approx 3\alpha V_0 \Delta T, \text{ or } \beta \approx 3\alpha.$$

# 59.

## *Summary*

- Interpret a phase diagram.
- State Dalton's law.
- Identify and describe the triple point of a gas from its phase diagram.
- Describe the state of equilibrium between a liquid and a gas, a liquid and a solid, and a gas and a solid.

Up to now, we have considered the behavior of ideal gases. Real gases are like ideal gases at high temperatures. At lower temperatures, however, the interactions between the molecules and their volumes cannot be ignored. The molecules are very close (condensation occurs) and there is a dramatic decrease in volume, as seen in Figure 1. The substance changes from a gas to a liquid. When a liquid is cooled to even lower temperatures, it becomes a solid. The volume never reaches zero because of the finite volume of the molecules.

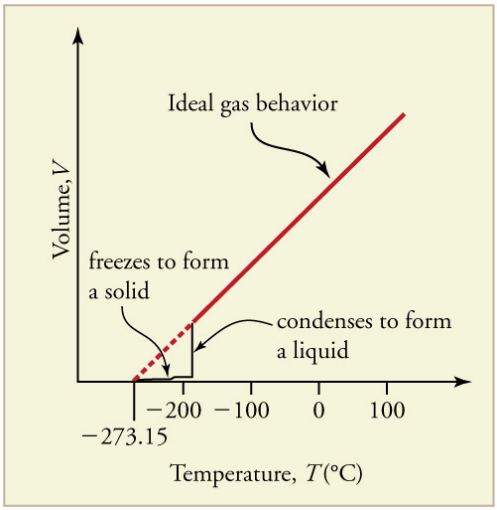



***Figure 1.*** *A sketch of volume versus temperature for a real gas at constant pressure. The linear (straight line) part of the graph represents ideal gas behavior—volume and temperature are directly and positively related and the line extrapolates to zero volume at* **−273.15°C**, *or absolute zero. When the gas becomes a liquid, however, the volume actually decreases precipitously at the liquefaction point. The volume decreases slightly once the substance is solid, but it never becomes zero.*

High pressure may also cause a gas to change phase to a liquid. Carbon dioxide, for example, is a gas at room temperature and atmospheric pressure, but becomes a liquid under sufficiently high pressure. If the pressure is reduced, the temperature drops and the liquid carbon dioxide solidifies into a snow-like substance at the temperature $-78^\circ\text{C}$. Solid $CO_2$ is called "dry ice." Another example of a gas that can be in a liquid phase is liquid nitrogen $(LN_2)$. $LN_2$ is made by liquefaction of atmospheric air (through compression and cooling). It boils at 77 K $(-196^\circ\text{C})$ at atmospheric pressure. $LN_2$ is useful as a refrigerant and allows

for the preservation of blood, sperm, and other biological materials. It is also used to reduce noise in electronic sensors and equipment, and to help cool down their current-carrying wires. In dermatology, $\mathbf{LN_2}$ is used to freeze and painlessly remove warts and other growths from the skin.

## *PV* Diagrams

We can examine aspects of the behavior of a substance by plotting a graph of pressure versus volume, called a **PV diagram**. When the substance behaves like an ideal gas, the ideal gas law describes the relationship between its pressure and volume. That is,

$$\boldsymbol{PV = NkT \textbf{ (ideal gas).}}$$

Now, assuming the number of molecules and the temperature are fixed,

$$\boldsymbol{PV = \textbf{ constant (ideal gas, constant temperature).}}$$

For example, the volume of the gas will decrease as the pressure increases. If you plot the relationship $\boldsymbol{PV = \textbf{constant}}$ on a $\boldsymbol{PV}$ diagram, you find a hyperbola. Figure 2 shows a graph of pressure versus volume. The hyperbolas represent ideal-gas behavior at various fixed temperatures, and are called *isotherms*. At lower temperatures, the curves begin to look less like hyperbolas—the gas is not behaving ideally and may even contain liquid. There is a **critical point**—that is, a **critical temperature**—above which liquid cannot exist. At sufficiently high pressure above the critical point, the gas will have the density of a liquid but will not condense. Carbon dioxide, for example, cannot be liquefied at a temperature above $\mathbf{31.0^\circ C}$. **Critical pressure** is the minimum pressure needed for liquid to exist at the critical temperature. Table 3 lists representative critical temperatures and pressures.

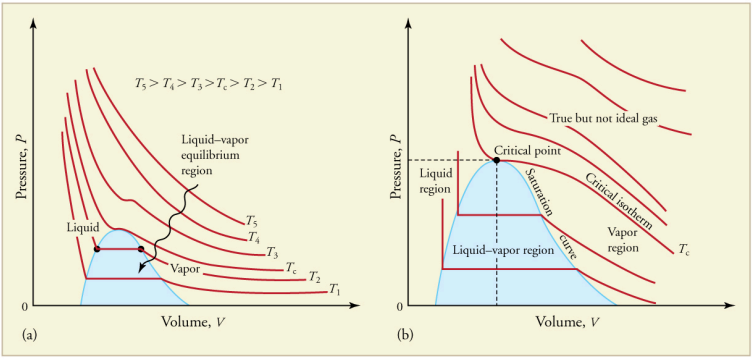



***Figure 2.*** **PV** *diagrams. (a) Each curve (isotherm) represents the relationship between* **P** *and* **V** *at a fixed temperature; the upper curves are at higher temperatures. The lower curves are not hyperbolas, because the gas is no longer an ideal gas. (b) An expanded portion of the* **PV** *diagram for low temperatures, where the phase can change from a gas to a liquid. The term "vapor" refers to the gas phase when it exists at a temperature below the boiling temperature.*

| **Substance** | **Critical temperature** | | **Critical pressure** | |
|---|---|---|---|---|
| | **K** | **°C** | **Pa** | **atm** |
| Water | 647.4 | 374.3 | $\mathbf{22.12 \times 10^6}$ | 219.0 |
| Sulfur dioxide | 430.7 | 157.6 | $\mathbf{7.88 \times 10^6}$ | 78.0 |
| Ammonia | 405.5 | 132.4 | $\mathbf{11.28 \times 10^6}$ | 111.7 |
| Carbon dioxide | 304.2 | 31.1 | $\mathbf{7.39 \times 10^6}$ | 73.2 |
| Oxygen | 154.8 | −118.4 | $\mathbf{5.08 \times 10^6}$ | 50.3 |
| Nitrogen | 126.2 | −146.9 | $\mathbf{3.39 \times 10^6}$ | 33.6 |
| Hydrogen | 33.3 | −239.9 | $\mathbf{1.30 \times 10^6}$ | 12.9 |
| Helium | 5.3 | −267.9 | $\mathbf{0.229 \times 10^6}$ | 2.27 |

**Table 3.** Critical Temperatures and Pressures

## Phase Diagrams

The plots of pressure versus temperatures provide considerable insight into thermal properties of substances. There are well-defined regions on these graphs that correspond to various phases of matter, so $PT$ graphs are called **phase diagrams**. Figure 3 shows the phase diagram for water. Using the graph, if you know the pressure and temperature you can determine the phase of water. The solid lines—boundaries between phases—indicate temperatures and pressures at which the phases coexist (that is, they exist together in ratios, depending on pressure and temperature). For example, the boiling point of water is $100^\circ\text{C}$ at 1.00 atm. As the pressure increases, the boiling temperature rises steadily to $374^\circ\text{C}$ at a pressure of 218 atm. A pressure cooker (or even a covered pot) will cook food faster because the water can exist as a liquid at temperatures greater than $100^\circ\text{C}$ without all boiling away. The curve ends at a point called the *critical point*, because at higher temperatures the liquid phase does not exist at any pressure. The critical point occurs at the critical temperature, as you can see for water from Table 3. The critical temperature for oxygen is $-118^\circ\text{C}$, so oxygen cannot be liquefied above this temperature.

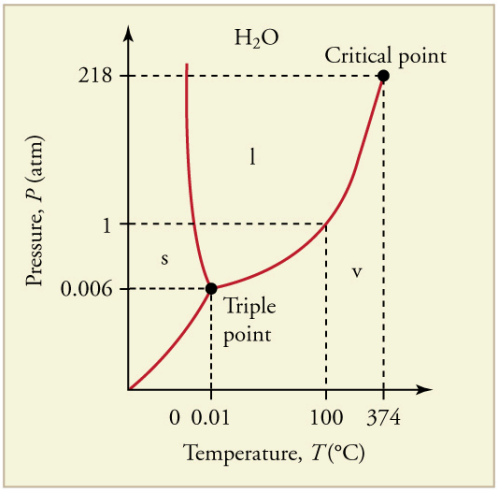



***Figure 3.*** *The phase diagram (**PT** graph) for water. Note that the axes are nonlinear and the graph is not to scale. This graph is simplified—there are several other exotic phases of ice at higher pressures.*

Similarly, the curve between the solid and liquid regions in Figure 3 gives the melting temperature at various pressures. For example, the melting point is $0^\circ C$ at 1.00 atm, as expected. Note that, at a fixed temperature, you can change the phase from solid (ice) to liquid (water) by increasing the pressure. Ice melts from pressure in the hands of a snowball maker. From the phase diagram, we can also say that the melting temperature of ice rises with increased pressure. When a car is driven over snow, the increased pressure from the tires melts the snowflakes; afterwards the water refreezes and forms an ice layer.

At sufficiently low pressures there is no liquid phase, but the substance can exist as either gas or solid. For water, there is no liquid phase at pressures below 0.00600 atm. The phase change from solid to gas is called **sublimation**. It accounts for large losses of snow pack that never make it into a river, the routine automatic

defrosting of a freezer, and the freeze-drying process applied to many foods. Carbon dioxide, on the other hand, sublimates at standard atmospheric pressure of 1 atm. (The solid form of $\mathbf{CO_2}$ is known as dry ice because it does not melt. Instead, it moves directly from the solid to the gas state.)

All three curves on the phase diagram meet at a single point, the triple point, where all three phases exist in equilibrium. For water, the triple point occurs at 273.16 K $\mathbf{(0.01^\circ C)}$, and is a more accurate calibration temperature than the melting point of water at 1.00 atm, or 273.15 K $\mathbf{(0.0^\circ C)}$. See Table 4 for the triple point values of other substances.

## Equilibrium

Liquid and gas phases are in equilibrium at the boiling temperature. (See Figure 4.) If a substance is in a closed container at the boiling point, then the liquid is boiling and the gas is condensing at the same rate without net change in their relative amount. Molecules in the liquid escape as a gas at the same rate at which gas molecules stick to the liquid, or form droplets and become part of the liquid phase. The combination of temperature and pressure has to be "just right"; if the temperature and pressure are increased, equilibrium is maintained by the same increase of boiling and condensation rates.

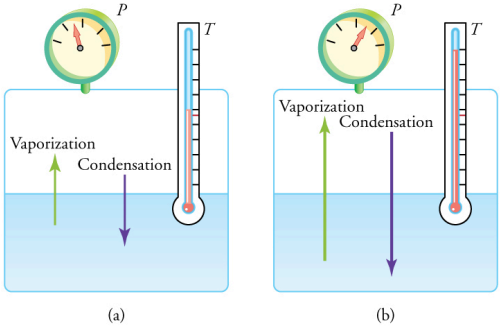



***Figure 4.*** *Equilibrium between liquid and gas at two different boiling points inside a closed container. (a) The rates of boiling and condensation are equal at this combination of temperature and pressure, so the liquid and gas phases are in equilibrium. (b) At a higher temperature, the boiling rate is faster and the rates at which molecules leave the liquid and enter the gas are also faster. Because there are more molecules in the gas, the gas pressure is higher and the rate at which gas molecules condense and enter the liquid is faster. As a result the gas and liquid are in equilibrium at this higher temperature.*

| Substance | Temperature | | Pressure | |
|---|---|---|---|---|
| | K | °C | Pa | atm |
| Water | 273.16 | 0.01 | $6.10 \times 10^2$ | 0.00600 |
| Carbon dioxide | 216.55 | −56.60 | $5.16 \times 10^5$ | 5.11 |
| Sulfur dioxide | 197.68 | −75.47 | $1.67 \times 10^3$ | 0.0167 |
| Ammonia | 195.40 | −77.75 | $6.06 \times 10^3$ | 0.0600 |
| Nitrogen | 63.18 | −210.0 | $1.25 \times 10^4$ | 0.124 |
| Oxygen | 54.36 | −218.8 | $1.52 \times 10^2$ | 0.00151 |
| Hydrogen | 13.84 | −259.3 | $7.04 \times 10^3$ | 0.0697 |

**Table 4.** Triple Point Temperatures and Pressures

One example of equilibrium between liquid and gas is that of water and steam at $100^\circ\text{C}$ and 1.00 atm. This temperature is the boiling point at that pressure, so they should exist in equilibrium. Why does an open pot of water at $100^\circ\text{C}$ boil completely away? The gas surrounding an open pot is not pure water: it is mixed with air. If pure water and steam are in a closed container at $100^\circ\text{C}$ and 1.00 atm, they would coexist—but with air over the pot, there are fewer water molecules to condense, and water boils. What about water at $20.0^\circ\text{C}$ and 1.00 atm? This temperature and pressure correspond to the liquid region, yet an open glass of water at this temperature will completely evaporate. Again, the gas around it is air and not pure water vapor, so that the reduced evaporation rate is greater than the condensation rate of water from dry air. If the glass is sealed, then the liquid phase remains. We call the gas phase a **vapor** when it exists, as it does for water at $20.0^\circ\text{C}$, at a temperature below the boiling temperature.

*Check Your Understanding 1*

Explain why a cup of water (or soda) with ice cubes stays at $0^\circ C$, even on a hot summer day.

## Vapor Pressure, Partial Pressure, and Dalton's Law

**Vapor pressure** is defined as the pressure at which a gas coexists with its solid or liquid phase. Vapor pressure is created by faster molecules that break away from the liquid or solid and enter the gas phase. The vapor pressure of a substance depends on both the substance and its temperature—an increase in temperature increases the vapor pressure.

**Partial pressure** is defined as the pressure a gas would create if it occupied the total volume available. In a mixture of gases, *the total pressure is the sum of partial pressures of the component gases*, assuming ideal gas behavior and no chemical reactions between the components. This law is known as **Dalton's law of partial pressures**, after the English scientist John Dalton (1766–1844), who proposed it. Dalton's law is based on kinetic theory, where each gas creates its pressure by molecular collisions, independent of other gases present. It is consistent with the fact that pressures add according to Chapter 9.5 Pascal's Principle. Thus water evaporates and ice sublimates when their vapor pressures exceed the partial pressure of water vapor in the surrounding mixture of gases. If their vapor pressures are less than the partial pressure of water vapor in the surrounding gas, liquid droplets or ice crystals (frost) form.

## *Check Your Understanding 2*

Is energy transfer involved in a phase change? If so, will energy have to be supplied to change phase from solid to liquid and liquid to gas? What about gas to liquid and liquid to solid? Why do they spray the orange trees with water in Florida when the temperatures are near or just below freezing?

## PHET EXPLORATIONS: STATES OF MATTER—BASICS

Heat, cool, and compress atoms and molecules and watch as they change between solid, liquid, and gas phases.



**PhET Interactive Simulation**

*Figure 5.* *States of Matter: Basics*

## Section Summary

- Most substances have three distinct phases: gas, liquid, and solid.
- Phase changes among the various phases of matter depend on temperature and pressure.
- The existence of the three phases with respect to pressure and temperature can be described in a phase diagram.
- Two phases coexist (i.e., they are in thermal equilibrium) at a set of pressures and temperatures. These are described as a line on a phase diagram.
- The three phases coexist at a single pressure and temperature. This is known as the triple point and is described by a single point on a phase diagram.
- A gas at a temperature below its boiling point is called a vapor.
- Vapor pressure is the pressure at which a gas coexists with its solid or liquid phase.
- Partial pressure is the pressure a gas would create if it existed alone.
- Dalton's law states that the total pressure is the sum of the partial pressures of all of the gases present.

### *Conceptual Questions*

**1:** A pressure cooker contains water and steam in equilibrium at a pressure greater than atmospheric pressure. How does this greater pressure increase cooking speed?

**2:** Why does condensation form most rapidly on the

coldest object in a room—for example, on a glass of ice water?

**3:** What is the vapor pressure of solid carbon dioxide (dry ice) at $-78.5^\circ\text{C}$?

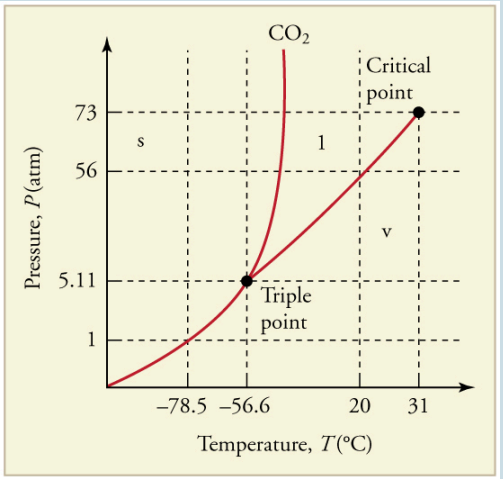



***Figure 6.*** *The phase diagram for carbon dioxide. The axes are nonlinear, and the graph is not to scale. Dry ice is solid carbon dioxide and has a sublimation temperature of* **–78.5°C**.

**4:** Can carbon dioxide be liquefied at room temperature ($20^\circ\text{C}$)? If so, how? If not, why not? (See Figure 6.)

**5:** Oxygen cannot be liquefied at room temperature by placing it under a large enough pressure to force its molecules together. Explain why this is.

**6:** What is the distinction between gas and vapor?

## Glossary

**PV diagram**
a graph of pressure vs. volume

**critical point**
the temperature above which a liquid cannot exist

**critical temperature**
the temperature above which a liquid cannot exist

**critical pressure**
the minimum pressure needed for a liquid to exist at the critical temperature

**vapor**
a gas at a temperature below the boiling temperature

**vapor pressure**
the pressure at which a gas coexists with its solid or liquid phase

**phase diagram**
a graph of pressure vs. temperature of a particular substance, showing at which pressures and temperatures the three phases of the substance occur

**triple point**
the pressure and temperature at which a substance exists in equilibrium as a solid, liquid, and gas

**sublimation**
the phase change from solid to gas

**partial pressure**
the pressure a gas would create if it occupied the total volume of space available

**Dalton's law of partial pressures**
the physical law that states that the total pressure of a gas is the sum of partial pressures of the component gases

## *Solutions*

**Check Your Understanding 1**

The ice and liquid water are in thermal equilibrium, so that the temperature stays at the freezing temperature as long as ice remains in the liquid. (Once all of the ice melts, the water temperature will start to rise.)

**Check Your Understanding 2**

Yes, energy transfer is involved in a phase change. We know that atoms and molecules in solids and liquids are bound to each other because we know that force is required to separate them. So in a phase change from solid to liquid and liquid to gas, a force must be exerted, perhaps by collision, to separate atoms and molecules. Force exerted through a distance is work, and energy is needed to do work to go from solid to liquid and liquid to gas. This is intuitively consistent with the need for energy to melt ice or boil water. The converse is also true. Going from gas to liquid or liquid to solid involves atoms and molecules pushing together, doing work and releasing energy.

PART XII

# CHAPTER 11 HEAT AND HEAT TRANSFER METHODS

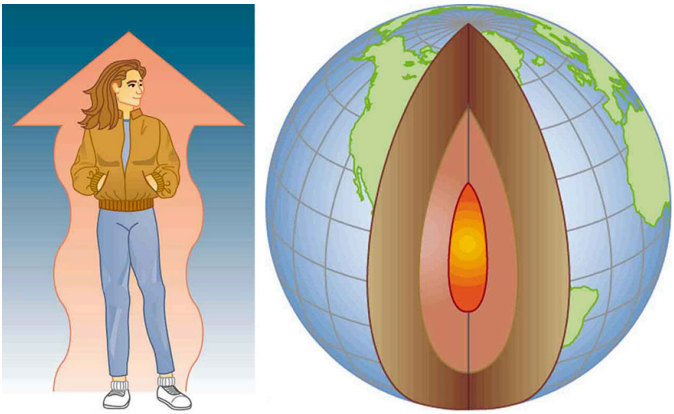



***Figure 1.*** *(a) The chilling effect of a clear breezy night is produced by the wind and by radiative heat transfer to cold outer space. (b) There was once great controversy about the Earth's age, but it is now generally accepted to be about 4.5 billion years old. Much of the debate is centered on the Earth's molten interior. According to our understanding of heat transfer, if the Earth is really that old, its center should have cooled off long ago. The discovery of radioactivity in rocks revealed the source of energy that keeps the Earth's interior molten, despite heat transfer to the surface, and from there to cold outer space.*

Energy can exist in many forms and heat is one of the most intriguing. Heat is often hidden, as it only exists when in transit, and is transferred by a number of distinctly different methods. Heat transfer touches every aspect of our lives and helps us understand

how the universe functions. It explains the chill we feel on a clear breezy night, or why Earth's core has yet to cool. This chapter defines and explores heat transfer, its effects, and the methods by which heat is transferred. These topics are fundamental, as well as practical, and will often be referred to in the chapters ahead.

# 60. 11.1 Heat

*Summary*

- Define heat as transfer of energy.

In Chapter 6 Work, Energy, and Energy Resources, we defined work as force times distance and learned that work done on an object changes its kinetic energy. We also saw in Chapter 10 Temperature and Thermal Expansion that temperature is proportional to the (average) kinetic energy of atoms and molecules. We say that a thermal system has a certain internal energy: its internal energy is higher if the temperature is higher. If two objects at different temperatures are brought in contact with each other, energy is transferred from the hotter to the colder object until equilibrium is reached and the bodies reach thermal equilibrium (i.e., they are at the same temperature). No work is done by either object, because no force acts through a distance. The transfer of energy is caused by the temperature difference, and ceases once the temperatures are equal. These observations lead to the following definition of **heat**: Heat is the spontaneous transfer of energy due to a temperature difference.

As noted in Chapter 10 Temperature and Thermal Expansion, heat is often confused with temperature. For example, we may say the heat was unbearable, when we actually mean that the temperature was high. Heat is a form of energy, whereas temperature is not. The misconception arises because we are sensitive to the flow of heat, rather than the temperature.

Owing to the fact that heat is a form of energy, it has the SI unit

of *joule* (J). The *calorie* (cal) is a common unit of energy, defined as the energy needed to change the temperature of 1.00 g of water by $\mathbf{1.00^\circ C}$ –specifically, between $\mathbf{14.5^\circ C}$ and $\mathbf{15.5^\circ C}$, since there is a slight temperature dependence. Perhaps the most common unit of heat is the **kilocalorie** (kcal), which is the energy needed to change the temperature of 1.00 kg of water by $\mathbf{1.00^\circ C}$. Since mass is most often specified in kilograms, kilocalorie is commonly used. Food calories (given the notation Cal, and sometimes called "big calorie") are actually kilocalories ( $\mathbf{1\ kilocalorie = 1000\ calories}$), a fact not easily determined from package labeling.

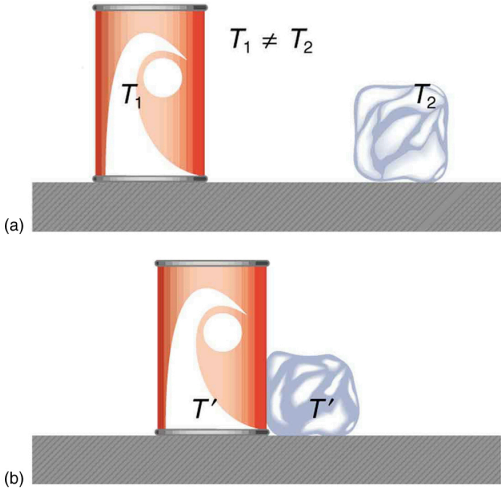



***Figure 1.*** *In figure (a) the soft drink and the ice have different temperatures,* $\mathbf{T_1}$ *and* $\mathbf{T_2}$*, and are not in thermal equilibrium. In figure (b), when the soft drink and ice are allowed to interact, energy is transferred until they reach the same temperature* $\mathbf{T'}$*, achieving equilibrium. Heat transfer occurs due to the difference in temperatures. In fact, since the soft drink and ice are both in contact with the surrounding air and bench, the equilibrium temperature will be the same for both.*

## Mechanical Equivalent of Heat

It is also possible to change the temperature of a substance by doing work. Work can transfer energy into or out of a system. This realization helped establish the fact that heat is a form of energy. James Prescott Joule (1818–1889) performed many experiments to establish the **mechanical equivalent of heat**—*the work needed to produce the same effects as heat transfer*. In terms of the units used for these two terms, the best modern value for this equivalence is

$$\mathbf{1.000\ kcal = 4186\ J}.$$

We consider this equation as the conversion between two different units of energy.

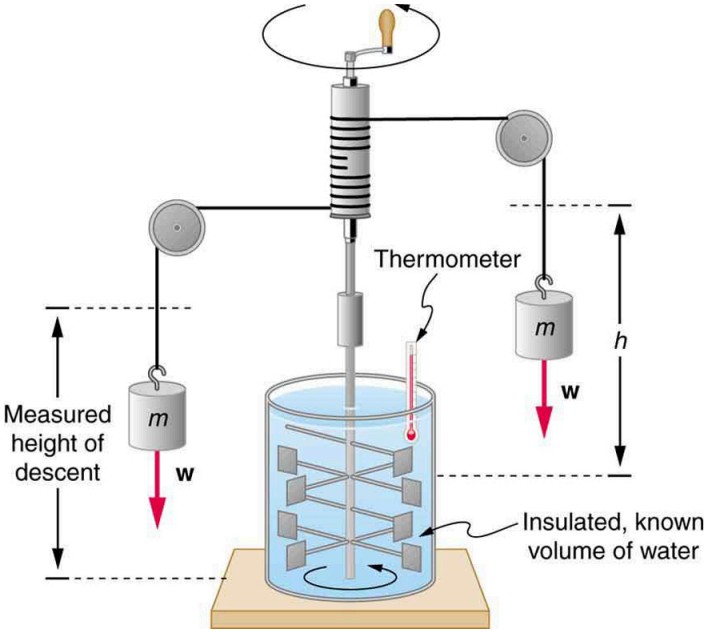



***Figure 2.*** *Schematic depiction of Joule's experiment that established the equivalence of heat and work.*

The figure above shows one of Joule's most famous experimental setups for demonstrating the mechanical equivalent of heat. It demonstrated that work and heat can produce the same effects, and helped establish the principle of conservation of energy. Gravitational potential energy (PE) (work done by the gravitational force) is converted into kinetic energy (KE), and then randomized by viscosity and turbulence into increased average kinetic energy of atoms and molecules in the system, producing a temperature increase. His contributions to the field of thermodynamics were so significant that the SI unit of energy was named after him.

Heat added or removed from a system changes its internal energy and thus its temperature. Such a temperature increase is observed while cooking. However, adding heat does not necessarily increase the temperature. An example is melting of ice; that is, when a substance changes from one phase to another. Work done on the system or by the system can also change the internal energy of the system. Joule demonstrated that the temperature of a system can be increased by stirring. If an ice cube is rubbed against a rough surface, work is done by the frictional force. A system has a well-defined internal energy, but we cannot say that it has a certain "heat content" or "work content". We use the phrase "heat transfer" to emphasize its nature.

## *Check Your Understanding*

**1:** Two samples (A and B) of the same substance are kept in a lab. Someone adds 10 kilojoules (kJ) of heat to one sample, while 10 kJ of work is done on the other sample. How can you tell to which sample the heat was added?

## Summary

- Heat and work are the two distinct methods of energy transfer.
- Heat is energy transferred solely due to a temperature difference.
- Any energy unit can be used for heat transfer, and the most common are kilocalorie (kcal) and joule (J).
- Kilocalorie is defined to be the energy needed to change the temperature of 1.00 kg of water between $\mathbf{14.5^\circ C}$ and $\mathbf{15.5^\circ C}$.
- The mechanical equivalent of this heat transfer is $\mathbf{1.00\ kcal = 4186\ J}$.

### *Conceptual Questions*

**1:** How is heat transfer related to temperature?

**2:** Describe a situation in which heat transfer occurs. What are the resulting forms of energy?

**3:** When heat transfers into a system, is the energy stored as heat? Explain briefly.

## Glossary

**heat**
the spontaneous transfer of energy due to a temperature difference

**kilocalorie**

1 kilocalorie =1000 calories

**mechanical equivalent of heat**

the work needed to produce the same effects as heat transfer

## *Solutions*

**Check Your Understanding**

**1:** Heat and work both change the internal energy of the substance. However, the properties of the sample only depend on the internal energy so that it is impossible to tell whether heat was added to sample A or B.

# 61. 11.2 Temperature Change and Specific Heat

*Summary*

- Observe heat transfer and change in temperature and mass.
- Calculate final temperature after heat transfer between two objects.

One of the major effects of heat transfer is temperature change: heating increases the temperature while cooling decreases it. We assume that there is no phase change and that no work is done on or by the system. Experiments show that the transferred heat depends on three factors—the change in temperature, the mass of the system, and the substance and phase of the substance.

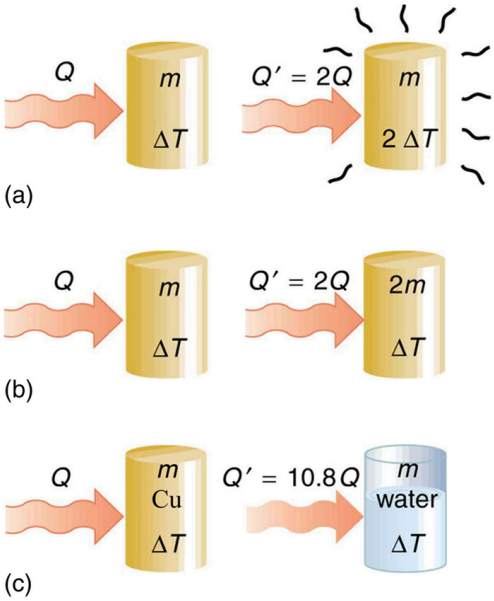



***Figure 1.*** *The heat* **Q** *transferred to cause a temperature change depends on the magnitude of the temperature change, the mass of the system, and the substance and phase involved. (a) The amount of heat transferred is directly proportional to the temperature change. To double the temperature change of a mass* **m**, *you need to add twice the heat. (b) The amount of heat transferred is also directly proportional to the mass. To cause an equivalent temperature change in a doubled mass, you need to add twice the heat. (c) The amount of heat transferred depends on the substance and its phase. If it takes an amount* **Q** *of heat to cause a temperature change* **ΔT** *in a given mass of copper, it will take 10.8 times that amount of heat to cause the equivalent temperature change in the same mass of water assuming no phase change in either substance.*

The dependence on temperature change and mass are easily understood. Owing to the fact that the (average) kinetic energy of an atom or molecule is proportional to the absolute temperature,

the internal energy of a system is proportional to the absolute temperature and the number of atoms or molecules. Owing to the fact that the transferred heat is equal to the change in the internal energy, the heat is proportional to the mass of the substance and the temperature change. The transferred heat also depends on the substance so that, for example, the heat necessary to raise the temperature is less for alcohol than for water. For the same substance, the transferred heat also depends on the phase (gas, liquid, or solid).

## HEAT TRANSFER AND ENERGY CHANGE

The quantitative relationship between heat transfer and temperature change contains all three factors:

$$Q = mc\Delta T,$$

where $Q$ is the symbol for heat transfer, $m$ is the mass of the substance, and $\Delta T$ is the change in temperature. The symbol $c$ stands for specific heat and depends on the material and phase. The specific heat is the amount of heat necessary to change the temperature of 1.00 kg of mass by $1.00^\circ\,\mathrm{C}$. The specific heat cc is a property of the substance; its SI unit is $\mathrm{J/(kg \cdot K)}$ or $\mathrm{J/(kg \cdot {}^\circ C)}$. Recall that the temperature change $(\Delta T)$ is the same in units of kelvin and degrees Celsius. If heat transfer is measured in kilocalories, then *the unit of specific heat* is $\mathrm{kcal/(kg \cdot {}^\circ C)}$.

Values of specific heat must generally be looked up in tables, because there is no simple way to calculate them. In general, the specific heat also depends on the temperature. Table 1 lists representative values of specific heat for various substances. Except for gases, the temperature and volume dependence of the specific heat of most substances is weak. We see from this table that the specific heat of water is five times that of glass and ten times that of iron, which means that it takes five times as much heat to raise the temperature of water the same amount as for glass and ten times as much heat to raise the temperature of water as for iron. In fact, water has one of the largest specific heats of any material, which is important for sustaining life on Earth.

## Example 1: Calculating the Required Heat: Heating Water in an Aluminum Pan

A 0.500 kg aluminum pan on a stove is used to heat 0.250 liters of water from $\mathbf{20.0^\circ C}$ to $\mathbf{80.0^\circ C}$. (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

**Strategy**

The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is increased by the same amount. We use the equation for the heat transfer for the given temperature change and mass of

water and aluminum. The specific heat values for water and aluminum are given in Table 1.

**Solution**

Because water is in thermal contact with the aluminum, the pan and the water are at the same temperature.

1. Calculate the temperature difference:
$$\Delta T = T_f - T_i = 60.0^\circ \text{C}.$$
2. Calculate the mass of water. Because the density of water is $1000 \text{ kg/m}^3$, one liter of water has a mass of 1 kg, and the mass of 0.250 liters of water is $m_w = 0.250 \text{ kg}$.
3. Calculate the heat transferred to the water. Use the specific heat of water in Table 1:
$$Q_w = m_w c_w \Delta T = (0.250 \text{ kg})(4186 \text{ J/kg}^\circ\text{C})(60.0^\circ\text{C}) = 62.8 \text{ kJ}.$$
4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in Table 1:
$$Q_{Al} = m_{Al} c_{Al} \Delta T = (0.500 \text{ kg})(900 \text{ J/kg}^\circ\text{C})(60.0^\circ\text{C}) = 27.0 \times 10^4 \text{ J} = 27.0 \text{ kJ}.$$
5. Compare the percentage of heat going into the pan versus that going into the water. First, find the total transferred heat:
$$Q_{\text{Total}} = Q_W + Q_{Al} = 62.8 \text{ kJ} + 27.0 \text{ kJ} = 89.8 \text{ kJ}.$$

Thus, the amount of heat going into heating the pan is
$$\frac{27.0 \text{ kJ}}{89.8 \text{ kJ}} \times 100\% = 30.1\%,$$
and the amount going into heating the water is
$$\frac{62.8 \text{ kJ}}{89.8 \text{ kJ}} \times 100\% = 69.9\%.$$

**Discussion**

In this example, the heat transferred to the container is a significant fraction of the total transferred heat. Although the mass of the pan is twice that of the water, the specific heat of water is over four times greater than that of aluminum. Therefore, it takes a bit more than twice the heat to achieve the given temperature change for the water as compared to the aluminum pan.



***Figure 2.*** *The smoking brakes on this truck are a visible evidence of the mechanical equivalent of heat.*

## Example 2: Calculating the Temperature Increase from the Work Done on a Substance: Truck Brakes Overheat on Downhill Runs

Truck brakes used to control speed on a downhill run do work, converting gravitational potential energy into increased internal energy (higher temperature) of the brake material. This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck. The problem is that the mass of the truck is large compared with that of the brake material absorbing the energy, and the temperature increase may occur too fast for sufficient heat to transfer from the brakes to the environment.

Calculate the temperature increase of 100 kg of brake material with an average specific heat of $\mathbf{800\ J/kg \cdot ^\circ C}$ if the material retains 10% of the energy from a 10,000-kg truck descending 75.0 m (in vertical displacement) at a constant speed.

**Strategy**

If the brakes are not applied, gravitational potential energy is converted into kinetic energy. When brakes are applied, gravitational potential energy is converted into internal energy of the brake material. We first calculate the gravitational potential energy $(\boldsymbol{Mgh})$

that the entire truck loses in its descent and then find the temperature increase produced in the brake material alone.

**Solution**

1. Calculate the change in gravitational potential energy as the truck goes downhill $Mgh = (10{,}000\ \text{kg})(9.80\ \text{m/s}^2)(75.0\ \text{m}) = 7.35 \times 10^6\ \text{J}$.
2. Calculate the temperature from the heat transferred using $Q = Mgh$ and

$$\Delta T = \frac{Q}{mc},$$

where $m$ is the mass of the brake material. Insert the values $m = 100\ \text{kg}$ and $c = 800\ \text{J/kg}\cdot^\circ\text{C}$ to find

$$\Delta T = \frac{(7.35 \times 10^6\ \text{J})}{(100\ \text{kg})(800\ \text{J/kg}^\circ\text{C})} = 92^\circ\text{C}.$$

**Discussion**

This temperature is close to the boiling point of water. If the truck had been traveling for some time, then just before the descent, the brake temperature would likely be higher than the ambient temperature. The temperature increase in the descent would likely raise the temperature of the brake material above the boiling point of water, so this technique is not practical. However, the same idea underlies the recent hybrid technology of cars, where mechanical energy

(gravitational potential energy) is converted by the brakes into electrical energy (battery).

| **Substances** | **Specific heat (*c*)** | |
|---|---|---|
| Solids | J/kg·°C | kcal/kg·°C[2] |
| Aluminum | 900 | 0.215 |
| Asbestos | 800 | 0.19 |
| Concrete, granite (average) | 840 | 0.20 |
| Copper | 387 | 0.0924 |
| Glass | 840 | 0.20 |
| Gold | 129 | 0.0308 |
| Human body (average at 37 °C) | 3500 | 0.83 |
| Ice (average, -50°C to 0°C) | 2090 | 0.50 |
| Iron, steel | 452 | 0.108 |
| Lead | 128 | 0.0305 |
| Silver | 235 | 0.0562 |
| Wood | 1700 | 0.4 |
| *Liquids* | | |
| Benzene | 1740 | 0.415 |
| Ethanol | 2450 | 0.586 |
| Glycerin | 2410 | 0.576 |
| Mercury | 139 | 0.0333 |
| Water (15.0 °C) | 4186 | 1.000 |
| *Gases*[3] | | |
| Air (dry) | 721 (1015) | 0.172 (0.242) |
| Ammonia | 1670 (2190) | 0.399 (0.523) |
| Carbon dioxide | 638 (833) | 0.152 (0.199) |
| Nitrogen | 739 (1040) | 0.177 (0.248) |
| Oxygen | 651 (913) | 0.156 (0.218) |
| Steam (100°C) | 1520 (2020) | 0.363 (0.482) |

**Table 1.** Specific Heats[1] of Various Substances

Note that Example 2 is an illustration of the mechanical equivalent

of heat. Alternatively, the temperature increase could be produced by a blow torch instead of mechanically.

## Example 3: Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan

Suppose you pour 0.250 kg of $\mathbf{20.0^\circ C}$ water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of $\mathbf{150^\circ C}$. Assume that the pan is placed on an insulated pad and that a negligible amount of water boils off. What is the temperature when the water and pan reach thermal equilibrium a short time later?

**Strategy**

The pan is placed on an insulated pad so that little heat transfer occurs with the surroundings. Originally the pan and water are not in thermal equilibrium: the pan is at a higher temperature than the water. Heat transfer then restores thermal equilibrium once the water and pan are in contact. Because heat transfer between the pan and water takes place rapidly, the mass of evaporated water is negligible and the magnitude of the heat lost by the pan is equal to the heat gained by the water. The exchange of heat stops once a thermal equilibrium between the pan and the water is achieved.

The heat exchange can be written as $|Q_{\text{hot}}| = Q_{\text{cold}}$.

**Solution**

1. Use the equation for heat transfer $Q = mc\Delta T$ to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:
$$Q_{\text{hot}} = m_{\text{Al}} c_{\text{Al}} (T_{\text{f}} - 150^{\circ}\text{C}).$$
2. Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water and the final temperature:
$$Q_{\text{cold}} = m_{\text{W}} c_{\text{W}} (T_{\text{f}} - 20.0^{\circ}\text{C}).$$
3. Note that $Q_{\text{hot}} < 0$ and $Q_{\text{cold}} > 0$0}" title="\boldsymbol{Q_{\textbf{cold}}>0}" class="latex mathjax">and that they must sum to zero because the heat lost by the hot pan must be the same as the heat gained by the cold water:
$$\begin{aligned} Q_{\text{cold}} + Q_{\text{hot}} &= 0, \\ Q_{\text{cold}} &= -Q_{\text{hot}}, \\ m_{\text{W}} c_{\text{W}} (T_{\text{f}} - 20.0^{\circ}\text{C}) &= -m_{\text{Al}} c_{\text{Al}} (T_{\text{f}} - 150^{\circ}\text{C}). \end{aligned}$$
4. This an equation for the unknown final temperature, $T_{\text{f}}$
5. Bring all terms involving $T_{\text{f}}$ on the left hand side and all other terms on the right hand side. Solve for $T_{\text{f}}$,
$$T_{\text{f}} =$$

$$\frac{m_{\text{Al}}c_{\text{Al}}(150^\circ\text{C}) + m_{\text{W}}c_{\text{W}}(20.0^\circ\text{C})}{m_{\text{Al}}c_{\text{Al}} + m_{\text{W}}c_{\text{W}}},$$

and insert the numerical values:

$$\begin{aligned} T_{\text{f}} &= \frac{(0.500\text{ kg})(900\text{ J/kg}^\circ\text{C})(150^\circ\text{C}) + (0.250\text{ kg})(4186\text{ J/kg}^\circ\text{C})(20.0^\circ\text{C})}{(0.500\text{ kg})(900\text{ J/kg}^\circ\text{C}) + (0.250\text{ kg})(4186\text{ J/kg}^\circ\text{C})} \\ &= \frac{88430\text{ J}}{1496.5\text{ J/}^\circ\text{C}} \\ &= 59.1^\circ\text{C}. \end{aligned}$$

**Discussion**

This is a typical *calorimetry* problem—two bodies at different temperatures are brought in contact with each other and exchange heat until a common temperature is reached. Why is the final temperature so much closer to $20.0^\circ\text{C}$ than $150^\circ\text{C}$? The reason is that water has a greater specific heat than most common substances and thus undergoes a small temperature change for a given heat transfer. A large body of water, such as a lake, requires a large amount of heat to increase its temperature appreciably. This explains why the temperature of a lake stays relatively constant during a day even when the temperature change of the air is large. However, the water temperature does change over longer times (e.g., summer to winter).

## TAKE-HOME EXPERIMENT:

## TEMPERATURE CHANGE OF LAND AND WATER

What heats faster, land or water?

To study differences in specific heat:

- Place equal masses of dry sand (or soil) and water at the same temperature into two small jars. (The average density of soil or sand is about 1.6 times that of water, so you can achieve approximately equal masses by using **50%** more water by volume.)
- Heat both (using an oven or a heat lamp) for the same amount of time.
- Record the final temperature of the two masses.
- Now bring both jars to the same temperature by heating for a longer period of time.
- Remove the jars from the heat source and measure their temperature every 5 minutes for about 30 minutes.

Which sample cools off the fastest? This activity replicates the phenomena responsible for land breezes and sea breezes.

### *Check Your Understanding*

**1:** If 25 kJ is necessary to raise the temperature of a block from $25^\circ\text{C}$ to $30^\circ\text{C}$, how much heat is necessary to heat the block from $45^\circ\text{C}$ to $50^\circ\text{C}$?

## Summary

- The transfer of heat $Q$ that leads to a change $\Delta T$ in the temperature of a body with mass $m$ is $Q = mc\Delta T$, where $c$ is the specific heat of the material. This relationship can also be considered as the definition of specific heat.

### *Conceptual Questions*

**1:** What three factors affect the heat transfer that is necessary to change an object's temperature?

**2:** The brakes in a car increase in temperature by $\Delta T$ when bringing the car to rest from a speed $v$. How much greater would $\Delta T$ be if the car initially had twice the speed? You may assume the car to stop sufficiently fast so that no heat transfers out of the brakes.

## *Problems & Exercises*

**1:** On a hot day, the temperature of an 80,000-L swimming pool increases by $\mathbf{1.50^\circ C}$. What is the net heat transfer during this heating? Ignore any complications, such as loss of water by evaporation.

**2:** Show that $\mathbf{1\ cal/g\cdot^\circ C = 1\ kcal/kg\cdot^\circ C}$.

**3:** To sterilize a 50.0-g glass baby bottle, we must raise its temperature from $\mathbf{22.0^\circ C}$ to $\mathbf{95.0^\circ C}$. How much heat transfer is required?

**4:** The same heat transfer into identical masses of different substances produces different temperature changes. Calculate the final temperature when 1.00 kcal of heat transfers into 1.00 kg of the following, originally at $\mathbf{20.0^\circ C}$: (a) water; (b) concrete; (c) steel; and (d) mercury.

**5:** Rubbing your hands together warms them by converting work into thermal energy. If a woman rubs her hands back and forth for a total of 20 rubs, at a distance of 7.50 cm per rub, and with an average frictional force of 40.0 N, what is the temperature increase? The mass of tissues warmed is only 0.100 kg, mostly in the palms and fingers.

**6:** A 0.250-kg block of a pure material is heated from $\mathbf{20.0^\circ C}$ to $\mathbf{65.0^\circ C}$ by the addition of 4.35 kJ of energy. Calculate its specific heat and identify the substance of which it is most likely composed.

**7:** Suppose identical amounts of heat transfer into different masses of copper and water, causing identical changes in temperature. What is the ratio of the mass of copper to water?

**8:** (a) The number of kilocalories in food is determined by calorimetry techniques in which the food is burned and the amount of heat transfer is measured. How many kilocalories per gram are there in a 5.00-g peanut if the energy from burning it is transferred to 0.500 kg of water held in a 0.100-kg aluminum cup, causing a $\mathbf{54.9^\circ C}$ temperature increase? (b) Compare your answer to labeling information found on a package of peanuts and comment on whether the values are consistent.

**9:** Following vigorous exercise, the body temperature of an 80.0-kg person is 40.0 $^{o}$C. At what rate in watts must the person transfer thermal energy to reduce the the body temperature to 37.0 $^{o}$C in 30.0 min, assuming the body continues to produce energy at the rate of 150 W? Remember that the specific heat capacity of a human is 3500 J/kg $^{o}$C and 1 watt = 1 joule / second or 1 W = 1 J/s.

**10:** Even when shut down after a period of normal use, a large commercial nuclear reactor transfers thermal energy at the rate of 150 MW by the radioactive decay of fission products. This heat transfer causes a rapid increase in temperature if the cooling system fails.

Remember 1 watt = 1 joule / second or 1 W = 1 J/s and MW = megawatt where M = 1,000,000 or 1 x $10^6$ . (a) Calculate the rate of temperature increase in degrees Celsius per second or $^{o}$C/s if the mass of the reactor core is

1.60 x $10^5$ kg and it has an average specific heat of 0.3349 kJ/kg $^{o}$C.

(b) How long would it take to obtain a temperature increase of 2000$^{o}$C which could cause some metals holding the radioactive materials to melt? (The initial rate of temperature increase would be greater than that calculated here because the heat transfer is concentrated in a smaller mass. Later, however, the temperature increase would slow down because the $\mathbf{5 \times 10^5\text{-kg}}$ steel containment vessel would also begin to heat up.)

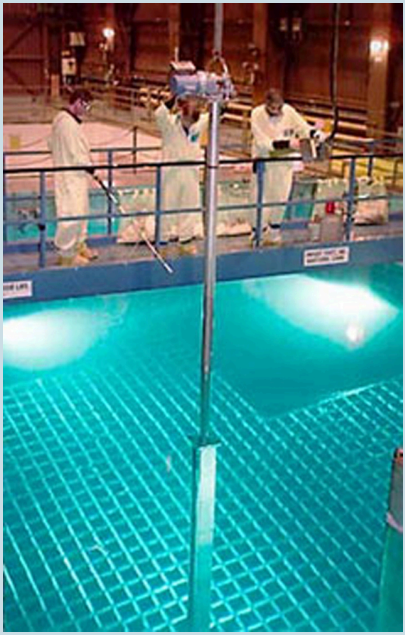


***Figure 3.*** *Radioactive spent-fuel pool at a nuclear power plant. Spent fuel stays hot for a long time. (credit: U.S. Department of Energy)*

## Footnotes

1. [1] The values for solids and liquids are at constant volume and at $\mathbf{25^\circ C}$, except as noted.
2. [2] These values are identical in units of $\mathbf{cal/g \cdot ^\circ C}$.
3. [3] $\mathbf{c_v}$ at constant volume and at $\mathbf{20.0^\circ C}$, except as noted,

and at 1.00 atm average pressure. Values in parentheses are $c_p$ a t a constant pressure of 1.00 atmosphere.

## Glossary

**specific heat**
the amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 °C

## *Solutions*

**Check Your Understanding**

**1:** The heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case.

**Problems & Exercises**

**1:** $5.02 \times 10^8$ J

**3:** $3.07 \times 10^3$ J

**5:** 0.171 degrees Celsius

**6**: $43.5 \times 10^3$ J = (0.250 kg) c ($45\ ^0$C) so c = 386 J/ kg $^o$C likely copper

**7:** 10.8

**9:** 617 W

# 62. 11.3 Phase Change and Latent Heat

*Summary*

- Examine heat transfer.
- Calculate final temperature from heat transfer.

So far we have discussed temperature change due to heat transfer. No temperature change occurs from heat transfer if ice melts and becomes liquid water (i.e., during a phase change). For example, consider water dripping from icicles melting on a roof warmed by the Sun. Conversely, water freezes in an ice tray cooled by lower-temperature surroundings.



***Figure 1.*** *Heat from the air transfers to the ice causing it to melt. (credit: Mike Brand)*

Energy is required to melt a solid because the cohesive bonds between the molecules in the solid must be broken apart such that, in the liquid, the molecules can move around at comparable kinetic energies; thus, there is no rise in temperature. Similarly, energy is needed to vaporize a liquid, because molecules in a liquid interact with each other via attractive forces. There is no temperature change until a phase change is complete. The temperature of a cup of soda initially at $\mathbf{0^\circ C}$ stays at $\mathbf{0^\circ C}$ until all the ice has melted. Conversely, energy is released during freezing and condensation, usually in the form of thermal energy. Work is done by cohesive forces when molecules are brought together. The corresponding energy must be given off (dissipated) to allow them to stay together Figure 2.

The energy involved in a phase change depends on two major factors: the number and strength of bonds or force pairs. The number of bonds is proportional to the number of molecules and thus to the mass of the sample. The strength of forces depends on the type of molecules. The heat $\boldsymbol{Q}$ required to change the phase of a sample of mass $\boldsymbol{m}$ is given by

$$\boldsymbol{Q = mL_f \ (\text{melting/freezing}),}$$
$$\boldsymbol{Q = mL_v \ (\text{vaporization/condensation}),}$$

where the latent heat of fusion, $\boldsymbol{L_f}$, and latent heat of vaporization, $\boldsymbol{L_v}$, are material constants that are determined experimentally. See (Table 2).

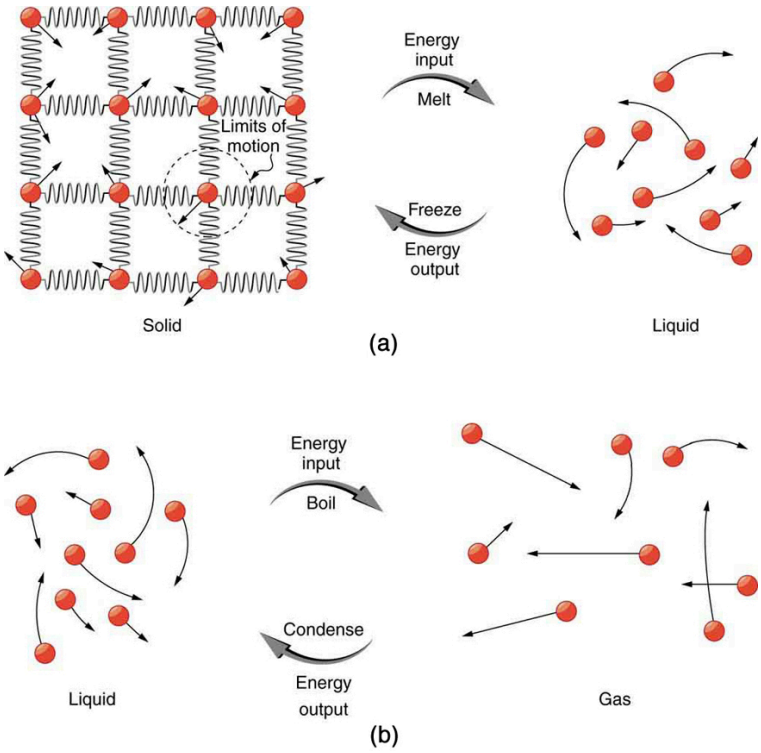



***Figure 2.*** *(a) Energy is required to partially overcome the attractive forces between molecules in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Molecules are separated by large distances when going from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is complete.*

Latent heat is measured in units of J/kg. Both $\boldsymbol{L_f}$ and $\boldsymbol{L_v}$ depend on the substance, particularly on the strength of its molecular forces as noted earlier. $\boldsymbol{L_f}$ and $\boldsymbol{L_v}$ are collectively called **latent heat coefficients**. They are *latent*, or hidden, because in phase changes, energy enters or leaves a system without causing a temperature change in the system; so, in effect, the energy is hidden. Table 2

lists representative values of $L_f$ and $L_v$, together with melting and boiling points.

The table shows that significant amounts of energy are involved in phase changes. Let us look, for example, at how much energy is needed to melt a kilogram of ice at $0^\circ C$ to produce a kilogram of water at $0^\circ C$. Using the equation for a change in temperature and the value for water from Table 2, we find that $Q = mL_f = (1.0\ kg)(334\ kJ/kg) = 334\ kJ$ is the energy to melt a kilogram of ice. This is a lot of energy as it represents the same amount of energy needed to raise the temperature of 1 kg of liquid water from $0^\circ C$ to $79.8^\circ C$. Even more energy is required to vaporize water; it would take 2256 kJ to change 1 kg of liquid water at the normal boiling point ($100^\circ C$ at atmospheric pressure) to steam (water vapor). This example shows that the energy for a phase change is enormous compared to energy associated with temperature changes without a phase change.

| Substance | Melting point (°C) | $L_f$ kJ/kg | $L_f$ kcal/kg | Boiling point (°C) | $L_v$ kJ/kg | $L_v$ kcal/kg |
|---|---|---|---|---|---|---|
| Helium | –269.7 | 5.23 | 1.25 | –268.9 | 20.9 | 4.99 |
| Hydrogen | –259.3 | 58.6 | 14.0 | –252.9 | 452 | 108 |
| Nitrogen | –210.0 | 25.5 | 6.09 | –195.8 | 201 | 48.0 |
| Oxygen | –218.8 | 13.8 | 3.30 | –183.0 | 213 | 50.9 |
| Ethanol | –114 | 104 | 24.9 | 78.3 | 854 | 204 |
| Ammonia | –75 | | 108 | –33.4 | 1370 | 327 |
| Mercury | –38.9 | 11.8 | 2.82 | 357 | 272 | 65.0 |
| Water | 0.00 | 334 | 79.8 | 100.0 | 2256[2] | 539[3] |
| Sulfur | 119 | 38.1 | 9.10 | 444.6 | 326 | 77.9 |
| Lead | 327 | 24.5 | 5.85 | 1750 | 871 | 208 |
| Antimony | 631 | 165 | 39.4 | 1440 | 561 | 134 |
| Aluminum | 660 | 380 | 90 | 2450 | 11400 | 2720 |
| Silver | 961 | 88.3 | 21.1 | 2193 | 2336 | 558 |
| Gold | 1063 | 64.5 | 15.4 | 2660 | 1578 | 377 |
| Copper | 1083 | 134 | 32.0 | 2595 | 5069 | 1211 |
| Uranium | 1133 | 84 | 20 | 3900 | 1900 | 454 |
| Tungsten | 3410 | 184 | 44 | 5900 | 4810 | 1150 |

**Table 2.** Heats of Fusion and Vaporization [1]

Phase changes can have a tremendous stabilizing effect even on temperatures that are not near the melting and boiling points, because evaporation and condensation (conversion of a gas into a liquid state) occur even at temperatures below the boiling point. Take, for example, the fact that air temperatures in humid climates rarely go above $\mathbf{35.0^\circ C}$, which is because most heat transfer goes into evaporating water into the air. Similarly, temperatures in humid weather rarely fall below the dew point because enormous heat is released when water vapor condenses.

We examine the effects of phase change more precisely by

considering adding heat into a sample of ice at $-20^\circ\text{C}$ (Figure 3). The temperature of the ice rises linearly, absorbing heat at a constant rate of $0.50\ \text{cal/g}\cdot^\circ\text{C}$ until it reaches $0^\circ\text{C}$. Once at this temperature, the ice begins to melt until all the ice has melted, absorbing 79.8 cal/g of heat. The temperature remains constant at $0^\circ\text{C}$ during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate of $1.00\ \text{cal/g}\cdot^\circ\text{C}$. At $100^\circ\text{C}$, the water begins to boil and the temperature again remains constant while the water absorbs 539 cal/g of heat during this phase change. When all the liquid has become steam vapor, the temperature rises again, absorbing heat at a rate of $0.482\ \text{cal/g}\cdot^\circ\text{C}$.

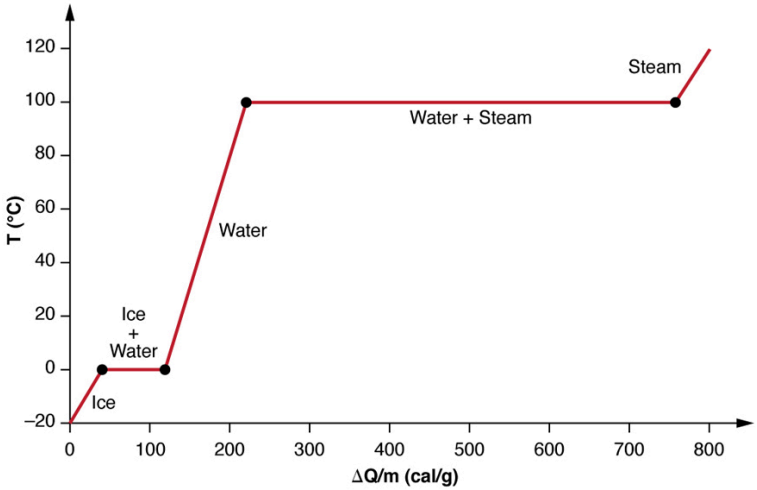



***Figure 3.*** *A graph of temperature versus energy added. The system is constructed so that no vapor evaporates while ice warms to become liquid water, and so that, when vaporization occurs, the vapor remains in of the system. The long stretches of constant temperature values at 0°C and 100°C reflect the large latent heat of melting and vaporization, respectively.*

Water can evaporate at temperatures below the boiling point. More energy is required than at the boiling point, because the kinetic

energy of water molecules at temperatures below $100^\circ\text{C}$ is less than that at $100^\circ\text{C}$, hence less energy is available from random thermal motions. Take, for example, the fact that, at body temperature, perspiration from the skin requires a heat input of 2428 kJ/kg, which is about 10 percent higher than the latent heat of vaporization at $100^\circ\text{C}$. This heat comes from the skin, and thus provides an effective cooling mechanism in hot weather. High humidity inhibits evaporation, so that body temperature might rise, leaving unevaporated sweat on your brow.

## Example 1: Calculate Final Temperature from Phase Change: Cooling Soda with Ice Cubes

Three ice cubes are used to chill a soda at $20^\circ\text{C}$ with mass $m_{\text{soda}} = 0.25\ \text{kg}$. The ice is at $0^\circ\text{C}$ and each ice cube has a mass of 6.0 g. Assume that the soda is kept in a foam container so that heat loss can be ignored. Assume the soda has the same heat capacity as water. Find the final temperature when all ice has melted.

**Strategy**

The ice cubes are at the melting temperature of $0^\circ\text{C}$. Heat is transferred from the soda to the ice for melting. Melting of ice occurs in two steps: first the phase change occurs and solid (ice) transforms into liquid water at the melting temperature, then the

temperature of this water rises. Melting yields water at $\mathbf{0^\circ C}$, so more heat is transferred from the soda to this water until the water plus soda system reaches thermal equilibrium,

$$Q_{\text{ice}} = -Q_{\text{soda}}.$$

The heat transferred to the ice is

$$Q_{\text{ice}} = m_{\text{ice}} L_{\text{f}} + m_{\text{ice}} c_{\text{W}} (T_{\text{f}} - 0^\circ \text{C}).$$

The heat given off by the soda is

$$Q_{\text{soda}} = m_{\text{soda}} c_{\text{W}} (T_{\text{f}} - 20^\circ \text{C}).$$

Since no heat is lost, $Q_{\text{ice}} = -Q_{\text{soda}}$, so that

$$m_{\text{ice}} L_{\text{f}} + m_{\text{ice}} c_{\text{W}} (T_{\text{f}} - 0^\circ \text{C}) = -m_{\text{soda}} c_{\text{W}} (T_{\text{f}} - 20^\circ \text{C}).$$

Bring all terms involving $T_{\text{f}}$ on the left-hand-side and all other terms on the right-hand-side. Solve for the unknown quantity $T_{\text{f}}$:

$$T_{\text{f}} = \frac{m_{\text{soda}} c_{\text{W}} (20^\circ \text{C}) - m_{\text{ice}} L_{\text{f}}}{(m_{\text{soda}} + m_{\text{ice}}) c_{\text{W}}}.$$

**Solution**

1. Identify the known quantities. The mass of ice is $m_{\text{ice}} = 3 \times 6.0 \text{ g} = 0.018 \text{ kg}$ and the mass of soda is $m_{\text{soda}} = 0.25 \text{ kg}$.
2. Calculate the terms in the numerator:
   $m_{\text{soda}} c_{\text{W}} (20^\circ \text{C}) = (0.25 \text{ kg})(4186 \text{ J/kg}\cdot^\circ \text{C})(20^\circ \text{C}) = 20,930 \text{ J}$
   and
   $m_{\text{ice}} L_{\text{f}} = (0.018 \text{ kg})(334,000 \text{ J/kg}) = 6012 \text{ J}.$
3. Calculate the denominator:
   $(m_{\text{soda}} + m_{\text{ice}}) c_{\text{W}} = (0.25 \text{ kg} + 0.018 \text{ kg})(4186 \text{ K/kg}\cdot^\circ \text{C}) = 1122 \text{ J}/^\circ \text{C}.$
4. Calculate the final temperature:

$$T_{\mathrm{f}} = \frac{20,930\ \mathrm{J} - 6012\ \mathrm{J}}{1122\ \mathrm{J}/^{\circ}\mathrm{C}} = 13^{\circ}\mathrm{C}.$$

**Discussion**

This example illustrates the enormous energies involved during a phase change. The mass of ice is about 7 percent the mass of water but leads to a noticeable change in the temperature of soda. Although we assumed that the ice was at the freezing temperature, this is incorrect: the typical temperature is $-6^{\circ}\mathrm{C}$. However, this correction gives a final temperature that is essentially identical to the result we found. Can you explain why?

We have seen that vaporization requires heat transfer to a liquid from the surroundings, so that energy is released by the surroundings. Condensation is the reverse process, increasing the temperature of the surroundings. This increase may seem surprising, since we associate condensation with cold objects—the glass in the figure, for example. However, energy must be removed from the condensing molecules to make a vapor condense. The energy is exactly the same as that required to make the phase change in the other direction, from liquid to vapor, and so it can be calculated from $Q = mL_{\mathrm{v}}$.



***Figure 4.*** *Condensation forms on this glass of iced tea because the temperature of the nearby air is reduced to below the dew point. The rate at which water molecules join together exceeds the rate at which they separate, and so water condenses. Energy is released when the water condenses, speeding the melting of the ice in the glass. (credit: Jenny Downing)*

## REAL-WORLD APPLICATION

Energy is also released when a liquid freezes. This phenomenon is used by fruit growers in Florida to protect oranges when the temperature is close to the

freezing point $0^\circ C$. Growers spray water on the plants in orchards so that the water freezes and heat is released to the growing oranges on the trees. This prevents the temperature inside the orange from dropping below freezing, which would damage the fruit.



***Figure 5.*** *The ice on these trees released large amounts of energy when it froze, helping to prevent the temperature of the trees from dropping below* **0ºC**. *Water is intentionally sprayed on orchards to help prevent hard frosts. (credit: Hermann Hammer)*

**Sublimation** is the transition from solid to vapor phase. You may have noticed that snow can disappear into thin air without a trace of liquid water, or the disappearance of ice cubes in a freezer. The reverse is also true: Frost can form on very cold windows without going through the liquid stage. A popular effect is the making of "smoke" from dry ice, which is solid carbon dioxide. Sublimation occurs because the equilibrium vapor pressure of solids is not zero. Certain air fresheners use the sublimation of a solid to inject a

perfume into the room. Moth balls are a slightly toxic example of a phenol (an organic compound) that sublimates, while some solids, such as osmium tetroxide, are so toxic that they must be kept in sealed containers to prevent human exposure to their sublimation-produced vapors.




(a)



(b)

***Figure 6.*** *Direct transitions between solid and vapor are common, sometimes useful, and even beautiful. (a) Dry ice sublimates directly to carbon dioxide*

*gas. The visible vapor is made of water droplets. (credit: Windell Oskay) (b) Frost forms patterns on a very cold window, an example of a solid formed directly from a vapor. (credit: Liz West)*

All phase transitions involve heat. In the case of direct solid-vapor transitions, the energy required is given by the equation $\boldsymbol{Q = mL_s}$, where $\boldsymbol{L_s}$ is the **heat of sublimation**, which is the energy required to change 1.00 kg of a substance from the solid phase to the vapor phase. $\boldsymbol{L_s}$ is analogous to $\boldsymbol{L_f}$ and $\boldsymbol{L_v}$, and its value depends on the substance. Sublimation requires energy input, so that dry ice is an effective coolant, whereas the reverse process (i.e., frosting) releases energy. The amount of energy required for sublimation is of the same order of magnitude as that for other phase transitions.

The material presented in this section and the preceding section allows us to calculate any number of effects related to temperature and phase change. In each case, it is necessary to identify which temperature and phase changes are taking place and then to apply the appropriate equation. Keep in mind that heat transfer and work can cause both temperature and phase changes.

## Problem-Solving Strategies for the Effects of Heat Transfer

1. *Examine the situation to determine that there is a change in the temperature or phase. Is there heat transfer into or out of the system?* When the presence or absence of a phase change is not obvious, you may wish to first solve the problem as if there were no phase changes, and examine the temperature change obtained. If it is sufficient to take you past a boiling or melting point, you should then go back and do the problem in

steps—temperature change, phase change, subsequent temperature change, and so on.

2. *Identify and list all objects that change temperature and phase.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful.
4. *Make a list of what is given or what can be inferred from the problem as stated (identify the knowns).*
5. *Solve the appropriate equation for the quantity to be determined (the unknown).* If there is a temperature change, the transferred heat depends on the specific heat whereas, for a phase change, the transferred heat depends on the latent heat. See Table 2.
6. *Substitute the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.* You will need to do this in steps if there is more than one stage to the process (such as a temperature change followed by a phase change).
7. *Check the answer to see if it is reasonable: Does it make sense?* As an example, be certain that the temperature change does not also cause a phase change that you have not taken into account.

## *Check Your Understanding*

**1:** Why does snow remain on mountain slopes even when daytime temperatures are higher than the freezing temperature?

## Summary

- Most substances can exist either in solid, liquid, and gas forms, which are referred to as "phases."
- Phase changes occur at fixed temperatures for a given substance at a given pressure, and these temperatures are called boiling and freezing (or melting) points.
- During phase changes, heat absorbed or released is given by:

$$Q = mL,$$

where $L$ is the latent heat coefficient.

### *Conceptual Questions*

**1:** Heat transfer can cause temperature and phase changes. What else can cause these changes?

**2:** How does the latent heat of fusion of water help slow the decrease of air temperatures, perhaps preventing temperatures from falling significantly below $0^\circ\text{C}$, in the vicinity of large bodies of water?

**3:** What is the temperature of ice right after it is formed by freezing water?

**4:** If you place $0^\circ\text{C}$ water in an insulated container, what will happen? Will some ice melt, will more water freeze, or will neither take place?

**5:** What effect does condensation on a glass of ice water have on the rate at which the ice melts? Will the

condensation speed up the melting process or slow it down?

**6:** In very humid climates where there are numerous bodies of water, such as in Florida, it is unusual for temperatures to rise above about $35^\circ C (95^\circ F)$. In deserts, however, temperatures can rise far above this. Explain how the evaporation of water helps limit high temperatures in humid climates.

**7:** In winters, it is often warmer in San Francisco than in nearby Sacramento, 150 km inland. In summers, it is nearly always hotter in Sacramento. Explain how the bodies of water surrounding San Francisco moderate its extreme temperatures.

**8:** Putting a lid on a boiling pot greatly reduces the heat transfer necessary to keep it boiling. Explain why.

**9:** Freeze-dried foods have been dehydrated in a vacuum. During the process, the food freezes and must be heated to facilitate dehydration. Explain both how the vacuum speeds up dehydration and why the food freezes as a result.

**10:** When still air cools by radiating at night, it is unusual for temperatures to fall below the dew point. Explain why.

**11:** In a physics classroom demonstration, an instructor inflates a balloon by mouth and then cools it in liquid nitrogen. When cold, the shrunken balloon has a small amount of light blue liquid in it, as well as some snow-like crystals. As it warms up, the liquid boils, and part of the crystals sublimate, with some crystals lingering for awhile

and then producing a liquid. Identify the blue liquid and the two solids in the cold balloon. Justify your identifications using data from Table 2.

## *Problems & Exercises*

**1:** How much heat transfer (in kilocalories) is required to thaw a 0.450-kg package of frozen vegetables originally at $0^\circ\text{C}$ if their heat of fusion is the same as that of water?

**2:** A bag containing $0^\circ\text{C}$ ice is much more effective in absorbing energy than one containing the same amount of $0^\circ\text{C}$ water.

1. How much heat transfer is necessary to raise the temperature of 0.800 kg of water from $0^\circ\text{C}$ to $30.0^\circ\text{C}$?
2. How much heat transfer is required to first melt 0.800 kg of $0^\circ\text{C}$ ice and then raise its temperature?
3. Explain how your answer supports the contention that the ice is more effective.

**3:** (a) How much heat transfer is required to raise the temperature of a 0.750-kg aluminum pot containing 2.50 kg of water from $30.0^\circ\text{C}$ to the boiling point and then boil away 0.750 kg of water? (b) How long does this take if the rate of heat transfer is 500 W $1\ \text{watt} = 1\ \text{joule/second}\ (1\ \text{W} = 1\ \text{J/s})$?

**4:** The formation of condensation on a glass of ice water causes the ice to melt faster than it would otherwise. If 8.00 g of condensation forms on a glass containing both water and 200 g of ice, how many grams of the ice will melt as a result? Assume no other heat transfer occurs.

**5:** On a trip, you notice that a 3.50-kg bag of ice lasts an average of one day in your cooler. What is the average power in watts entering the ice if it starts at $\mathbf{0^\circ C}$ and completely melts to $\mathbf{0^\circ C}$ water in exactly one day $\mathbf{(1\ watt\ =\ 1\ joule/second (1\ W\ =\ 1\ J/s)}$?

**6:** On a certain dry sunny day, a swimming pool's temperature would rise by $\mathbf{1.50^\circ C}$ if not for evaporation. What fraction of the water must evaporate to carry away precisely enough energy to keep the temperature constant?

**7:** (a) How much heat transfer is necessary to raise the temperature of a 0.200-kg piece of ice from $\mathbf{-20.0^\circ C}$ to $\mathbf{130^\circ C}$, including the energy needed for phase changes?

(b) How much time is required for each stage, assuming a constant 20.0 kJ/s rate of heat transfer?

(c) Make a graph of temperature versus time for this process.

**8:** In 1986, a gargantuan iceberg broke away from the Ross Ice Shelf in Antarctica. It was approximately a rectangle 160 km long, 40.0 km wide, and 250 m thick.

(a) What is the mass of this iceberg, given that the density of ice is $\mathbf{917\ kg/m^3}$?

(b) How much heat transfer (in joules) is needed to melt it?

(c) How many years would it take sunlight alone to melt ice this thick, if the ice absorbs an average of $\mathbf{100\ W/m^2}$, 12.00 h per day?

**9:** How many grams of coffee must evaporate from 350 g of coffee in a 100-g glass cup to cool the coffee from $\mathbf{95.0^\circ C}$ to $\mathbf{45.0^\circ C}$? You may assume the coffee has the same thermal properties as water and that the average heat of vaporization is 2340 kJ/kg (560 cal/g). (You may neglect the change in mass of the coffee as it cools, which will give you an answer that is slightly larger than correct.)

**10:** (a) It is difficult to extinguish a fire on a crude oil tanker, because each liter of crude oil releases $\mathbf{2.80 \times 10^7\ J}$ of energy when burned. To illustrate this difficulty, calculate the number of liters of water that must be expended to absorb the energy released by burning 1.00 L of crude oil, if the water has its temperature raised from $\mathbf{20.0^\circ C}$ to $\mathbf{100^\circ C}$, it boils, and the resulting steam is raised to $\mathbf{300^\circ C}$. (b) Discuss additional complications caused by the fact that crude oil has a smaller density than water.

**11:** The energy released from condensation in thunderstorms can be very large. Calculate the energy released into the atmosphere for a small storm of radius 1 km, assuming that 1.0 cm of rain is precipitated uniformly over this area.

**12:** To help prevent frost damage, 4.00 kg of $0^\circ\text{C}$ water is sprayed onto a fruit tree.

(a) How much heat transfer occurs as the water freezes?

(b) How much would the temperature of the 200-kg tree decrease if this amount of heat transferred from the tree? Take the specific heat to be $3.35\ \text{kJ/kg}\cdot^\circ\text{C}$, and assume that no phase change occurs.

**13:** A 0.250-kg aluminum bowl holding 0.800 kg of soup at $25.0^\circ\text{C}$ is placed in a freezer. What is the final temperature if 377 kJ of energy is transferred from the bowl and soup, assuming the soup's thermal properties are the same as that of water? Explicitly show how you follow the steps in Problem-Solving Strategies for the Effects of Heat Transfer.

**14:** A 0.0500-kg ice cube at $-30.0^\circ\text{C}$ is placed in 0.400 kg of $35.0^\circ\text{C}$ water in a very well-insulated container. What is the final temperature?

**15:** If you pour 0.0100 kg of $20.0^\circ\text{C}$ water onto a 1.20-kg block of ice (which is initially at $-15.0^\circ\text{C}$), what is the final temperature? You may assume that the water cools so rapidly that effects of the surroundings are negligible.

**16:** Indigenous people sometimes cook in watertight baskets by placing hot rocks into water to bring it to a boil. What mass of $500^\circ\text{C}$ rock must be placed in 4.00 kg of $15.0^\circ\text{C}$ water to bring its temperature to $100^\circ\text{C}$, if 0.0250 kg of water escapes as vapor from the initial sizzle?

You may neglect the effects of the surroundings and take the average specific heat of the rocks to be that of granite.

**17:** What would be the final temperature of the pan and water in Chapter 14.2 Example 3 if 0.260 kg of water was placed in the pan and 0.0100 kg of the water evaporated immediately, leaving the remainder to come to a common temperature with the pan?

**18:** In some countries, liquid nitrogen is used on dairy trucks instead of mechanical refrigerators. A 3.00-hour delivery trip requires 200 L of liquid nitrogen, which has a density of $808\ \mathrm{kg/m^3}$.

(a) Calculate the heat transfer necessary to evaporate this amount of liquid nitrogen and raise its temperature to $3.00^\circ\mathrm{C}$. (Use $c_p$ and assume it is constant over the temperature range.) This value is the amount of cooling the liquid nitrogen supplies.

(b) What is this heat transfer rate in kilowatt-hours?

(c) Compare the amount of cooling obtained from melting an identical mass of $0^\circ\mathrm{C}$ ice with that from evaporating the liquid nitrogen.

**19:** Some gun fanciers make their own bullets, which involves melting and casting the lead slugs. How much heat transfer is needed to raise the temperature and melt 0.500 kg of lead, starting from $25.0^\circ\mathrm{C}$?

## Footnotes

1. 1 Values quoted at the normal melting and boiling temperatures at standard atmospheric pressure (1 atm).
2. 2 At 37.0°C (body temperature), the heat of vaporization $L_v$ for water is 2430 kJ/kg or 580 kcal/kg
3. 3 At 37.0°C (body temperature), the heat of vaporization $L_v$ for water is 2430 kJ/kg or 580 kcal/kg

## Glossary

**heat of sublimation**
: the energy required to change a substance from the solid phase to the vapor phase

**latent heat coefficient**
: a physical constant equal to the amount of heat transferred for every 1 kg of a substance during the change in phase of the substance

**sublimation**
: the transition from the solid phase to the vapor phase

## *Solutions*

**Check Your Understanding**

**1:** Snow is formed from ice crystals and thus is the solid phase of water. Because enormous heat is necessary for phase changes, it takes a certain amount of time for this heat to be accumulated from the air, even if the air is above

$0^\circ C$. The warmer the air is, the faster this heat exchange occurs and the faster the snow melts.

**Problems & Exercises**

**1:**

35.9 kcal

**3:**

(a) 591 kcal

(b) $4.94 \times 10^3$ s

**5:**

13.5 W

**7:**

(a) 148 kcal

(b) 0.418 s, 3.34 s, 4.19 s, 22.6 s, 0.456 s

**9:**

33.0 g

**10:**

(a) 9.67 L

(b) Crude oil is less dense than water, so it floats on top of the water, thereby exposing it to the oxygen in the air, which it uses to burn. Also, if the water is under the oil, it is less efficient in absorbing the heat generated by the oil.

**12:**

a) 319 kcal

b) $2.00^\circ C$

**14:**

$\mathbf{20.6^\circ C}$

**16:**

4.38 kg

**18:**

(a) $\mathbf{1.57 \times 10^4\ kcal}$

(b) $\mathbf{18.3\ kW \cdot h}$

(c) $\mathbf{1.29 \times 10^4\ kcal}$

# 63. 11.4 Heat Transfer Methods

*Summary*

- Discuss the different methods of heat transfer.

Equally as interesting as the effects of heat transfer on a system are the methods by which this occurs. Whenever there is a temperature difference, heat transfer occurs. Heat transfer may occur rapidly, such as through a cooking pan, or slowly, such as through the walls of a picnic ice chest. We can control rates of heat transfer by choosing materials (such as thick wool clothing for the winter), controlling air movement (such as the use of weather stripping around doors), or by choice of color (such as a white roof to reflect summer sunlight). So many processes involve heat transfer, so that it is hard to imagine a situation where no heat transfer occurs. Yet every process involving heat transfer takes place by only three methods:

1. **Conduction** is heat transfer through stationary matter by physical contact. (The matter is stationary on a macroscopic scale—we know there is thermal motion of the atoms and molecules at any temperature above absolute zero.) Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction.
2. **Convection** is the heat transfer by the macroscopic movement of a fluid. This type of transfer takes place in a forced-air

furnace and in weather systems, for example.

3. Heat transfer by **radiation** occurs when microwaves, infrared radiation, visible light, or another form of electromagnetic radiation is emitted or absorbed. An obvious example is the warming of the Earth by the Sun. A less obvious example is thermal radiation from the human body.

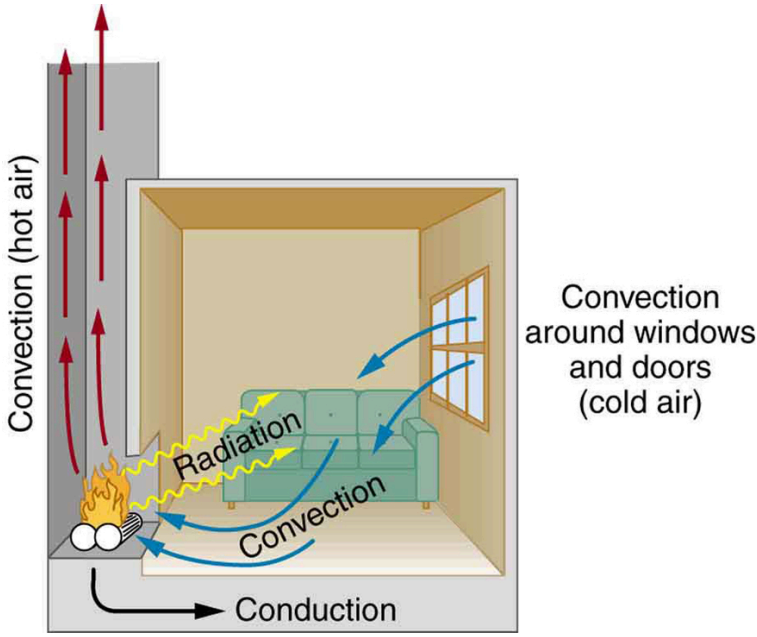



***Figure 1.*** *In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but at a much slower rate. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney.*

We examine these methods in some detail in the three following modules. Each method has unique and interesting characteristics, but all three do have one thing in common: they transfer heat solely because of a temperature difference Figure 1.

### *Check Your Understanding*

**1:** Name an example from daily life (different from the text) for each mechanism of heat transfer.

## Summary

- Heat is transferred by three different methods: conduction, convection, and radiation.

### *Conceptual Questions*

**1:** What are the main methods of heat transfer from the hot core of Earth to its surface? From Earth's surface to outer space?

**2:** When our bodies get too warm, they respond by sweating and increasing blood circulation to the surface to transfer thermal energy away from the core. What effect will this have on a person in a $\mathbf{40.0^{\circ}C}$ hot tub?

**3:** Figure 2 shows a cut-away drawing of a thermos bottle (also known as a Dewar flask), which is a device designed specifically to slow down all forms of heat transfer. Explain the functions of the various parts, such as the vacuum, the

silvering of the walls, the thin-walled long glass neck, the rubber support, the air layer, and the stopper.

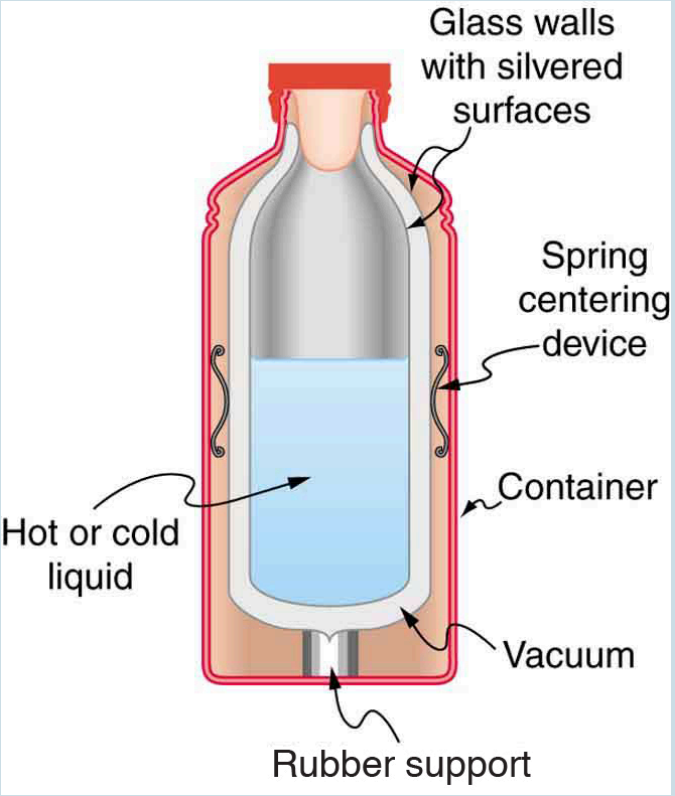



***Figure 2.*** *The construction of a thermos bottle is designed to inhibit all methods of heat transfer.*

## Glossary

**conduction**
heat transfer through stationary matter by physical contact

**convection**
heat transfer by the macroscopic movement of fluid

**radiation**
heat transfer which occurs when microwaves, infrared radiation, visible light, or other electromagnetic radiation is emitted or absorbed

## *Solutions*

**Check Your Understanding**

**1:** Conduction: Heat transfers into your hands as you hold a hot cup of coffee.

Convection: Heat transfers as the barista "steams" cold milk to make hot *cocoa*.

Radiation: Reheating a cold cup of coffee in a microwave oven.

# 64. 11.5 Conduction

*Summary*

- Calculate thermal conductivity.
- Observe conduction of heat in collisions.
- Study thermal conductivities of common substances.



***Figure 1.*** *Insulation is used to limit the conduction of heat from the inside to the outside (in winters) and from the outside to the inside (in summers). (credit: Giles Douglas)*

Your feet feel cold as you walk barefoot across the living room carpet in your cold house and then step onto the kitchen tile floor.

This result is intriguing, since the carpet and tile floor are both at the same temperature. The different sensation you feel is explained by the different rates of heat transfer: the heat loss during the same time interval is greater for skin in contact with the tiles than with the carpet, so the temperature drop is greater on the tiles.

Some materials conduct thermal energy faster than others. In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors. Figure 2 shows molecules in two bodies at different temperatures. The (average) kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, an energy transfer from the molecule with greater kinetic energy to the molecule with less kinetic energy occurs. The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. The heat flux thus depends on the temperature difference $\boldsymbol{\Delta T = T_{\text{hot}} - T_{\text{cold}}}$. Therefore, you will get a more severe burn from boiling water than from hot tap water. Conversely, if the temperatures are the same, the net heat transfer rate falls to zero, and equilibrium is achieved. Owing to the fact that the number of collisions increases with increasing area, heat conduction depends on the cross-sectional area. If you touch a cold wall with your palm, your hand cools faster than if you just touch it with your fingertip.

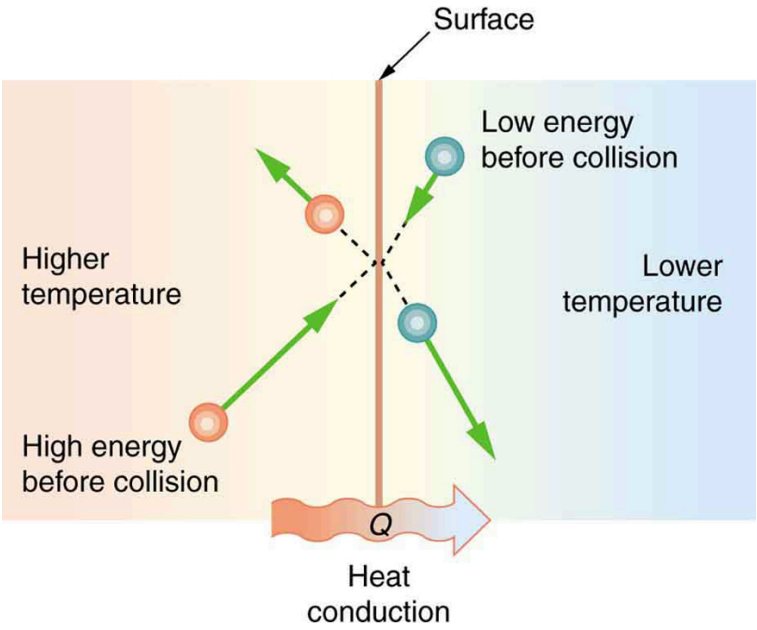



***Figure 2.*** *The molecules in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions. In this illustration, a molecule in the lower temperature region (right side) has low energy before collision, but its energy increases after colliding with the contact surface. In contrast, a molecule in the higher temperature region (left side) has high energy before collision, but its energy decreases after colliding with the contact surface.*

A third factor in the mechanism of conduction is the thickness of the material through which heat transfers. The figure below shows a slab of material with different temperatures on either side. Suppose that $T_2$ is greater than $T_1$, so that heat is transferred from left to right. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. This model explains why thick clothing is warmer than thin clothing in winters, and why Arctic mammals protect themselves with thick blubber.

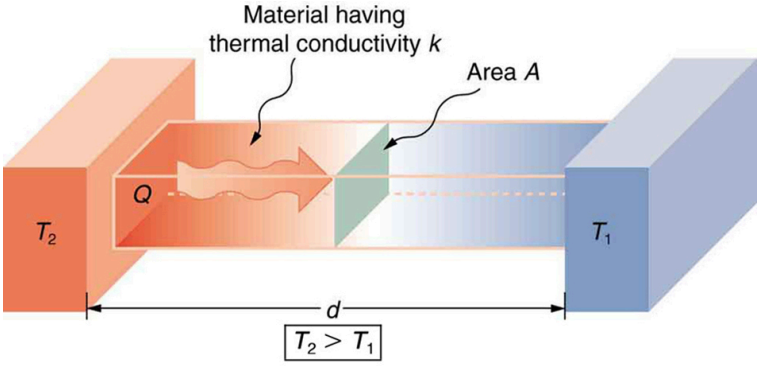



*Figure 3. Heat conduction occurs through any material, represented here by a rectangular bar, whether window glass or walrus blubber. The temperature of the material is* $\mathbf{T_2}$ *on the left and* $\mathbf{T_1}$ *on the right, where* $\mathbf{T_2}$ *is greater than* $\mathbf{T_1}$*. The rate of heat transfer by conduction is directly proportional to the surface area* **A**, *the temperature difference* $\mathbf{T_2 - T_1}$, *and the substance's conductivity* **k**. *The rate of heat transfer is inversely proportional to the thickness* **d**.

Lastly, the heat transfer rate depends on the material properties described by the coefficient of thermal conductivity. All four factors are included in a simple equation that was deduced from and is confirmed by experiments. The **rate of conductive heat transfer** through a slab of material, such as the one in Figure 3, is given by

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d},$$

where Q/t is the rate of heat transfer in watts or kilocalories per second, $k$ is the thermal conductivity of the material, $A$ and $d$ are its surface area and thickness, as shown in Figure 3, and $(T_2 - T_1)$ is the temperature difference across the slab. Table 3 gives representative values of thermal conductivity.

## Example 1: Calculating Heat Transfer Through Conduction: Conduction Rate Through an Ice Box

A Styrofoam ice box has a total area of 0.950 m20.950 m2 and walls with an average thickness of 2.50 cm. The box contains ice, water, and canned beverages at $0^\circ\text{C}$. The inside of the box is kept cold by melting ice. How much ice melts in one day if the ice box is kept in the trunk of a car at $35.0^\circ\text{C}$?

**Strategy**

This question involves both heat for a phase change (melting of ice) and the transfer of heat by conduction. To find the amount of ice melted, we must find the net heat transferred. This value can be obtained by calculating the rate of heat transfer by conduction and multiplying by time.

**Solution**

1. Identify the knowns.

   $A = 0.950\ \text{m}^2$; $d = 2.50\ \text{cm} = 0.0250\ \text{m}$; $T_1 = 0^\circ\text{C}$; $T_2 = 35.0^\circ\text{C}$, $t = 1\ \text{day} = 24\ \text{hours} = 86,400\ \text{s}$.

2. Identify the unknowns. We need to solve for the mass of the ice, $m$. We will also need to solve for the net heat transferred to melt the ice, $Q$.

3. Determine which equations to use. The rate of heat transfer by conduction is given by

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}.$$

4. The heat is used to melt the ice: $Q = mL_f$.
5. Insert the known values:

$$\frac{Q}{t} = \frac{(0.010\ \text{J/s}\cdot\text{m}\cdot^\circ\text{C})(0.950\ \text{m}^2)(35.0^\circ\text{C} - 0^\circ\text{C})}{0.0250\ \text{m}} = 13.3\ \text{J/s}.$$

6. Multiply the rate of heat transfer by the time ($1\ \text{day} = 86,400\ \text{s}$):
   $Q = (Q/t)t = (13.3\ \text{J/s})(86,400\ \text{s}) = 1.15 \times 10^6\ \text{J}$.
7. Set this equal to the heat transferred to melt the ice: $Q = mL_f$. Solve for the mass $m$:

$$m = \frac{Q}{L_f} = \frac{1.15 \times 10^6\ \text{J}}{334 \times 10^3\ \text{J/kg}} = 3.44\ \text{kg}.$$

**Discussion**

The result of 3.44 kg, or about 7.6 lbs, seems about right, based on experience. You might expect to use about a 4 kg (7–10 lb) bag of ice per day. A little extra ice is required if you add any warm food or beverages.

Inspecting the conductivities in Table 3 shows that Styrofoam is a very poor conductor and thus a good insulator. Other good insulators include fiberglass, wool, and goose-down feathers. Like Styrofoam, these all incorporate many small pockets of air, taking advantage of air's poor thermal conductivity.

| Substance | Thermal conductivity k (J/s·m·°C) |
|---|---|
| Silver | 420 |
| Copper | 390 |
| Gold | 318 |
| Aluminum | 220 |
| Steel iron | 80 |
| Steel (stainless) | 14 |
| Ice | 2.2 |
| Glass (average) | 0.84 |
| Concrete brick | 0.84 |
| Water | 0.6 |
| Fatty tissue (without blood) | 0.2 |
| Asbestos | 0.16 |
| Plasterboard | 0.16 |
| Wood | 0.08–0.16 |
| Snow (dry) | 0.10 |
| Cork | 0.042 |
| Glass wool | 0.042 |
| Wool | 0.04 |
| Down feathers | 0.025 |
| Air | 0.023 |
| Styrofoam | 0.010 |

**Table 3.** Thermal Conductivities of Common Substances[1]

A combination of material and thickness is often manipulated to develop good insulators—the smaller the conductivity $k$ and the larger the thickness $d$, the better. The ratio of $d/k$ will thus be large for a good insulator. The ratio $d/k$ is called the $R$ factor. The rate of conductive heat transfer is inversely proportional to $R$. The larger the value of $R$, the better the insulation. $R$ factors are most

commonly quoted for household insulation, refrigerators, and the like—unfortunately, it is still in non-metric units of $ft^2 \cdot °F \cdot h/Btu$, although the unit usually goes unstated (1 British thermal unit [Btu] is the amount of energy needed to change the temperature of 1.0 lb of water by 1.0 °F). A couple of representative values are an $R$ factor of 11 for 3.5-in-thick fiberglass batts (pieces) of insulation and an $R$ factor of 19 for 6.5-in-thick fiberglass batts. Walls are usually insulated with 3.5-in batts, while ceilings are usually insulated with 6.5-in batts. In cold climates, thicker batts may be used in ceilings and walls.

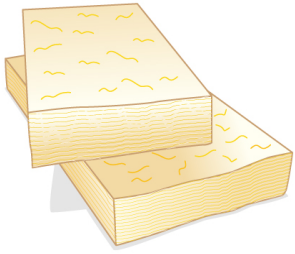


***Figure 4.*** *The fiberglass batt is used for insulation of walls and ceilings to prevent heat transfer between the inside of the building and the outside environment.*

Note that in Table 3, the best thermal conductors—silver, copper, gold, and aluminum—are also the best electrical conductors, again related to the density of free electrons in them. Cooking utensils are typically made from good conductors.

## Example 2: Calculating the Temperature Difference Maintained by a Heat Transfer: Conduction Through an Aluminum Pan

Water is boiling in an aluminum pan placed on an electrical element on a stovetop. The sauce pan has a bottom that is 0.800 cm thick and 14.0 cm in diameter. The boiling water is evaporating at the rate of 1.00 g/s. What is the temperature difference across (through) the bottom of the pan?

**Strategy**

Conduction through the aluminum is the primary method of heat transfer here, and so we use the equation for the rate of heat transfer and solve for the temperature difference.

$$T_2 - T_1 = \frac{Q}{t}\left(\frac{d}{kA}\right).$$

**Solution**

1. Identify the knowns and convert them to the SI units.

   The thickness of the pan,
   $d = 0.800\ \text{cm} = 8.0 \times 10^{-3}\ \text{m}$,
   the area of the pan,

$$A = \pi(0.14/2)^2\ \mathrm{m^2} = 1.54 \times 10^{-2}\ \mathrm{m^2},$$

and the thermal conductivity,

$$k = 220\ \mathrm{J/s \cdot m \cdot ^\circ C}.$$

2. Calculate the necessary heat of vaporization of 1 g of water:

$$Q = mL_\mathrm{v} = (1.00 \times 10^{-3}\ \mathrm{kg})(2256 \times 10^3\ \mathrm{J/kg}) = 2256\ \mathrm{J}.$$

3. Calculate the rate of heat transfer given that 1 g of water melts in one second:

$$Q/t = 2256\ \mathrm{J/s\ or\ 2.26\ kW}.$$

4. Insert the knowns into the equation and solve for the temperature difference:

$$\begin{aligned} T_2 - T_1 &= \frac{Q}{t}\left(\frac{d}{kA}\right) \\ &= (2256\ \mathrm{J/s}) \\ &\quad \frac{8.00 \times 10^{-3}\ \mathrm{m}}{(220\ \mathrm{J/s \cdot m \cdot ^\circ C})(1.54 \times 10^{-2}\ \mathrm{m^2})} \\ &= 5.33^\circ \mathrm{C}. \end{aligned}$$

**Discussion**

The value for the heat transfer $\boldsymbol{Q/t = 2.26\ \mathrm{kW\ or\ }2256\ \mathrm{J/s}}$ is typical for an electric stove. This value gives a remarkably small temperature difference between the stove and the pan. Consider that the stove burner is red hot while the inside of the pan is nearly $100^\circ \mathrm{C}$ because of its contact with boiling water. This contact effectively cools the bottom of the pan in spite of its

proximity to the very hot stove burner. Aluminum is such a good conductor that it only takes this small temperature difference to produce a heat transfer of 2.26 kW into the pan.

Conduction is caused by the random motion of atoms and molecules. As such, it is an ineffective mechanism for heat transport over macroscopic distances and short time distances. Take, for example, the temperature on the Earth, which would be unbearably cold during the night and extremely hot during the day if heat transport in the atmosphere was to be only through conduction. In another example, car engines would overheat unless there was a more efficient way to remove excess heat from the pistons.

### *Check Your Understanding*

**1:** How does the rate of heat transfer by conduction change when all spatial dimensions are doubled?

## Summary

- Heat conduction is the transfer of heat between two objects in direct contact with each other.
- The rate of heat transfer $Q/t$ (energy per unit time) is

proportional to the temperature difference $T_2 - T_1$ and the contact area $A$ and inversely proportional to the distance $d$ between the objects:

$$\frac{Q}{t} = \frac{kA(T_2 - T_1}{d}.$$

## *Conceptual Questions*

**1:** Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A pot placed over a heating element will be heated, while it is safe to touch the surface only a few centimetres away. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?

**2:** Loose-fitting white clothing covering most of the body is ideal for desert dwellers, both in the hot Sun and during cold evenings. Explain how such clothing is advantageous during both day and night.



***Figure 5.*** *A jellabiya is worn by many men in Egypt. (credit: Zerida)*

## *Problems & Exercises*

**1:** (a) Calculate the rate of heat conduction through house walls that are 13.0 cm thick and that have an average thermal conductivity twice that of glass wool. Assume there are no windows or doors. The surface area of the walls is $\mathbf{120\ m^2}$ and their inside surface is at $\mathbf{18.0^\circ C}$, while their outside surface is a t$\mathbf{5.00^\circ C}$. (b) How many 1-kW room heaters would be needed to balance the heat transfer due to conduction?

**2:** The rate of heat conduction out of a window on a winter day is rapid enough to chill the air next to it. To see just how rapidly the windows transfer heat by conduction, calculate the rate of conduction in watts through a $\mathbf{3.00\text{-}m^2}$ window that is$\mathbf{0.635\ cm}$ thick (1/4 in) if the temperatures of the inner and outer surfaces are $\mathbf{5.00^\circ C}$ and $\mathbf{-10.0^\circ C}$, respectively. This rapid rate will not be maintained–the inner surface will cool, and even result in frost formation.

**3:** Calculate the rate of heat conduction out of the human body, assuming that the core internal temperature is $\mathbf{37.0^\circ C}$, the skin temperature is $\mathbf{34.0^\circ C}$, the thickness of the tissues between averages $\mathbf{1.00\ cm}$, and the surface area is $\mathbf{1.40\ m^2}$.

**4:** Suppose you stand with one foot on ceramic flooring and one foot on a wool carpet, making contact over an area of $\mathbf{80.0\ cm^2}$ with each foot. Both the ceramic and the carpet are 2.00 cm thick and are $\mathbf{10.0^\circ C}$ on their

bottom sides. At what rate must heat transfer occur from each foot to keep the top of the ceramic and carpet at $33.0^\circ\text{C}$?

**5:** A man consumes 3000 kcal of food in one day, converting most of it to maintain body temperature. If he loses half this energy by evaporating water (through breathing and sweating), how many kilograms of water evaporate?

**6:** (a) A firewalker runs across a bed of hot coals without sustaining burns. Calculate the heat transferred by conduction into the sole of one foot of a firewalker given that the bottom of the foot is a 3.00-mm-thick callus with a conductivity at the low end of the range for wood and its density is $300\ \text{kg/m}^3$. The area of contact is $25.0\ \text{cm}^2$, the temperature of the coals is $700^\circ\text{C}$, and the time in contact is 1.00 s.

(b) What temperature increase is produced in the $25.0\ \text{cm}^3$ of tissue affected?

(c) What effect do you think this will have on the tissue, keeping in mind that a callus is made of dead cells?

**7:** (a) What is the rate of heat conduction through the 3.00-cm-thick fur of a large animal having a $1.40\text{-m}^2$ surface area? Assume that the animal's skin temperature is $32.0^\circ\text{C}$, that the air temperature is $-5.00^\circ\text{C}$, and that fur has the same thermal conductivity as air. (b) What food intake will the animal need in one day to replace this heat transfer?

**8:** A walrus transfers energy by conduction through its blubber at the rate of 150 W when immersed in $-1.00^\circ\text{C}$ water. The walrus's internal core temperature is $37.0^\circ\text{C}$, and it has a surface area of $2.00\ \text{m}^2$. What is the average thickness of its blubber, which has the conductivity of fatty tissues without blood?



***Figure 6.*** *Walrus on ice. (credit: Captain Budd Christman, NOAA Corps)*

**9:** Compare the rate of heat conduction through a 13.0-cm-thick wall that has an area of $10.0\ \text{m}^2$ and a thermal conductivity twice that of glass wool with the rate of heat conduction through a window that is 0.750 cm thick and that has an area o f$2.00\ \text{m}^2$, assuming the same temperature difference across each.

**10:** Suppose a person is covered head to foot by wool clothing with average thickness of 2.00 cm and is transferring energy by conduction through the clothing at

the rate of 50.0 W. What is the temperature difference across the clothing, given the surface area is $\mathbf{1.40\ m^2}$?

**11:** Some stove tops are smooth ceramic for easy cleaning. If the ceramic is 0.600 cm thick and heat conduction occurs through the same area and at the same rate as computed in Example 2, what is the temperature difference across it? Ceramic has the same thermal conductivity as glass and brick.

**12:** One easy way to reduce heating (and cooling) costs is to add extra insulation in the attic of a house. Suppose the house already had 15 cm of fiberglass insulation in the attic and in all the exterior surfaces. If you added an extra 8.0 cm of fiberglass to the attic, then by what percentage would the heating cost of the house drop? Take the single story house to be of dimensions 10 m by 15 m by 3.0 m. Ignore air infiltration and heat loss through windows and doors.

**13:** (a) Calculate the rate of heat conduction through a double-paned window that has a $\mathbf{1.50\text{-}m^2}$ area and is made of two panes of 0.800-cm-thick glass separated by a 1.00-cm air gap. The inside surface temperature is $\mathbf{15.0^\circ C}$, while that on the outside is $\mathbf{-10.0^\circ C}$. (Hint: There are identical temperature drops across the two glass panes. First find these and then the temperature drop across the air gap. This problem ignores the increased heat transfer in the air gap due to convection.)

(b) Calculate the rate of heat conduction through a 1.60-cm-thick window of the same area and with the same temperatures. Compare your answer with that for part (a).

**14:** Many decisions are made on the basis of the payback period: the time it will take through savings to equal the capital cost of an investment. Acceptable payback times depend upon the business or philosophy one has. (For some industries, a payback period is as small as two years.) Suppose you wish to install the extra insulation in Exercise 12. If energy cost $1.00 per million joules and the insulation was $4.00 per square meter, then calculate the simple payback time. Take the average $\Delta T$ for the 120 day heating season to be $15.0^\circ \text{C}$.

**15:** For the human body, what is the rate of heat transfer by conduction through the body's tissue with the following conditions: the tissue thickness is 3.00 cm, the change in temperature is $2.00^\circ \text{C}$, and the skin area is $1.50\ \text{m}^2$. How does this compare with the average heat transfer rate to the body resulting from an energy intake of about 2400 kcal per day? (No exercise is included.)

## Footnotes

1. 1 At temperatures near 0°C.

## Glossary

**R factor**
: the ratio of thickness to the conductivity of a material

**rate of conductive heat transfer**
rate of heat transfer from one material to another

**thermal conductivity**
the property of a material's ability to conduct heat

## *Solutions*

**Check Your Understanding**

**1:** Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubled $(A_{\text{final}} = (2d)^2 = 4d^2 = 4A_{\text{initial}})$. The distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two, or two:

$$\left(\frac{Q}{t}\right)_{\text{final}} = \frac{kA_{\text{final}}(T_2 - T_1)}{d_{\text{final}}} = \frac{k(4A_{\text{initial}})(T_2 - T_1)}{2d_{\text{initial}}} = 2\frac{kA_{\text{initial}}(T_2 - T_1}{d_{\text{initial}}} = 2\left(\frac{Q}{t}\right)_{\text{initial}}.$$

**Problems & Exercises**

**1:** (a) $1.01 \times 10^3$ W (b) One

**3:** 84.0 W

**5:** 2.59 kg

**7:** (a) 39.7 W  (b) 820 kcal

**9:** 35 to 1, window to wall

**11:** 1.05 x $10^3$ K

**13:** (a) 83 W (b) 24 times that of a double pane window.

**15:** 20.0 W, 17.2% of 2400 kcal per day

# 65. 11.6 Convection

*Summary*

- Discuss the method of heat transfer by convection.

Convection is driven by large-scale flow of matter. In the case of Earth, the atmospheric circulation is caused by the flow of hot air from the tropics to the poles, and the flow of cold air from the poles toward the tropics. (Note that Earth's rotation causes the observed easterly flow of air in the northern hemisphere). Car engines are kept cool by the flow of water in the cooling system, with the water pump maintaining a flow of cool water to the pistons. The circulatory system is used the body: when the body overheats, the blood vessels in the skin expand (dilate), which increases the blood flow to the skin where it can be cooled by sweating. These vessels become smaller when it is cold outside and larger when it is hot (so more fluid flows, and more energy is transferred).

The body also loses a significant fraction of its heat through the breathing process.

While convection is usually more complicated than conduction, we can describe convection and do some straightforward, realistic calculations of its effects. Natural convection is driven by buoyant forces: hot air rises because density decreases as temperature increases. The house in Figure 1 is kept warm in this manner, as is the pot of water on the stove in Figure 2. Ocean currents and large-scale atmospheric circulation transfer energy from one part of the globe to another. Both are examples of natural convection.

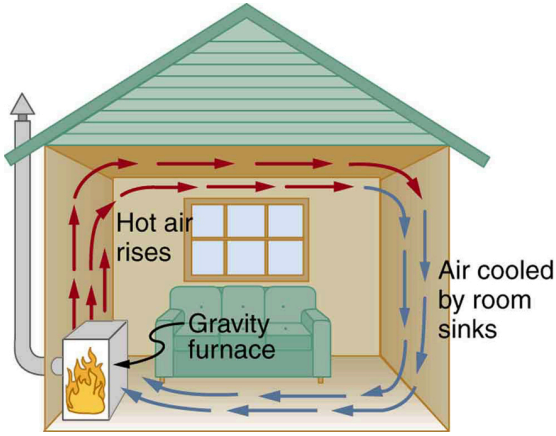



***Figure 1.*** *Air heated by the so-called gravity furnace expands and rises, forming a convective loop that transfers energy to other parts of the room. As the air is cooled at the ceiling and outside walls, it contracts, eventually becoming denser than room air and sinking to the floor. A properly designed heating system using natural convection, like this one, can be quite efficient in uniformly heating a home.*

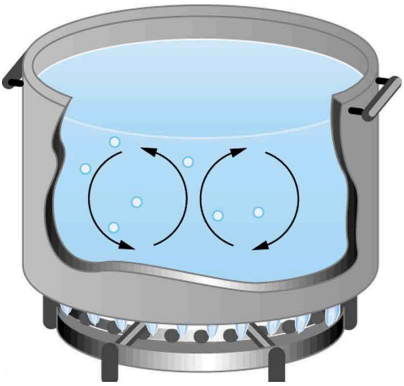


***Figure 2.*** *Convection plays an important role in heat transfer inside this pot of water. Once conducted to the inside, heat transfer to other parts of the pot is mostly by convection. The hotter water expands, decreases in density, and rises to transfer heat to other regions of the water, while colder water sinks to the bottom. This process keeps repeating.*

## TAKE-HOME EXPERIMENT: CONVECTION ROLLS IN A HEATED PAN

Take two small pots of water and use an eye dropper to place a drop of food coloring near the bottom of each. Leave one on a bench top and heat the other over a stovetop. Watch how the color spreads and how long it

takes the color to reach the top. Watch how convective loops form.

## Example 1: Calculating Heat Transfer by Convection: Convection of Air Through the Walls of a House

Most houses are not airtight: air goes in and out around doors and windows, through cracks and crevices, following wiring to switches and outlets, and so on. The air in a typical house is completely replaced in less than an hour. Suppose that a moderately-sized house has inside dimensions $\mathbf{12.0\ m \times 18.0\ m \times 3.00\ m}$ high, and that all air is replaced in 30.0 min. Calculate the heat transfer per unit time in watts needed to warm the incoming cold air by $\mathbf{10.0^\circ C}$, thus replacing the heat transferred by convection alone.

**Strategy**

Heat is used to raise the temperature of air so that $\boldsymbol{Q = mc\Delta T}$. The rate of heat transfer is then $\boldsymbol{Q/t}$, where tt is the time for air turnover. We are given that $\boldsymbol{\Delta T}$ is $\mathbf{10.0^\circ C}$, but we must still find values for the mass of air and its specific heat before we can

calculate $Q$. The specific heat of air is a weighted average of the specific heats of nitrogen and oxygen, which gives $c = cp \cong 1000\ \text{J/kg}\cdot^\circ\text{C}$ from Table 4 (note that the specific heat at constant pressure must be used for this process).

**Solution**

1. Determine the mass of air from its density and the given volume of the house. The density is given from the density $\rho$ and the volume
$m = \rho V = (1.29\ \text{kg/m}^3)(12.0\ \text{m} \times\ 18.0\ \text{m} \times\ 3.00\ \text{m}) = 836\ \text{kg}.$
2. Calculate the heat transferred from the change in air temperature: $Q = mc\Delta T$ so that
$Q = (836\ \text{kg})(1000\ \text{J/kg}\cdot^\circ\text{C})(10.0^\circ\text{C}) = 8.36 \times 10^6\ \text{J}.$
3. Calculate the heat transfer from the heat $Q$ and the turnover time $t$. Since air is turned over in $t = 0.500\ \text{h} = 1800\ \text{s}$, the heat transferred per unit time is
$$\frac{Q}{t} = \frac{8.36 \times 10^6\ \text{J}}{1800\ \text{s}} = 4.64\ \text{kW}.$$

**Discussion**

This rate of heat transfer is equal to the power consumed by about forty-six 100-W light bulbs. Newly constructed homes are designed for a turnover time of 2 hours or more, rather than 30 minutes for the house of this example. Weather stripping, caulking, and improved window seals are commonly employed. More extreme measures are sometimes taken in very cold (or hot) climates to achieve a tight standard of more than 6 hours for one air turnover. Still longer turnover times

are unhealthy, because a minimum amount of fresh air is necessary to supply oxygen for breathing and to dilute household pollutants. The term used for the process by which outside air leaks into the house from cracks around windows, doors, and the foundation is called "air infiltration."

A cold wind is much more chilling than still cold air, because convection combines with conduction in the body to increase the rate at which energy is transferred away from the body. The table below gives approximate wind-chill factors, which are the temperatures of still air that produce the same rate of cooling as air of a given temperature and speed. Wind-chill factors are a dramatic reminder of convection's ability to transfer heat faster than conduction. For example, a 15.0 m/s wind at $0^\circ C$ has the chilling equivalent of still air at about $-18^\circ C$.

| **Moving air temperature** | **Wind speed (m/s)** | | | | |
|---|---|---|---|---|---|
| °C | 2 | 5 | 10 | 15 | 20 |
| 5 | 3 | –1 | –8 | –10 | –12 |
| 2 | 0 | –7 | –12 | –16 | –18 |
| 0 | –2 | –9 | –15 | –18 | –20 |
| –5 | –7 | –15 | –22 | –26 | –29 |
| –10 | –12 | –21 | –29 | –34 | –36 |
| –20 | –23 | –34 | –44 | –50 | –52 |
| –40 | –44 | –59 | –73 | –82 | –84 |

**Table 4.** Wind-Chill Factors

Although air can transfer heat rapidly by convection, it is a poor conductor and thus a good insulator. The amount of available space

for airflow determines whether air acts as an insulator or conductor. The space between the inside and outside walls of a house, for example, is about 9 cm (3.5 in) –large enough for convection to work effectively. The addition of wall insulation prevents airflow, so heat loss (or gain) is decreased. Similarly, the gap between the two panes of a double-paned window is about 1 cm, which prevents convection and takes advantage of air's low conductivity to prevent greater loss. Fur, fiber, and fiberglass also take advantage of the low conductivity of air by trapping it in spaces too small to support convection, as shown in the figure. Fur and feathers are lightweight and thus ideal for the protection of animals.

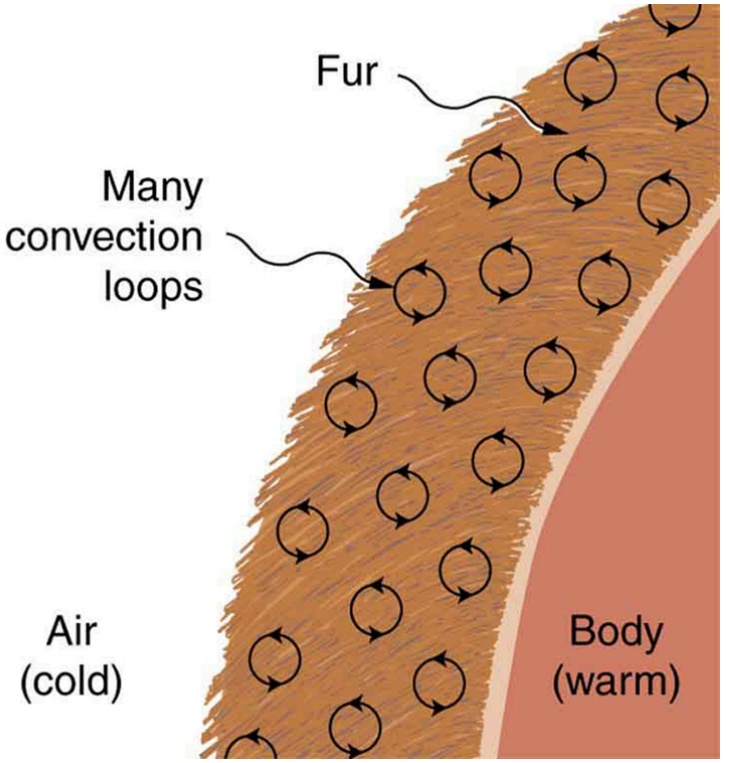



***Figure 3.*** *Fur is filled with air, breaking it up into many small pockets. Convection is very slow here, because the loops are so small. The low conductivity of air makes fur a very good lightweight insulator.*

Some interesting phenomena happen *when convection is accompanied by a phase change*. It allows us to cool off by sweating, even if the temperature of the surrounding air exceeds body temperature. Heat from the skin is required for sweat to evaporate from the skin, but without air flow, the air becomes saturated and evaporation stops. Air flow caused by convection replaces the saturated air by dry air and evaporation continues.

## Example 2: Calculate the Flow of Mass during Convection: Sweat-Heat Transfer away from the Body

The average person produces heat at the rate of about 120 W when at rest. At what rate must water evaporate from the body to get rid of all this energy? (This evaporation might occur when a person is sitting in the shade and surrounding temperatures are the same as skin temperature, eliminating heat transfer by other methods.)

**Strategy**

Energy is needed for a phase change ($\boldsymbol{Q = mL_{\mathrm{v}}}$). Thus, the energy loss per unit time is

$$\frac{\boldsymbol{Q}}{\boldsymbol{t}} = \frac{\boldsymbol{mL_{\mathrm{v}}}}{\boldsymbol{t}} = \mathbf{120\ W = 120\ J/s.}$$

We divide both sides of the equation by $\boldsymbol{L_{\mathrm{v}}}$ to find that the mass evaporated per unit time is

$$\frac{m}{t} = \frac{120\ \text{J/s}}{L_\text{v}}.$$

**Solution**

(1) Insert the value of the latent heat from Table 2, $L_\text{v} = 2430\ \text{kJ/kg} = 2430\ \text{J/g}$. This yields

$$\frac{m}{t} = \frac{120\ \text{J/s}}{2430\ \text{J/g}}$$
$$= 0.0494\ \text{g/s} = 2.96\ \text{g/min}.$$

**Discussion**

Evaporating about 3 g/min seems reasonable. This would be about 180 g (about 7 oz) per hour. If the air is very dry, the sweat may evaporate without even being noticed. A significant amount of evaporation also takes place in the lungs and breathing passages.

Another important example of the combination of phase change and convection occurs when water evaporates from the oceans. Heat is removed from the ocean when water evaporates. If the water vapor condenses in liquid droplets as clouds form, heat is released in the atmosphere. Thus, there is an overall transfer of heat from the ocean to the atmosphere. This process is the driving power behind thunderheads, those great cumulus clouds that rise as much as 20.0 km into the stratosphere. Water vapor carried in by convection condenses, releasing tremendous amounts of energy. This energy causes the air to expand and rise, where it is colder. More condensation occurs in these colder regions, which in turn drives the cloud even higher. Such a mechanism is called positive feedback, since the process reinforces and accelerates itself. These

systems sometimes produce violent storms, with lightning and hail, and constitute the mechanism driving hurricanes.



***Figure 4.*** *Cumulus clouds are caused by water vapor that rises because of convection. The rise of clouds is driven by a positive feedback mechanism. (credit: Mike Love)*



***Figure 5.*** *Convection accompanied by a phase change releases the energy needed to drive this thunderhead into the stratosphere. (credit: Gerardo García Moretti )*



***Figure 6.*** *The phase change that occurs when this iceberg melts involves tremendous heat transfer. (credit: Dominic Alves)*

The movement of icebergs is another example of convection accompanied by a phase change. Suppose an iceberg drifts from Greenland into warmer Atlantic waters. Heat is removed from the warm ocean water when the ice melts and heat is released to the land mass when the iceberg forms on Greenland.

### *Check Your Understanding*

**1:** Explain why using a fan in the summer feels refreshing!

## Summary

- Convection is heat transfer by the macroscopic movement of mass. Convection can be natural or forced and generally transfers thermal energy faster than conduction. Table 4 gives wind-chill factors, indicating that moving air has the same chilling effect of much colder stationary air. *Convection that occurs along with a phase change* can transfer energy from cold regions to warm ones.

### *Conceptual Questions*

**1:** One way to make a fireplace more energy efficient is to have an external air supply for the combustion of its fuel. Another is to have room air circulate around the outside of

the fire box and back into the room. Detail the methods of heat transfer involved in each.

**2:** On cold, clear nights horses will sleep under the cover of large trees. How does this help them keep warm?

## *Problems & Exercises*

**1:** At what wind speed does $-10^\circ\text{C}$ air cause the same chill factor as still air at $-29^\circ\text{C}$?

**2:** At what temperature does still air cause the same chill factor as $-5^\circ\text{C}$ air moving at 15 m/s?

**3:** The "steam" above a freshly made cup of instant coffee is really water vapor droplets condensing after evaporating from the hot coffee. What is the final temperature of 250 g of hot coffee initially at $90.0^\circ\text{C}$ if 2.00 g evaporates from it? The coffee is in a Styrofoam cup, so other methods of heat transfer can be neglected.

**4:** (a) How many kilograms of water must evaporate from a 60.0-kg woman to lower her body temperature by $0.750^\circ\text{C}$?

(b) Is this a reasonable amount of water to evaporate in the form of perspiration, assuming the relative humidity of the surrounding air is low?

**5:** On a hot dry day, evaporation from a lake has just enough heat transfer to balance the $1.00\ \text{kW/m}^2$ of incoming heat from the Sun. What mass of water evaporates in 1.00 h from each square meter? Explicitly show how you follow the steps in the Chapter 14.3 Problem-Solving Strategies for the Effects of Heat Transfer.

**6:** One winter day, the climate control system of a large university classroom building malfunctions. As a result, $500\ \text{m}^3$ of excess cold air is brought in each minute. At what rate in kilowatts must heat transfer occur to warm this air by $10.0^\circ\text{C}$ (that is, to bring the air to room temperature)?

**7:** The Kilauea volcano in Hawaii is the world's most active, disgorging about $5 \times 10^5\ \text{m}^3$ of $1200^\circ\text{C}$ lava per day. What is the rate of heat transfer out of Earth by convection if this lava has a density of $2700\ \text{kg/m}^3$ and eventually cools to $30^\circ\text{C}$? Assume that the specific heat of lava is the same as that of granite.

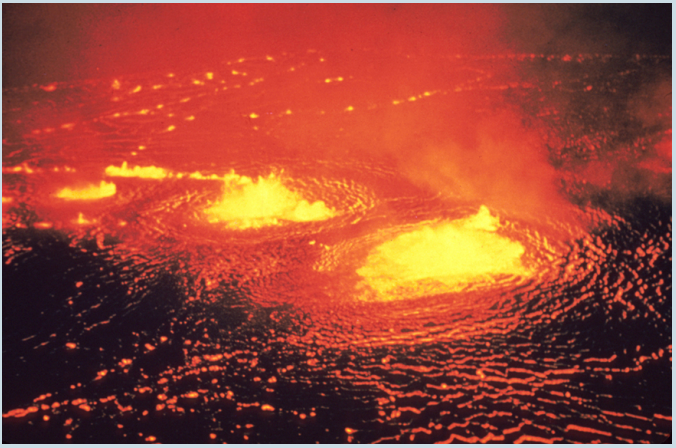


***Figure 7.*** *Lava flow on Kilauea volcano in Hawaii. (credit: J. P. Eaton, U.S. Geological Survey)*

**8:** During heavy exercise, the body pumps 2.00 L of blood per minute to the surface, where it is cooled by $\mathbf{2.00^\circ C}$. What is the rate of heat transfer from this forced convection alone, assuming blood has the same specific heat as water and its density is $\mathbf{1050\ kg/m^3}$?

**9:** A person inhales and exhales 2.00 L of $\mathbf{37.0^\circ C}$ air, evaporating $\mathbf{4.00 \times 10^{-2}\ g}$ of water from the lungs and breathing passages with each breath.

(a) How much heat transfer occurs due to evaporation in each breath?

(b) What is the rate of heat transfer in watts if the person is breathing at a moderate rate of 18.0 breaths per minute?

(c) If the inhaled air had a temperature of $20.0^\circ\text{C}$, what is the rate of heat transfer for warming the air?

(d) Discuss the total rate of heat transfer as it relates to typical metabolic rates. Will this breathing be a major form of heat transfer for this person?

**10:** A glass coffee pot has a circular bottom with a 9.00-cm diameter in contact with a heating element that keeps the coffee warm with a continuous heat transfer rate of 50.0 W

(a) What is the temperature of the bottom of the pot, if it is 3.00 mm thick and the inside temperature is $60.0^\circ\text{C}$?

(b) If the temperature of the coffee remains constant and all of the heat transfer is removed by evaporation, how many grams per minute evaporate? Take the heat of vaporization to be 2340 kJ/kg.

## *Solutions*

**Check Your Understanding**

**1:** Using a fan increases the flow of air: warm air near your body is replaced by cooler air from elsewhere. Convection increases the rate of heat transfer so that moving air "feels" cooler than still air.

**Problems & Exercises**

**1:** 10 m/s

**3:** $\mathbf{85.7^{\circ}C}$

**5:** 1.48 kg

**7:** 2 X 10 $^{4}$ MW

**9: (**a) 97.2 J (b) 29.2 W (c) 9.49 W (d) The total rate of heat loss would be $\mathbf{29.2\ W + 9.49\ W = 38.7\ W.}$ While sleeping, our body consumes 83 W of power, while sitting it consumes 120 to 210 W. Therefore, the total rate of heat loss from breathing will not be a major form of heat loss for this person.

# 66. 11.7 Radiation

*Summary*

- Discuss heat transfer by radiation.
- Explain the power of different materials.

You can feel the heat transfer from a fire and from the Sun. Similarly, you can sometimes tell that the oven is hot without touching its door or looking inside—it may just warm you as you walk by. The space between the Earth and the Sun is largely empty, without any possibility of heat transfer by convection or conduction. In these examples, heat is transferred by radiation. That is, the hot body emits electromagnetic waves that are absorbed by our skin: no medium is required for electromagnetic waves to propagate. Different names are used for electromagnetic waves of different wavelengths: radio waves, microwaves, infrared **radiation**, visible light, ultraviolet radiation, X-rays, and gamma rays.



***Figure 1.*** *Most of the heat transfer from this fire to the observers is through infrared radiation. The visible light, although dramatic, transfers relatively little thermal energy. Convection transfers energy away from the observers as hot air rises, while conduction is negligibly slow here. Skin is very sensitive to infrared radiation, so that you can sense the presence of a fire without looking at it directly. (credit: Daniel X. O'Neil)*

The energy of electromagnetic radiation depends on the wavelength (color) and varies over a wide range: a smaller wavelength (or higher frequency) corresponds to a higher energy. Because more heat is radiated at higher temperatures, a temperature change is accompanied by a color change. Take, for example, an electrical element on a stove, which glows from red to orange, while the higher-temperature steel in a blast furnace glows from yellow to white. The radiation you feel is mostly infrared, which corresponds to a lower temperature than that of the electrical element and the steel. The radiated energy depends on its intensity, which is represented in the figure below by the height of the distribution.

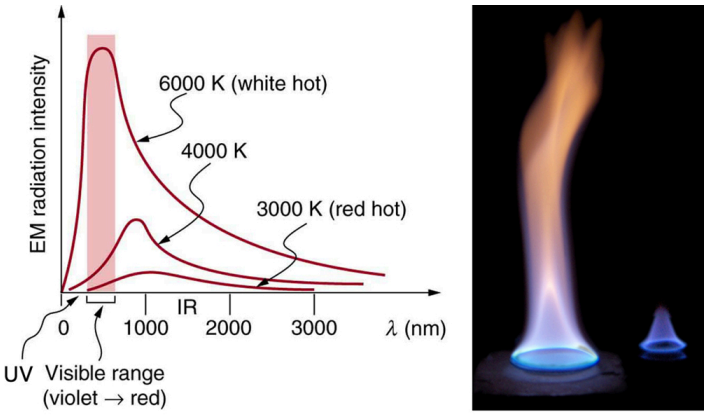



***Figure 2.*** *(a) A graph of the spectra of electromagnetic waves emitted from an ideal radiator at three different temperatures. The intensity or rate of radiation emission increases dramatically with temperature, and the spectrum shifts toward the visible and ultraviolet parts of the spectrum. The shaded portion denotes the visible part of the spectrum. It is apparent that the shift toward the ultraviolet with temperature makes the visible appearance shift from red to white to blue as temperature increases. (b) Note the variations in color corresponding to variations in flame temperature. (credit: Tuohirulla)*

All objects absorb and emit electromagnetic radiation. The rate of heat transfer by radiation is largely determined by the color of the object. Black is the most effective, and white is the least effective. People living in hot climates generally avoid wearing black clothing, for instance (see Note). Similarly, black asphalt in a parking lot will be hotter than adjacent gray sidewalk on a summer day, because black absorbs better than gray. The reverse is also true—black radiates better than gray. Thus, on a clear summer night, the asphalt will be colder than the gray sidewalk, because black radiates the energy more rapidly than gray. An *ideal radiator* is the same color as an *ideal absorber*, and captures all the radiation that falls on it. In contrast, white is a poor absorber and is also a poor radiator. A white object reflects all radiation, like a mirror. (A perfect, polished

white surface is mirror-like in appearance, and a crushed mirror looks white.)

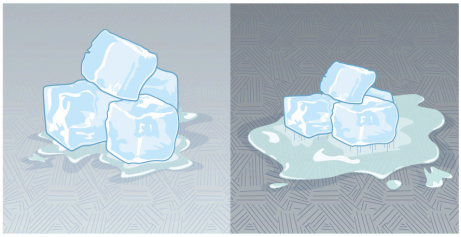


***Figure 3.*** *This illustration shows that the darker pavement is hotter than the lighter pavement (much more of the ice on the right has melted), although both have been in the sunlight for the same time. The thermal conductivities of the pavements are the same.*

Gray objects have a uniform ability to absorb all parts of the electromagnetic spectrum. Colored objects behave in similar but more complex ways, which gives them a particular color in the visible range and may make them special in other ranges of the nonvisible spectrum. Take, for example, the strong absorption of infrared radiation by the skin, which allows us to be very sensitive to it.

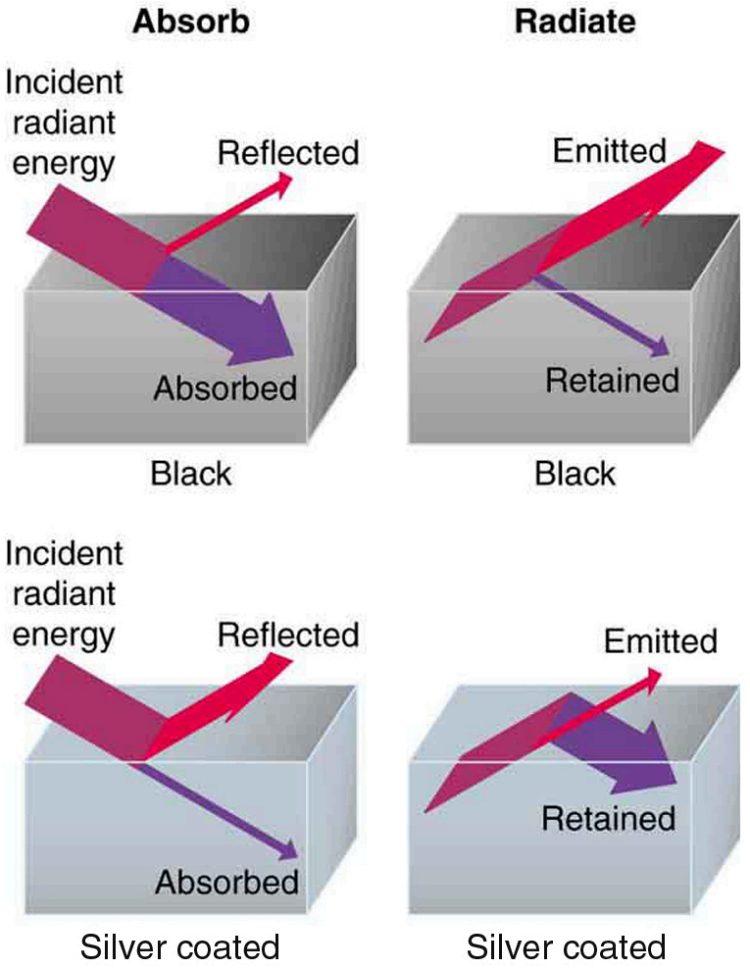



***Figure 4.*** *A black object is a good absorber and a good radiator, while a white (or silver) object is a poor absorber and a poor radiator. It is as if radiation from the inside is reflected back into the silver object, whereas radiation from the inside of the black object is "absorbed" when it hits the surface and finds itself on the outside and is strongly emitted.*

The rate of heat transfer by emitted radiation is determined by the **Stefan-Boltzmann law of radiation**:

$$\frac{Q}{t} = \sigma e A T^4,$$

where $\sigma = 5.67 \times 10^{-8}\ \mathrm{J/s \cdot m^2 \cdot K^4}$ is the Stefan-Boltzmann constant, $A$ is the surface area of the object, and $T$ is its absolute temperature in kelvin. The symbol $e$ stands for the **emissivity** of the object, which is a measure of how well it radiates. An ideal jet-black (or black body) radiator has $e = 1$, whereas a perfect reflector has $e = 0$. Real objects fall between these two values. Take, for example, tungsten light bulb filaments which have an $e$ of about $0.5$, and carbon black (a material used in printer toner), which has the (greatest known) emissivity of about $0.99$.

The radiation rate is directly proportional to the *fourth power* of the absolute temperature—a remarkably strong temperature dependence. Furthermore, the radiated heat is proportional to the surface area of the object. If you knock apart the coals of a fire, there is a noticeable increase in radiation due to an increase in radiating surface area.



***Figure 5.*** *A thermograph of part of a building shows temperature variations, indicating where heat transfer to the outside is most severe. Windows are a major region of heat transfer to the outside of homes. (credit: U.S. Army)*

Skin is a remarkably good absorber and emitter of infrared radiation, having an emissivity of 0.97 in the infrared spectrum. Thus, we are all nearly (jet) black in the infrared, in spite of the obvious variations in skin color. This high infrared emissivity is why we can so easily feel radiation on our skin. It is also the basis for the use of night scopes used by law enforcement and the military to detect human beings. Even small temperature variations can be detected because of the $T^4$ dependence. Images, called *thermographs*, can be used medically to detect regions of abnormally high temperature in the body, perhaps indicative of disease. Similar techniques can be used to detect heat leaks in homes Figure 5, optimize performance of blast furnaces, improve comfort levels in work environments, and even remotely map the Earth's temperature profile.

All objects emit and absorb radiation. The *net* rate of heat transfer

by radiation (absorption minus emission) is related to both the temperature of the object and the temperature of its surroundings. Assuming that an object with a temperature $T_1$ is surrounded by an environment with uniform temperature $T_2$, the **net rate of heat transfer by radiation** is

$$\frac{Q_{\text{net}}}{t} = \sigma e A(T_2^4 - T_1^4),$$

where $e$ is the emissivity of the object alone. In other words, it does not matter whether the surroundings are white, gray, or black; the balance of radiation into and out of the object depends on how well it emits and absorbs radiation. When $T_2 > T_1$,T_1}," title="\boldsymbol{T_2>T_1}," class="latex mathjax">the quantity $Q_{\text{net}}/t$ is positive; that is, the net heat transfer is from hot to cold.

## TAKE-HOME EXPERIMENT: TEMPERATURE IN THE SUN

Place a thermometer out in the sunshine and shield it from direct sunlight using an aluminum foil. What is the reading? Now remove the shield, and note what the thermometer reads. Take a handkerchief soaked in nail polish remover, wrap it around the thermometer and place it in the sunshine. What does the thermometer read?

## Example 1: Calculate the Net Heat Transfer of a Person: Heat Transfer by Radiation

What is the rate of heat transfer by radiation, with an unclothed person standing in a dark room whose ambient temperature is $\mathbf{22.0^\circ C}$. The person has a normal skin temperature of $\mathbf{33.0^\circ C}$ and a surface area of $\mathbf{1.50\ m^2}$. The emissivity of skin is 0.97 in the infrared, where the radiation takes place.

**Strategy**

We can solve this by using the equation for the rate of radiative heat transfer.

**Solution**

Insert the temperatures values $\boldsymbol{T_2 = 295\ K}$ and $\boldsymbol{T_1 = 306\ K}$, so that

$$\frac{Q}{t} = \sigma e A(T_2^4 - T_1^4)$$

$$= (5.67 \times 10^{-8}\ \mathrm{J/s \cdot m^2 \cdot K^4})(0.97)(1.50\ \mathrm{m^2})[(295\ \mathrm{K})^4 - (306\ \mathrm{K})^4]$$

$$= -99\ \mathrm{J/s} = -99\ \mathrm{W}.$$

**Discussion**

This value is a significant rate of heat transfer to the environment (note the minus sign), considering that a person at rest may produce energy at the rate of 125 W and that conduction and convection will also be

transferring energy to the environment. Indeed, we would probably expect this person to feel cold. Clothing significantly reduces heat transfer to the environment by many methods, because clothing slows down both conduction and convection, and has a lower emissivity (especially if it is white) than skin.

The Earth receives almost all its energy from radiation of the Sun and reflects some of it back into outer space. Because the Sun is hotter than the Earth, the net energy flux is from the Sun to the Earth. However, the rate of energy transfer is less than the equation for the radiative heat transfer would predict because the Sun does not fill the sky. The average emissivity ($e$) of the Earth is about 0.65, but the calculation of this value is complicated by the fact that the highly reflective cloud coverage varies greatly from day to day. There is a negative feedback (one in which a change produces an effect that opposes that change) between clouds and heat transfer; greater temperatures evaporate more water to form more clouds, which reflect more radiation back into space, reducing the temperature. The often mentioned **greenhouse effect** is directly related to the variation of the Earth's emissivity with radiation type (see the figure given below). The greenhouse effect is a natural phenomenon responsible for providing temperatures suitable for life on Earth. The Earth's relatively constant temperature is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by carbon dioxide ($\mathbf{CO_2}$) and water ($\mathbf{H_2O}$) in the atmosphere and then re-radiated back to the Earth or into outer space. Re-radiation back to the Earth maintains its surface temperature about $\mathbf{40^\circ C}$ higher than it would be if there was no atmosphere, similar to the way glass increases temperatures in a greenhouse.

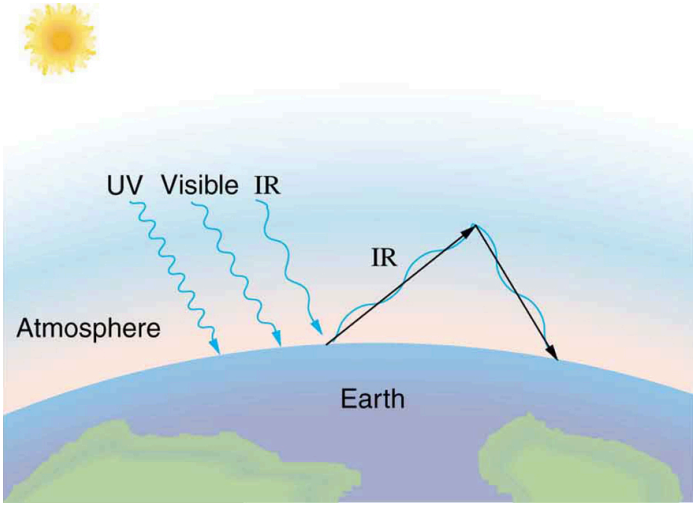



***Figure 6.*** *The greenhouse effect is a name given to the trapping of energy in the Earth's atmosphere by a process similar to that used in greenhouses. The atmosphere, like window glass, is transparent to incoming visible radiation and most of the Sun's infrared. These wavelengths are absorbed by the Earth and re-emitted as infrared. Since Earth's temperature is much lower than that of the Sun, the infrared radiated by the Earth has a much longer wavelength. The atmosphere, like glass, traps these longer infrared rays, keeping the Earth warmer than it would otherwise be. The amount of trapping depends on concentrations of trace gases like carbon dioxide, and a change in the concentration of these gases is believed to affect the Earth's surface temperature.*

The greenhouse effect is also central to the discussion of global warming due to emission of carbon dioxide and methane (and other so-called greenhouse gases) into the Earth's atmosphere from industrial production and farming. Changes in global climate could lead to more intense storms, precipitation changes (affecting agriculture), reduction in rain forest biodiversity, and rising sea levels.

Heating and cooling are often significant contributors to energy use in individual homes. Current research efforts into developing environmentally friendly homes quite often focus on reducing

conventional heating and cooling through better building materials, strategically positioning windows to optimize radiation gain from the Sun, and opening spaces to allow convection. It is possible to build a zero-energy house that allows for comfortable living in most parts of the United States with hot and humid summers and cold winters.

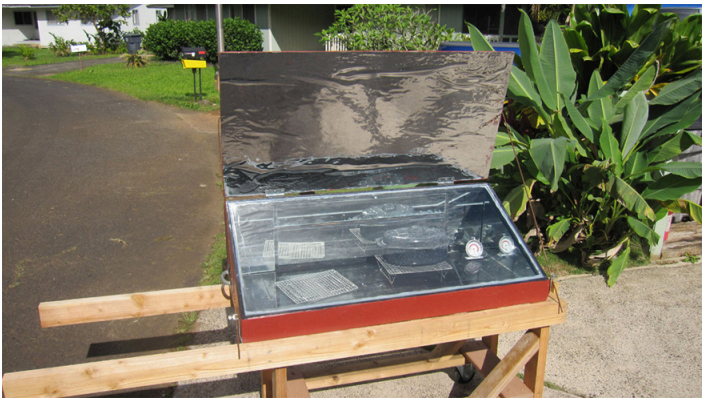


***Figure 7.*** *This simple but effective solar cooker uses the greenhouse effect and reflective material to trap and retain solar energy. Made of inexpensive, durable materials, it saves money and labor, and is of particular economic value in energy-poor developing countries. (credit: E.B. Kauai)*

Conversely, dark space is very cold, about $\mathbf{3K\ (-454^{\circ}F)}$, so that the Earth radiates energy into the dark sky. Owing to the fact that clouds have lower emissivity than either oceans or land masses, they reflect some of the radiation back to the surface, greatly reducing heat transfer into dark space, just as they greatly reduce heat transfer into the atmosphere during the day. The rate of heat transfer from soil and grasses can be so rapid that frost may occur on clear summer evenings, even in warm latitudes.

## *Check Your Understanding*

**1:** What is the change in the rate of the radiated heat by a body at the temperature $T_1 = 20^\circ\text{C}$ compared to when the body is at the temperature $T_2 = 40^\circ\text{C}$?

## CAREER CONNECTION: ENERGY CONSERVATION CONSULTATION

The cost of energy is generally believed to remain very high for the foreseeable future. Thus, passive control of heat loss in both commercial and domestic housing will become increasingly important. Energy consultants measure and analyze the flow of energy into and out of houses and ensure that a healthy exchange of air is maintained inside the house. The job prospects for an energy consultant are strong.

## PROBLEM-SOLVING STRATEGIES FOR THE METHODS OF HEAT TRANSFER

1. *Examine the situation to determine what type of heat transfer is involved.*
2. *Identify the type(s) of heat transfer—conduction, convection, or radiation.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is very useful.*
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).*
5. *Solve the appropriate equation for the quantity to be determined (the unknown).*
6. For conduction, equation $\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}$ is appropriate. Table 3 lists thermal conductivities. For convection, determine the amount of matter moved and use equation $Q = mc\Delta T$, to calculate the heat transfer involved in the temperature change of the fluid. If a phase change accompanies convection, equation $Q = mL_f$ or $Q = mL_v$ is appropriate to find the heat transfer involved in the phase change. Table 2 lists information relevant to phase change. For radiation, equation

$\frac{Q_{\text{net}}}{t} = \sigma e A(T_2^4 - T_1^4)$ gives the net heat transfer rate.

7. *Insert the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.*
8. *Check the answer to see if it is reasonable. Does it make sense?*

## Summary

- Radiation is the rate of heat transfer through the emission or absorption of electromagnetic waves.
- The rate of heat transfer depends on the surface area and the fourth power of the absolute temperature:

$$\frac{Q}{t} = \sigma e A T^4,$$

where $\sigma = 5.67 \times 10^{-8}\ \text{J/s} \cdot\ \text{m}^2 \cdot\ \text{K}^4$ is the Stefan-Boltzmann constant and $e$ is the emissivity of the body. For a black body, $e = 1$ whereas a shiny white or perfect reflector has $e = 0$, with real objects having values of $e$ between 1 and 0. The net rate of heat transfer by radiation is

$$\frac{Q_{\text{net}}}{t} = \sigma e A(T_2^4 - T_1^4)$$

where $T_1$ is the temperature of an object surrounded by an

environment with uniform temperature $T_2$ and $e$ is the emissivity of the *object*.

## *Conceptual Questions*

**1:** When watching a daytime circus in a large, dark-colored tent, you sense significant heat transfer from the tent. Explain why this occurs.

**2:** Satellites designed to observe the radiation from cold (3 K) dark space have sensors that are shaded from the Sun, Earth, and Moon and that are cooled to very low temperatures. Why must the sensors be at low temperature?

**3:** Why are cloudy nights generally warmer than clear ones?

**4:** Why are thermometers that are used in weather stations shielded from the sunshine? What does a thermometer measure if it is shielded from the sunshine and also if it is not?

**5:** On average, would Earth be warmer or cooler without the atmosphere? Explain your answer.

## *Problems & Exercises*

**1:** At what net rate does heat radiate from a $275\text{-m}^2$ black roof on a night when the roof's temperature is $30.0^\circ\text{C}$ and the surrounding temperature is $15.0^\circ\text{C}$? The emissivity of the roof is 0.900.

**2:** (a) Cherry-red embers in a fireplace are at $850^\circ\text{C}$ and have an exposed area of $0.200\ \text{m}^2$ and an emissivity of 0.980. The surrounding room has a temperature of $18.0^\circ\text{C}$. If 50% of the radiant energy enters the room, what is the net rate of radiant heat transfer in kilowatts? (b) Does your answer support the contention that most of the heat transfer into a room by a fireplace comes from infrared radiation?

**3:** Radiation makes it impossible to stand close to a hot lava flow. Calculate the rate of heat transfer by radiation from $1.00\ \text{m}^2$ of $1200^\circ\text{C}$ fresh lava into $30.0^\circ\text{C}$ surroundings, assuming lava's emissivity is 1.00.

**4:** (a) Calculate the rate of heat transfer by radiation from a car radiator at $110^\circ\text{C}$ into a $50.0^\circ\text{C}$ environment, if the radiator has an emissivity of 0.750 and a $1.20\text{-m}^2$ surface area. (b) Is this a significant fraction of the heat transfer by an automobile engine? To answer this, assume a horsepower of $200\ \text{hp}\ (1.5\ \text{kW})$ and the efficiency of automobile engines as 25%.

**5:** Find the net rate of heat transfer by radiation from a skier standing in the shade, given the following. She is

completely clothed in white (head to foot, including a ski mask), the clothes have an emissivity of 0.200 and a surface temperature of $10.0^\circ\text{C}$, the surroundings are at $-15.0^\circ\text{C}$, and her surface area is $1.60\ \text{m}^2$.

**6:** Suppose you walk into a sauna that has an ambient temperature of $50.0^\circ\text{C}$. (a) Calculate the rate of heat transfer to you by radiation given your skin temperature is $37.0^\circ\text{C}$, the emissivity of skin is 0.98, and the surface area of your body is $1.50\ \text{m}^2$. (b) If all other forms of heat transfer are balanced (the net heat transfer is zero), at what rate will your body temperature increase if your mass is 75.0 kg?

**7:** Thermography is a technique for measuring radiant heat and detecting variations in surface temperatures that may be medically, environmentally, or militarily meaningful. (a) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of $34.0^\circ\text{C}$ compared with that at $33.0^\circ\text{C}$, such as on a person's skin? (b) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of $34.0^\circ\text{C}$ compared with that at $20.0^\circ\text{C}$, such as for warm and cool automobile hoods?

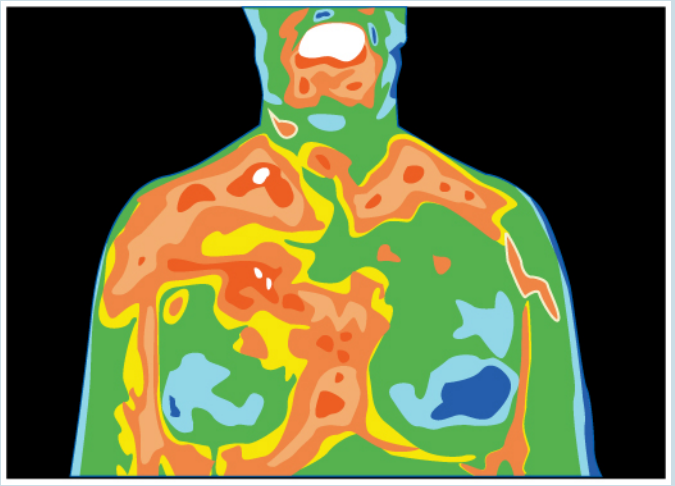


***Figure 8.*** *Artist's rendition of a thermograph of a patient's upper body, showing the distribution of heat represented by different colors.*

**8:** The Sun radiates like a perfect black body with an emissivity of exactly 1. (a) Calculate the surface temperature of the Sun, given that it is a sphere with a $\mathbf{7.00 \times 10^{8}\text{-m}}$ radius that radiates $\mathbf{3.80 \times 10^{26}\ W}$ into 3-K space. (b) How much power does the Sun radiate per square meter of its surface? (c) How much power in watts per square meter is that value at the distance of Earth, $\mathbf{1.50 \times 10^{11}\ m}$ away? (This number is called the solar constant.)

**9:** A large body of lava from a volcano has stopped flowing and is slowly cooling. The interior of the lava is at $\mathbf{1200^{\circ}\,C}$, its surface is at $\mathbf{450^{\circ}\,C}$, and the surroundings are at $\mathbf{27.0^{\circ}\,C}$. (a) Calculate the rate at

which energy is transferred by radiation from $1.00\ \mathbf{m^2}$ of surface lava into the surroundings, assuming the emissivity is 1.00. (b) Suppose heat conduction to the surface occurs at the same rate. What is the thickness of the lava between the $450^\circ\ \mathbf{C}$ surface and the $1200^\circ\ \mathbf{C}$ interior, assuming that the lava's conductivity is the same as that of brick?

**10:** Calculate the temperature the entire sky would have to be in order to transfer energy by radiation at $1000\ \mathbf{W/m^2}$—about the rate at which the Sun radiates when it is directly overhead on a clear day. This value is the effective temperature of the sky, a kind of average that takes account of the fact that the Sun occupies only a small part of the sky but is much hotter than the rest. Assume that the body receiving the energy has a temperature of $27.0^\circ\ \mathbf{C}$.

**11:** (a) A shirtless rider under a circus tent feels the heat radiating from the sunlit portion of the tent. Calculate the temperature of the tent canvas based on the following information: The shirtless rider's skin temperature is $34.0^\circ\ \mathbf{C}$ and has an emissivity of 0.970. The exposed area of skin is $0.400\ \mathbf{m^2}$. He receives radiation at the rate of 20.0 W—half what you would calculate if the entire region behind him was hot. The rest of the surroundings are at $34.0^\circ\ \mathbf{C}$. (b) Discuss how this situation would change if the sunlit side of the tent was nearly pure white and if the rider was covered by a white tunic.

**12: Integrated Concepts**

One $30.0^\circ C$ day the relative humidity is $75.0\%$, and that evening the temperature drops to $20.0^\circ C$, well below the dew point. (a) How many grams of water condense from each cubic meter of air? (b) How much heat transfer occurs by this condensation? (c) What temperature increase could this cause in dry air?

**13: Integrated Concepts**

Large meteors sometimes strike the Earth, converting most of their kinetic energy into thermal energy. (a) What is the kinetic energy of a $10^9$ kg meteor moving at 25.0 km/s? (b) If this meteor lands in a deep ocean and $80\%$ of its kinetic energy goes into heating water, how many kilograms of water could it raise by $5.0^\circ C$? (c) Discuss how the energy of the meteor is more likely to be deposited in the ocean and the likely effects of that energy.

**14: Integrated Concepts**

Frozen waste from airplane toilets has sometimes been accidentally ejected at high altitude. Ordinarily it breaks up and disperses over a large area, but sometimes it holds together and strikes the ground. Calculate the mass of $0^\circ C$ ice that can be melted by the conversion of kinetic and gravitational potential energy when a 20.0 kg piece of frozen waste is released at 12.0 km altitude while moving at 250 m/s and strikes the ground at 100 m/s (since less than 20.0 kg melts, a significant mess results).

**15: Integrated Concepts**

(a) A large electrical power facility produces 1600 MW of "waste heat," which is dissipated to the environment in

cooling towers by warming air flowing through the towers by $5.00^\circ C$. What is the necessary flow rate of air in $m^3/s$? (b) Is your result consistent with the large cooling towers used by many large electrical power plants?

**16: Integrated Concepts**

(a) Suppose you start a workout on a Stairmaster, producing power at the same rate as climbing 116 stairs per minute. Assuming your mass is 76.0 kg and your efficiency is $20.0\%$, how long will it take for your body temperature to rise $1.00^\circ C$ if all other forms of heat transfer in and out of your body are balanced? (b) Is this consistent with your experience in getting warm while exercising?

**17: Integrated Concepts**

A 76.0-kg person suffering from hypothermia comes indoors and shivers vigorously. How long does it take the heat transfer to increase the person's body temperature by $2.00^\circ C$ if all other forms of heat transfer are balanced?

**18: Integrated Concepts**

In certain large geographic regions, the underlying rock is hot. Wells can be drilled and water circulated through the rock for heat transfer for the generation of electricity. (a) Calculate the heat transfer that can be extracted by cooling $1.00\ km^3$ of granite by $100^\circ C$. (b) How long will it take for heat transfer at the rate of 300 MW, assuming no heat transfers back into the $1.00\ km^3$ of rock by its surroundings?

**19: Integrated Concepts**

Heat transfers from your lungs and breathing passages by evaporating water. (a) Calculate the maximum number of grams of water that can be evaporated when you inhale 1.50 L of $37^\circ C$ air with an original relative humidity of 40.0%. (Assume that body temperature is also $37^\circ C$.) (b) How many joules of energy are required to evaporate this amount? (c) What is the rate of heat transfer in watts from this method, if you breathe at a normal resting rate of 10.0 breaths per minute?

**20: Integrated Concepts**

(a) What is the temperature increase of water falling 55.0 m over Niagara Falls? (b) What fraction must evaporate to keep the temperature constant?

**21: Integrated Concepts**

Hot air rises because it has expanded. It then displaces a greater volume of cold air, which increases the buoyant force on it. (a) Calculate the ratio of the buoyant force to the weight of $50.0^\circ C$ air surrounded by $20.0^\circ C$ air. (b) What energy is needed to cause $1.00\ m^3$ of air to go from $20.0^\circ C$ to $50.0^\circ C$? (c) What gravitational potential energy is gained by this volume of air if it rises 1.00 m? Will this cause a significant cooling of the air?

**22: Unreasonable Results**

(a) What is the temperature increase of an 80.0 kg person who consumes 2500 kcal of food in one day with 95.0% of the energy transferred as heat to the body? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

**23: Unreasonable Results**

A slightly deranged Arctic inventor surrounded by ice thinks it would be much less mechanically complex to cool a car engine by melting ice on it than by having a water-cooled system with a radiator, water pump, antifreeze, and so on. (a) If $80.0\%$ of the energy in 1.00 gal of gasoline is converted into "waste heat" in a car engine, how many kilograms of $0^\circ\text{C}$ ice could it melt? (b) Is this a reasonable amount of ice to carry around to cool the engine for 1.00 gal of gasoline consumption? (c) What premises or assumptions are unreasonable?

**24:Unreasonable Results**

(a) Calculate the rate of heat transfer by conduction through a window with an area of $1.00\ \text{m}^2$ that is 0.750 cm thick, if its inner surface is at $22.0^\circ\text{C}$ and its outer surface is at $35.0^\circ\text{C}$. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

**25: Unreasonable Results**

A meteorite 1.20 cm in diameter is so hot immediately after penetrating the atmosphere that it radiates 20.0 kW of power. (a) What is its temperature, if the surroundings are at $20.0^\circ\text{C}$ and it has an emissivity of 0.800? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

**26: Construct Your Own Problem**

Consider a new model of commercial airplane having its brakes tested as a part of the initial flight permission procedure. The airplane is brought to takeoff speed and

then stopped with the brakes alone. Construct a problem in which you calculate the temperature increase of the brakes during this process. You may assume most of the kinetic energy of the airplane is converted to thermal energy in the brakes and surrounding materials, and that little escapes. Note that the brakes are expected to become so hot in this procedure that they ignite and, in order to pass the test, the airplane must be able to withstand the fire for some time without a general conflagration.

**27: Construct Your Own Problem**

Consider a person outdoors on a cold night. Construct a problem in which you calculate the rate of heat transfer from the person by all three heat transfer methods. Make the initial circumstances such that at rest the person will have a net heat transfer and then decide how much physical activity of a chosen type is necessary to balance the rate of heat transfer. Among the things to consider are the size of the person, type of clothing, initial metabolic rate, sky conditions, amount of water evaporated, and volume of air breathed. Of course, there are many other factors to consider and your instructor may wish to guide you in the assumptions made as well as the detail of analysis and method of presenting your results.

## Glossary

**emissivity**
measure of how well an object radiates

**greenhouse effect**
warming of the Earth that is due to gases such as carbon

dioxide and methane that absorb infrared radiation from the Earth's surface and reradiate it in all directions, thus sending a fraction of it back toward the surface of the Earth

**net rate of heat transfer by radiation**
is $\frac{Q_{\text{net}}}{t} = \sigma e A(T_2^4 - T_1^4)$

**radiation**
energy transferred by electromagnetic waves directly as a result of a temperature difference

**Stefan-Boltzmann law of radiation**
$\frac{Q}{t} = \sigma e A T^4$, where $\sigma$ is the Stefan-Boltzmann constant, $A$ is the surface area of the object, $T$ is the absolute temperature, and $e$ is the emissivity

## *Solutions*

**Check Your Understanding**

**1:** The radiated heat is proportional to the fourth power of the *absolute temperature*. Because $T_1 = 293\ \text{K}$ and $T_2 = 313\ \text{K}$, the rate of heat transfer increases by about 30 percent of the original rate.

**Problems & Exercises**

**1:** -21.7 kW  Note that the negative answer implies heat loss to the surroundings.

**3:** -266 kW

**5:** -36.0 W

**7:** (a) 1.31%  (b) 20.5%

**9:** (a) – 15.0 kW (b) 4.2 cm

**11:** (a) $48.5^\circ C$ (b) A pure white object reflects more of the radiant energy that hits it, so a white tent would prevent more of the sunlight from heating up the inside of the tent, and the white tunic would prevent that heat which entered the tent from heating the rider. Therefore, with a white tent, the temperature would be lower than $48.5^\circ C$, and the rate of radiant heat transferred to the rider would be less than 20.0 W.

**13:** (a) $3 \times 10^{17}$ J (b) $1 \times 10^{13}$ kg (c) When a large meteor hits the ocean, it causes great tidal waves, dissipating large amount of its energy in the form of kinetic energy of the water.

**15:** (a) $3.44 \times 10^5 \ m^3/s$ (b) This is equivalent to 12 million cubic feet of air per second. That is tremendous. This is too large to be dissipated by heating the air by only $5^\circ C$. Many of these cooling towers use the circulation of cooler air over warmer water to increase the rate of evaporation. This would allow much smaller amounts of air necessary to remove such a large amount of heat because evaporation removes larger quantities of heat than was considered in part (a).

**17:** 20.9 min

**19:** (a) $3.96 \times 10^{-2}$ g (b) 96.2 J (c) 16.0 W

**21:** (a) 1.102 (b) $2.79 \times 10^4$ J (c) 12.6 J. This will not cause a significant cooling of the air because it is much less than the energy found in part (b), which is the energy required to warm the air from $20.0^\circ C$ to $50.0^\circ C$.

**22:** (a) $36^\circ C$ (b) Any temperature increase greater than

about $3^\circ C$ would be unreasonably large. In this case the final temperature of the person would rise to $73^\circ C\ (163^\circ F)$.

(c) The assumption of $95\%$ heat retention is unreasonable.

**24:** (a) 1.46 kW (b) Very high power loss through a window. An electric heater of this power can keep an entire room warm. (c) The surface temperatures of the window do not differ by as great an amount as assumed. The inner surface will be warmer, and the outer surface will be cooler.

PART XIII

# CHAPTER 12 ELECTRIC CHARGE AND ELECTRIC FIELD



***Figure 1.*** *Static electricity from this plastic slide causes the child's hair to stand on end. The sliding motion stripped electrons away from the child's body, leaving an excess of positive charges, which repel each other along each strand of hair. (credit: Ken Bosma/Wikimedia Commons)*



***Figure 2.*** *Styrofoam peanuts clinging to a cat's fur due to static electricity. The triboelectric effect causes an electrostatic charge to build up on the fur due to the cat's motions. The electric field of the charge causes polarization of the molecules of the styrofoam due to electrostatic induction, resulting in a slight attraction of the light plastic pieces to the charged fur. This effect is also the cause of static cling in clothes. Image Credit: Sean McGrath, CCBY, Wikimedia Commons*

The image of American politician and scientist Benjamin Franklin (1706–1790) flying a kite in a thunderstorm is familiar to most schoolchild. (See Figure 3.) In this experiment, Franklin demonstrated a connection between lightning and **static electricity**. Sparks were drawn from a key hung on a kite string during an electrical storm. These sparks were like those produced by static electricity, such as the spark that jumps from your finger to a metal doorknob after you walk across a wool carpet. What Franklin demonstrated in his dangerous experiment was a connection between phenomena on two different scales: one the grand power of an electrical storm, the other an effect of more human

proportions. Connections like this one reveal the underlying unity of the laws of nature, an aspect we humans find particularly appealing.

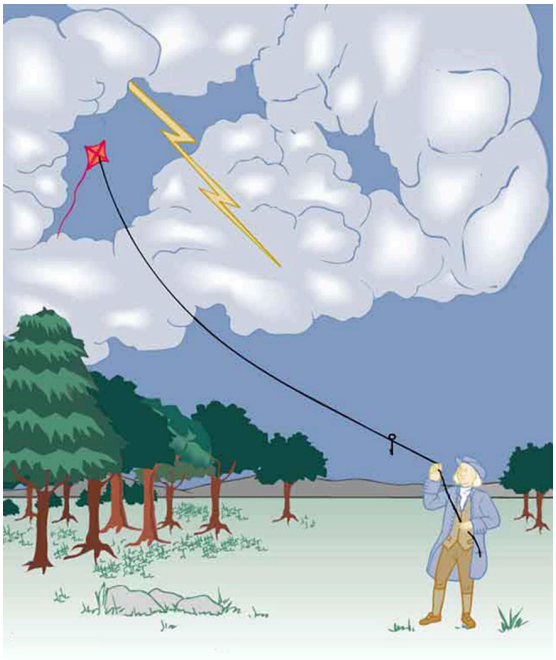


***Figure 3.*** *When Benjamin Franklin demonstrated that lightning was related to static electricity, he made a connection that is now part of the evidence that all directly experienced forces except the gravitational force are manifestations of the electromagnetic force.*

Much has been written about Franklin. His experiments were only part of the life of a man who was a scientist, inventor, revolutionary, statesman, and writer. Franklin's experiments were not performed in isolation, nor were they the only ones to reveal connections.

For example, the Italian scientist Luigi Galvani (1737–1798) performed a series of experiments in which static electricity was used to stimulate contractions of leg muscles of dead frogs, an effect already known in humans subjected to static discharges. But

Galvani also found that if he joined two metal wires (say copper and zinc) end to end and touched the other ends to muscles, he produced the same effect in frogs as static discharge. Alessandro Volta (1745–1827), partly inspired by Galvani's work, experimented with various combinations of metals and developed the battery.

During the same era, other scientists made progress in discovering fundamental connections. The periodic table was developed as the systematic properties of the elements were discovered. This influenced the development and refinement of the concept of atoms as the basis of matter. Such submicroscopic descriptions of matter also help explain a great deal more.

Atomic and molecular interactions, such as the forces of friction, cohesion, and adhesion, are now known to be manifestations of the **electromagnetic force**. Static electricity is just one aspect of the electromagnetic force, which also includes moving electricity and magnetism.

All the macroscopic forces that we experience directly, such as the sensations of touch and the tension in a rope, are due to the electromagnetic force, one of the four fundamental forces in nature. The gravitational force, another fundamental force, is actually sensed through the electromagnetic interaction of molecules, such as between those in our feet and those on the top of a bathroom scale. (The other two fundamental forces, the strong nuclear force and the weak nuclear force, cannot be sensed on the human scale.)

This chapter begins the study of electromagnetic phenomena at a fundamental level. The next several chapters will cover static electricity, moving electricity, and magnetism—collectively known as electromagnetism. In this chapter, we begin with the study of electric phenomena due to charges that are at least temporarily stationary, called electrostatics, or static electricity.

## Glossary

**static electricity**
: a buildup of electric charge on the surface of an object

**electromagnetic force**
: one of the four fundamental forces of nature; the electromagnetic force consists of static electricity, moving electricity and magnetism

# 67. 12.1 Static Electricity and Charge: Conservation of Charge

*Summary*

- Define electric charge, and describe how the two types of charge interact.
- Describe three common situations that generate static electricity.
- State the law of conservation of charge.



***Figure 1.*** *Borneo amber was mined in Sabah, Malaysia, from shale-sandstone-mudstone veins. When a piece of amber is rubbed with a piece of silk, the amber gains more electrons, giving it a net negative charge. At the same time, the silk, having lost electrons, becomes positively charged. (credit: Sebakoamber, Wikimedia Commons)*

What makes plastic wrap cling? Static electricity. Not only are applications of static electricity common these days, its existence has been known since ancient times. The first record of its effects dates to ancient Greeks who noted more than 500 years B.C. that polishing amber temporarily enabled it to attract bits of straw (see Figure 1). The very word *electric* derives from the Greek word for amber (*electron*).

Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Static cling generated in a clothes dryer and the attraction of straw to recently polished amber also result from rubbing. Similarly, lightning results from air movements under certain weather conditions. You can also rub a balloon on your hair, and the static electricity created can then make the balloon cling to a wall. We also have to be cautious of static

electricity, especially in dry climates. When we pump gasoline, we are warned to discharge ourselves (after sliding across the seat) on a metal surface before grabbing the gas nozzle. Attendants in hospital operating rooms must wear booties with aluminum foil on the bottoms to avoid creating sparks which may ignite the oxygen being used.

Some of the most basic characteristics of static electricity include:

- The effects of static electricity are explained by a physical quantity not previously introduced, called electric charge.
- There are only two types of charge, one called positive and the other called negative.
- Like charges repel, whereas unlike charges attract.
- The force between charges decreases with distance.

How do we know there are two types of **electric charge**? When various materials are rubbed together in controlled ways, certain combinations of materials always produce one type of charge on one material and the opposite type on the other. By convention, we call one type of charge "positive", and the other type "negative." For example, when glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged. Since the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, since each rod has positive charge on it. Similarly, two silk cloths so rubbed will repel, since both cloths have negative charge. Figure 2 shows how these simple materials can be used to explore the nature of the force between charges.

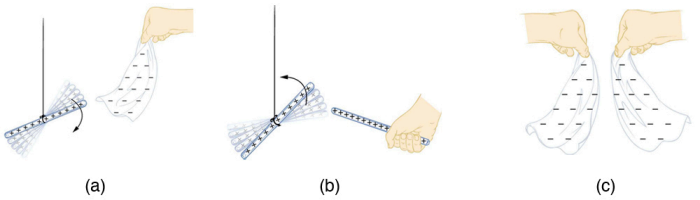



***Figure 2.*** *A glass rod becomes positively charged when rubbed with silk, while the silk becomes negatively charged. (a) The glass rod is attracted to the silk because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel.*

More sophisticated questions arise. Where do these charges come from? Can you create or destroy charge? Is there a smallest unit of charge? Exactly how does the force depend on the amount of charge and the distance between charges? Such questions obviously occurred to Benjamin Franklin and other early researchers, and they interest us even today.

## Charge Carried by Electrons and Protons

Franklin wrote in his letters and books that he could see the effects of electric charge but did not understand what caused the phenomenon. Today we have the advantage of knowing that normal matter is made of atoms, and that atoms contain positive and negative charges, usually in equal amounts.

Figure 3 shows a simple model of an atom with negative **electrons** orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged **protons**. Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.) Other charge-carrying particles are observed in cosmic rays and nuclear decay, and are created in particle

accelerators. All but the electron and proton survive only a short time and are quite rare by comparison.

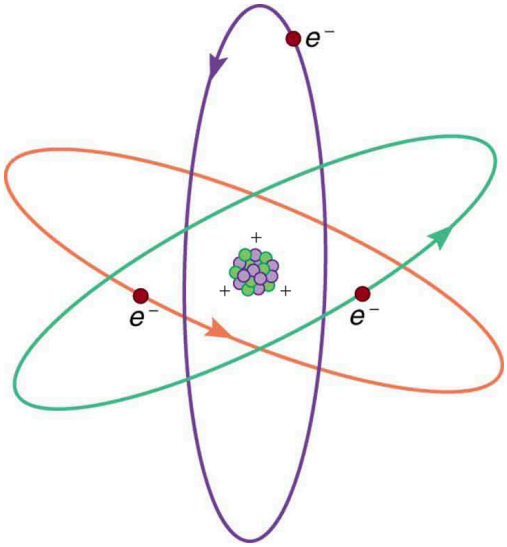



***Figure 3.*** *This simplified (and not to scale) view of an atom is called the planetary model of the atom. Negative electrons orbit a much heavier positive nucleus, as the planets orbit the much heavier sun. There the similarity ends, because forces in the atom are electromagnetic, whereas those in the planetary system are gravitational. Normal macroscopic amounts of matter contain immense numbers of atoms and molecules and, hence, even greater numbers of individual negative and positive charges.*

The charges of electrons and protons are identical in magnitude but opposite in sign. Furthermore, all charged objects in nature are integral multiples of this basic quantity of charge, meaning that all charges are made of combinations of a basic unit of charge. Usually, charges are formed by combinations of electrons and protons. The magnitude of this basic charge is

$$|q_e| = 1.60 \times 10^{-19} \text{ C}.$$

The symbol $q$ is commonly used for charge and the subscript $e$ indicates the charge of a single electron (or proton).

The SI unit of charge is the coulomb (C). The number of protons needed to make a charge of 1.00 C is

$$1.00\ \text{C} \times \frac{1\ \text{proton}}{1.60 \times 10^{-19}\ \text{C}} = 6.25 \times 10^{18}\ \text{protons.}$$

Similarly, $6.25 \times 10^{18}$ electrons have a combined charge of –1.00 coulomb. Just as there is a smallest bit of an element (an atom), there is a smallest bit of charge. There is no directly observed charge smaller than $|q_e|$ (see Things Great and Small: The Submicroscopic Origin of Charge), and all observed charges are integral multiples of $|q_e|$.

## Things Great and Small: The Submicroscopic Origin of Charge

With the exception of exotic, short-lived particles, all charge in nature is carried by electrons and protons. Electrons carry the charge we have named negative. Protons carry an equal-magnitude charge that we call positive. (See Figure 4.) Electron and proton charges are considered fundamental building blocks, since all other charges are integral multiples of those carried by electrons and protons. Electrons and protons are also two of the three fundamental building blocks of

ordinary matter. The neutron is the third and has zero total charge.

Figure 4 shows a person touching a Van de Graaff generator and receiving excess positive charge. The expanded view of a hair shows the existence of both types of charges but an excess of positive. The repulsion of these positive like charges causes the strands of hair to repel other strands of hair and to stand up. The further blowup shows an artist's conception of an electron and a proton perhaps found in an atom in a strand of hair.

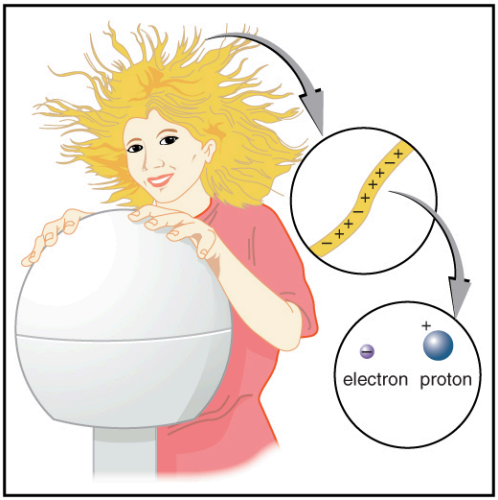



***Figure 4.*** *When this person touches a Van de Graaff generator, she receives an excess of positive charge, causing her hair to stand on end. The charges in one hair are shown. An artist's conception of an electron and a proton illustrate the particles carrying the negative and positive charges. We cannot really see these particles with visible light because they are so small (the electron seems to be an infinitesimal point), but we know a great deal about their measurable properties, such as the charges they carry.*

The electron seems to have no substructure; in contrast, when the substructure of protons is explored by scattering extremely energetic electrons from them, it appears that there are point-like particles inside the proton. These sub-particles, named quarks, have never been directly observed, but they are believed to carry fractional charges as seen in Figure 5. Charges on electrons and protons and all other directly observable particles are unitary, but these quark substructures carry charges of either $-\frac{1}{3}$ or $+\frac{2}{3}$. There are continuing attempts to observe fractional charge directly and to learn of the properties of quarks, which are perhaps the ultimate substructure of matter.

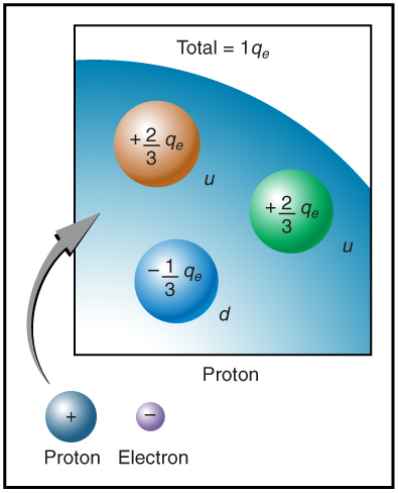



***Figure 5.*** *Artist's conception of fractional quark charges inside a proton. A group of three quark charges add up to the single positive charge on the proton:* $-1/3q_e + 2/3q_e + 2/3q_e = +1q_e$ .

## Separation of Charge in Atoms

Charges in atoms and molecules can be separated–for example, by rubbing materials together. Some atoms and molecules have a greater affinity for electrons than others and will become negatively charged by close contact in rubbing, leaving the other material positively charged. (See Figure 6.) Positive charge can similarly be induced by rubbing. Methods other than rubbing can also separate charges. Batteries, for example, use combinations of substances that interact in such a way as to separate charges. Chemical interactions may transfer negative charge from one substance to the other, making one battery terminal negative and leaving the first one positive.

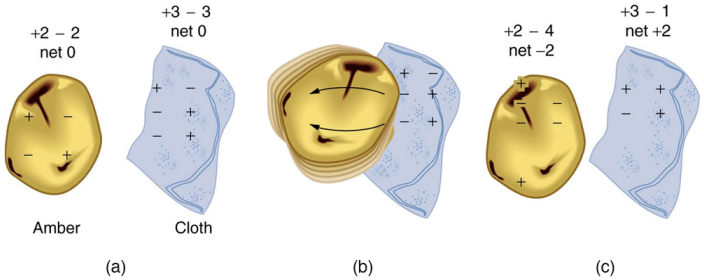



***Figure 6.*** *When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.*

No charge is actually created or destroyed when charges are separated as we have been discussing. Rather, existing charges are moved about. In fact, in all situations the total amount of charge is

always constant. This universally obeyed law of nature is called the **law of conservation of charge**.

### Law of Conservation of Charge

Total charge is constant in any process.

In more exotic situations, such as in particle accelerators, mass, $\Delta m$, can be created from energy in the amount $\Delta m = \frac{E}{c^2}$. Sometimes, the created mass is charged, such as when an electron is created. Whenever a charged particle is created, another having an opposite charge is always created along with it, so that the total charge created is zero. Usually, the two particles are "matter-antimatter" counterparts. For example, an antielectron would usually be created at the same time as an electron. The antielectron has a positive charge (it is called a positron), and so the total charge created is zero. (See Figure 7.) All particles have antimatter counterparts with opposite signs. When matter and antimatter counterparts are brought together, they completely annihilate one another. By annihilate, we mean that the mass of the two particles is converted to energy $E$, again obeying the relationship $\Delta m = \frac{E}{c^2}$. Since the two particles have equal and opposite charge, the total charge is zero before and after the annihilation; thus, total charge is conserved.

## Making Connections: Conservation Laws

Only a limited number of physical quantities are universally conserved. Charge is one—energy, momentum, and angular momentum are others. Because they are conserved, these physical quantities are used to explain more phenomena and form more connections than other, less basic quantities. We find that conserved quantities give us great insight into the rules followed by nature and hints to the organization of nature. Discoveries of conservation laws have led to further discoveries, such as the weak nuclear force and the quark substructure of protons and other particles.

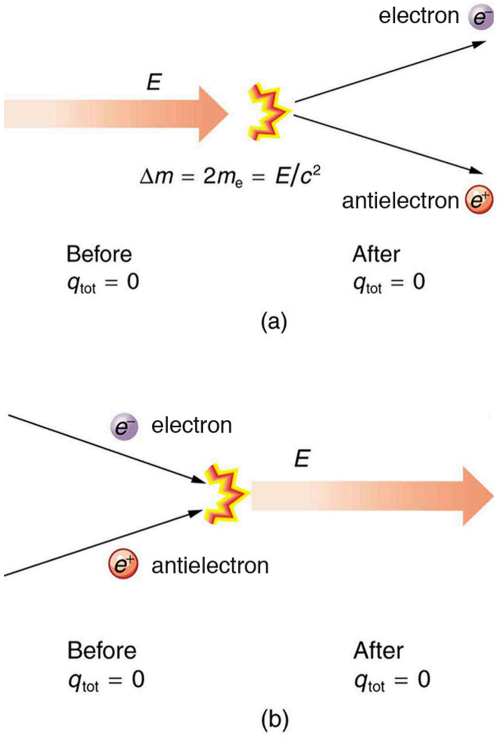



***Figure 7.*** *(a) When enough energy is present, it can be converted into matter. Here the matter created is an electron–antielectron pair. ($m_e$ is the electron's mass.) The total charge before and after this event is zero. (b) When matter and antimatter collide, they annihilate each other; the total charge is conserved at zero before and after the annihilation.*

The law of conservation of charge is absolute—it has never been observed to be violated. Charge, then, is a special physical quantity, joining a very short list of other quantities in nature that are always conserved. Other conserved quantities include energy, momentum, and angular momentum.

PhET Explorations: Balloons and Static Electricity

Why does a balloon stick to your sweater? Rub a balloon on a sweater, then let go of the balloon and it flies over and sticks to the sweater. View the charges in the sweater, balloons, and the wall.



PhET Interactive Simulation

***Figure 8.*** *Balloons and Static Electricity*

## Section Summary

- There are only two types of charge, which we call positive and negative.
- Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, while the vast majority of negative charge is carried by electrons.
- The electric charge of one electron is equal in magnitude and

opposite in sign to the charge of one proton.

- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge $|q_e|$ is $|q_e| = 1.60 \times 10^{-19}\ \text{C}$.
- Whenever charge is created or destroyed, equal amounts of positive and negative are involved.
- Most often, existing charges are separated from neutral objects to obtain some net charge.
- Both positive and negative charges exist in neutral objects and can be separated by rubbing one object with another. For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge ensures that whenever a charge is created, an equal charge of the opposite sign is created at the same time.

## *Conceptual Questions*

**1:** There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

**2:** Why do most objects tend to contain nearly equal numbers of positive and negative charges?

## *Problems & Exercises*

**1:** Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of $\mathbf{-2.00\ nC}$ (b) How many electrons must be removed from a neutral object to leave a net charge of $\mathbf{0.500\ \mu C}$ ?

**2:** If $\mathbf{1.80 \times 10^{20}}$ electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

**3:** To start a car engine, the car battery moves $\mathbf{3.75 \times 10^{21}}$ electrons through the starter motor. How many coulombs of charge were moved?

**4:** A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge $|\boldsymbol{q_e}|$ is this?

# Glossary

**electric charge**
: a physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electromagnetic force

**law of conservation of charge**
: states that whenever a charge is created, an equal amount of charge with the opposite sign is created simultaneously

**electron**
: a particle orbiting the nucleus of an atom and carrying the smallest unit of negative charge

**proton**
: a particle in the nucleus of an atom and carrying a positive charge equal in magnitude and opposite in sign to the amount of negative charge carried by an electron

## *Solutions*

**1:**

(a) $\mathbf{1.25 \times 10^{10}}$

(b) $\mathbf{3.13 \times 10^{12}}$

**2:**

-600 C

# 68. 12.2 Conductors and Insulators

## *Summary*

- Define conductor and insulator, explain the difference, and give examples of each.
- Describe three methods for charging an object.
- Explain what happens to an electric force as you move farther from the source.
- Define polarization.



***Figure 1.*** *This power adapter uses metal wires and connectors to conduct electricity from the wall socket to a laptop computer. The conducting wires allow electrons to move freely through the cables, which are shielded by rubber and plastic. These materials act as insulators that don't allow electric charge to escape outward. (credit: Evan-Amos, Wikimedia Commons)*

Some substances, such as metals and salty water, allow charges to move through them with relative ease. Some of the electrons in metals and similar conductors are not bound to individual atoms or sites in the material. These **free electrons** can move through the material much as air moves through loose sand. Any substance that has free electrons and allows charge to move relatively freely through it is called a **conductor**. The moving electrons may collide with fixed atoms and molecules, losing some energy, but they can move in a conductor. Superconductors allow the movement of charge without any loss of energy. Salty water and other similar conducting materials contain free ions that can move through them. An ion is an atom or molecule having a positive or negative (nonzero) total charge. In other words, the total number of electrons is not equal to the total number of protons.

Other substances, such as glass, do not allow charges to move through them. These are called **insulators**. Electrons and ions in insulators are bound in the structure and cannot move easily—as much as $\mathbf{10^{23}}$ times more slowly than in conductors. Pure water and dry table salt are insulators, for example, whereas molten salt and salty water are conductors.

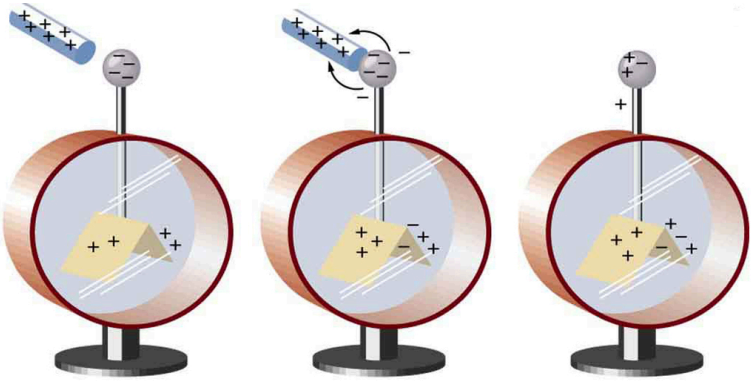



***Figure 2.*** *An electroscope is a favorite instrument in physics demonstrations and student laboratories. It is typically made with gold foil leaves hung from a (conducting) metal stem and is insulated from the room air in a glass-walled container. (a) A positively charged glass rod is brought near the tip of the electroscope, attracting electrons to the top and leaving a net positive charge on the leaves. Like charges in the light flexible gold leaves repel, separating them. (b) When the rod is touched against the ball, electrons are attracted and transferred, reducing the net charge on the glass rod but leaving the electroscope positively charged. (c) The excess charges are evenly distributed in the stem and leaves of the electroscope once the glass rod is removed.*

## Charging by Contact

Figure 2 shows an electroscope being charged by touching it with a positively charged glass rod. Because the glass rod is an insulator, it must actually touch the electroscope to transfer charge to or from it. (Note that the extra positive charges reside on the surface of the glass rod as a result of rubbing it with silk before starting the experiment.) Since only electrons move in metals, we see that they are attracted to the top of the electroscope. There, some are transferred to the positive rod by touch, leaving the electroscope with a net positive charge.

**Electrostatic repulsion** in the leaves of the charged electroscope separates them. The electrostatic force has a horizontal component

that results in the leaves moving apart as well as a vertical component that is balanced by the gravitational force. Similarly, the electroscope can be negatively charged by contact with a negatively charged object.

## Charging by Induction

It is not necessary to transfer excess charge directly to an object in order to charge it. Figure 3 shows a method of **induction** wherein a charge is created in a nearby object, without direct contact. Here we see two neutral metal spheres in contact with one another but insulated from the rest of the world. A positively charged rod is brought near one of them, attracting negative charge to that side, leaving the other sphere positively charged.

This is an example of induced **polarization** of neutral objects. Polarization is the separation of charges in an object that remains neutral. If the spheres are now separated (before the rod is pulled away), each sphere will have a net charge. Note that the object closest to the charged rod receives an opposite charge when charged by induction. Note also that no charge is removed from the charged rod, so that this process can be repeated without depleting the supply of excess charge.

Another method of charging by induction is shown in Figure 4. The neutral metal sphere is polarized when a charged rod is brought near it. The sphere is then grounded, meaning that a conducting wire is run from the sphere to the ground. Since the earth is large and most ground is a good conductor, it can supply or accept excess charge easily. In this case, electrons are attracted to the sphere through a wire called the ground wire, because it supplies a conducting path to the ground. The ground connection is broken before the charged rod is removed, leaving the sphere with an excess charge opposite to that of the rod. Again, an opposite charge

is achieved when charging by induction and the charged rod loses none of its excess charge.

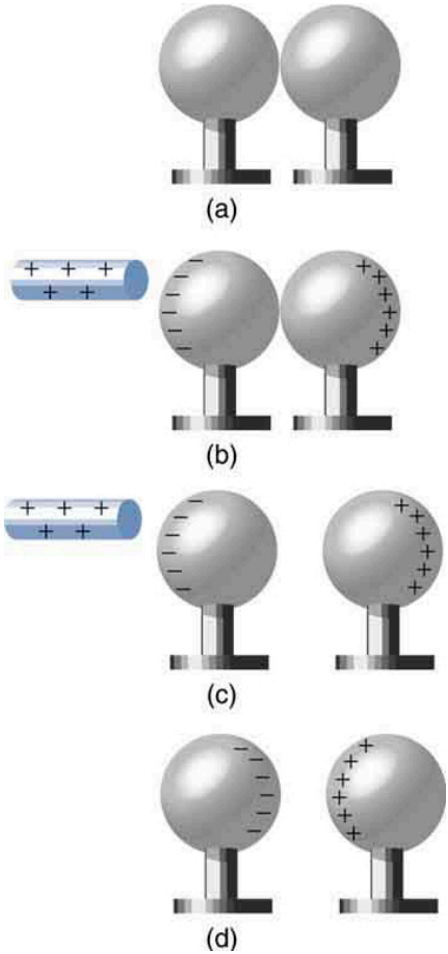



***Figure 3.*** *Charging by induction. (a) Two uncharged or neutral metal spheres are in contact with each other but insulated from the rest of the world. (b) A positively charged glass rod is brought near the sphere on the left, attracting negative charge and leaving the other sphere positively charged. (c) The spheres are separated before the rod is removed, thus separating negative and positive charge. (d) The spheres retain net charges after the inducing rod is removed—without ever having been touched by a charged object.*

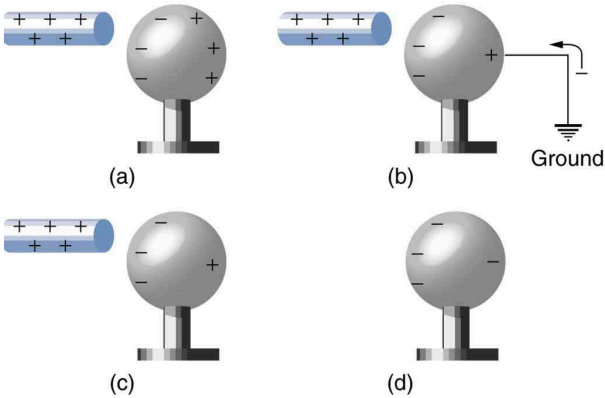



***Figure 4.*** *Charging by induction, using a ground connection. (a) A positively charged rod is brought near a neutral metal sphere, polarizing it. (b) The sphere is grounded, allowing electrons to be attracted from the earth's ample supply. (c) The ground connection is broken. (d) The positive rod is removed, leaving the sphere with an induced negative charge.*

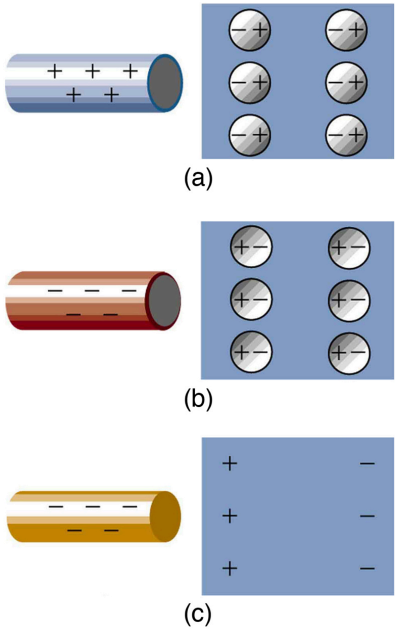



***Figure 5.*** *Both positive and negative objects attract a neutral object by polarizing its molecules. (a) A positive object brought near a neutral insulator polarizes its molecules. There is a slight shift in the distribution of the electrons orbiting the molecule, with unlike charges being brought nearer and like charges moved away. Since the electrostatic force decreases with distance, there is a net attraction. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction.*

Neutral objects can be attracted to any charged object. The pieces of straw attracted to polished amber are neutral, for example. If you run a plastic comb through your hair, the charged comb can pick up neutral pieces of paper. Figure 5 shows how the polarization of

atoms and molecules in neutral objects results in their attraction to a charged object.

When a charged rod is brought near a neutral substance, an insulator in this case, the distribution of charge in atoms and molecules is shifted slightly. Opposite charge is attracted nearer the external charged rod, while like charge is repelled. Since the electrostatic force decreases with distance, the repulsion of like charges is weaker than the attraction of unlike charges, and so there is a net attraction. Thus a positively charged glass rod attracts neutral pieces of paper, as will a negatively charged rubber rod. Some molecules, like water, are polar molecules. Polar molecules have a natural or inherent separation of charge, although they are neutral overall. Polar molecules are particularly affected by other charged objects and show greater polarization effects than molecules with naturally uniform charge distributions.

## *Check Your Understanding*

Can you explain the attraction of water to the charged rod in the figure below?

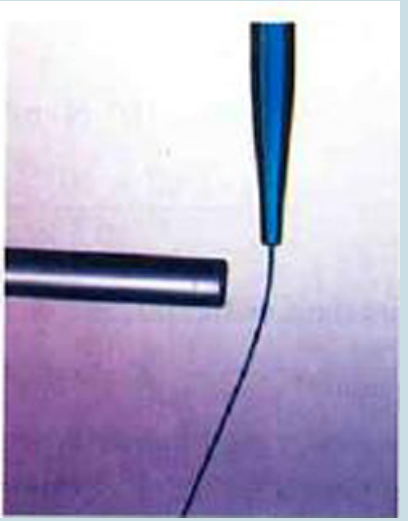


***Figure 6.***

## PhET Explorations: John Travoltage

Make sparks fly with John Travoltage. Wiggle Johnnie's foot and he picks up charges from the carpet. Bring his hand close to the door knob and get rid of the excess charge.

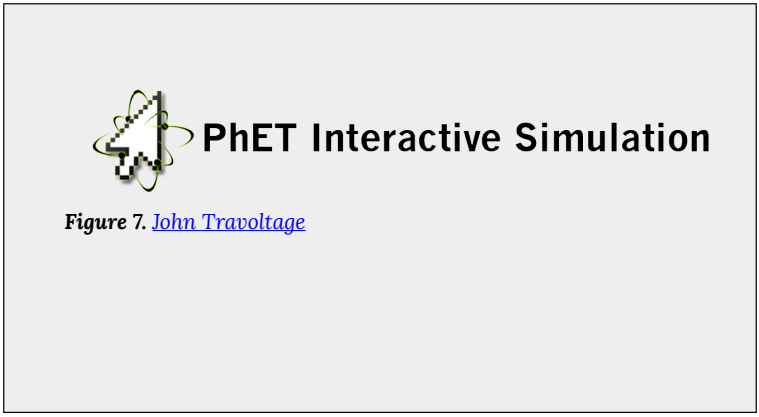



***Figure 7.*** *John Travoltage*

## Section Summary

- Polarization is the separation of positive and negative charges in a neutral object.
- A conductor is a substance that allows charge to flow freely through its atomic structure.
- An insulator holds charge within its atomic structure.
- Objects with like charges repel each other, while those with unlike charges attract each other.
- A conducting object is said to be grounded if it is connected to the Earth through a conductor. Grounding allows transfer of charge to and from the earth's large reservoir.
- Objects can be charged by contact with another charged object and obtain the same sign charge.
- If an object is temporarily grounded, it can be charged by induction, and obtains the opposite sign charge.
- Polarized objects have their positive and negative charges concentrated in different areas, giving them a non-symmetrical charge.
- Polar molecules have an inherent separation of charge.

## *Conceptual Questions*

**1:** An eccentric inventor attempts to levitate by first placing a large negative charge on himself and then putting a large positive charge on the ceiling of his workshop. Instead, while attempting to place a large negative charge on himself, his clothes fly off. Explain.

**2:** If you have charged an electroscope by contact with a positively charged object, describe how you could use it to determine the charge of other objects. Specifically, what would the leaves of the electroscope do if other charged objects were brought near its knob?

**3:** When a glass rod is rubbed with silk, it becomes positive and the silk becomes negative—yet both attract dust. Does the dust have a third type of charge that is attracted to both positive and negative? Explain.

**4:** Why does a car always attract dust right after it is polished? (Note that car wax and car tires are insulators.)

**5:** Describe how a positively charged object can be used to give another object a negative charge. What is the name of this process?

**6:** What is grounding? What effect does it have on a charged conductor? On a charged insulator?

## *Problems & Exercises*

**1:** Suppose a speck of dust in an electrostatic precipitator has $\mathbf{1.0000 \times 10^{12}}$ protons in it and has a net charge of −5.00 nC (a very large charge for a small speck). How many electrons does it have?

**2:** An amoeba has $\mathbf{1.00 \times 10^{16}}$ protons and a net charge of 0.300 pC. (a) How many fewer electrons are there than protons? (b) If you paired them up, what fraction of the protons would have no electrons?

**3:** A 50.0 g ball of copper has a net charge of $\mathbf{2.00\ \mu C}$. What fraction of the copper's electrons has been removed? (Each copper atom has 29 protons, and copper has an atomic mass of 63.5.)

**4:** What net charge would you place on a 100 g piece of sulfur if you put an extra electron on 1 in $\mathbf{10^{12}}$ of its atoms? (Sulfur has an atomic mass of 32.1.)

**5:** How many coulombs of positive charge are there in 4.00 kg of plutonium, given its atomic mass is 244 and that each plutonium atom has 94 protons?

## Glossary

**free electron**
an electron that is free to move away from its atomic orbit

**conductor**
a material that allows electrons to move separately from their

atomic orbits

**insulator**
: a material that holds electrons securely within their atomic orbits

**grounded**
: when a conductor is connected to the Earth, allowing charge to freely flow to and from Earth's unlimited reservoir

**induction**
: the process by which an electrically charged object brought near a neutral object creates a charge in that object

**polarization**
: slight shifting of positive and negative charges to opposite sides of an atom or molecule

**electrostatic repulsion**
: the phenomenon of two objects with like charges repelling each other

## *Solutions*

**Check Your Understanding**

Answer

Water molecules are polarized, giving them slightly positive and slightly negative sides. This makes water even more susceptible to a charged rod's attraction. As the water flows downward, due to the force of gravity, the charged conductor exerts a net attraction to the opposite charges in the stream of water, pulling it closer.

**Problems & Exercises**

$$\mathbf{1.03 \times 10^{12}}$$

$$\mathbf{9.09 \times 10^{-13}}$$

$$1.48 \times 10^8 \text{ C}$$

# 69. 12.3 Coulomb's Law

## *Summary*

- State Coulomb's law in terms of how the electrostatic force changes with the distance between two objects.
- Calculate the electrostatic force between two charged point forces, such as electrons or protons.
- Compare the electrostatic force to the gravitational attraction for a proton and an electron; for a human and the Earth.

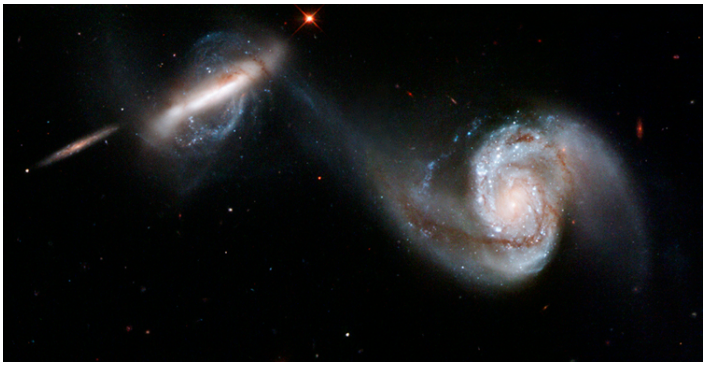


***Figure 1.*** *This NASA image of Arp 87 shows the result of a strong gravitational attraction between two galaxies. In contrast, at the subatomic level, the electrostatic attraction between two objects, such as an electron and a proton, is far greater than their mutual attraction due to gravity. (credit: NASA/HST)*

Through the work of scientists in the late 18th century, the main features of the **electrostatic force**—the existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called **Coulomb's law** after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.

### Coulomb's Law

$$F = k\frac{|q_1 q_2|}{r2}.$$

Coulomb's law calculates the magnitude of the force FF between two point charges,

$q_1$ and $q_2$, separated by a distance $r$. In SI units, the constant $k$ is equal to

$$k = 8.988 \times 10^9 \frac{N \cdot m^2}{C^2}$$
$$\approx 8.99 \times 10^9 \frac{N \cdot m^2}{C^2}.$$

The electrostatic force is a vector quantity and is expressed in units of newtons. The force is understood to be along the line joining the two charges. (See Figure 2.)

Although the formula for Coulomb's law is simple, it was no mean

task to prove it. The experiments Coulomb did, with the primitive equipment then available, were difficult. Modern experiments have verified Coulomb's law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared $\left(F \propto 1/r^2\right)$ to an accuracy of 1 part in $10^{16}$. No exceptions have ever been found, even at the small distances within the atom.

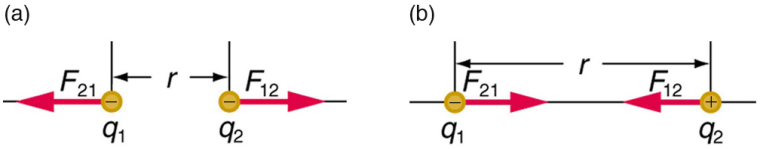



***Figure 2.*** *The magnitude of the electrostatic force* **F** *between point charges* $\mathbf{q}_1$ *and* $\mathbf{q}_2$ *separated by a distance* **r** *is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on* $\mathbf{q}_1$ *is equal in magnitude and opposite in direction to the force it exerts on* $\mathbf{q}_2$*. (a) Like charges. (b) Unlike charges.*

## How Strong is the Coulomb Force Relative to the Gravitational Force?

Compare the electrostatic force between an electron and proton separated by $\mathbf{0.530 \times 10^{-10}\ m}$ with the gravitational force between them. This distance is their average separation in a hydrogen atom.

**Strategy**

To compare the two forces, we first compute the

electrostatic force using Coulomb's law, $F = k \frac{|q_1 q_2|}{r2}$. We then calculate the gravitational force using Newton's universal law of gravitation. Finally, we take a ratio to see how the forces compare in magnitude.

**Solution**

Entering the given and known information about the charges and separation of the electron and proton into the expression of Coulomb's law yields

$$F = k \frac{|q_1 q_2|}{r2}$$
$$= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{(1.60 \times 10^{-19} \text{ C})(1.60 \times 10^{-19} \text{ C})}{(0.530 \times 10^{-10} \text{ m})^2}$$

Thus the Coulomb force is

$$F = 8.19 \times 10^{-8} \text{ N}.$$

The charges are opposite in sign, so this is an attractive force. This is a very large force for an electron–it would cause an acceleration of $8.99 \times 10^{22} \text{ m/s}^2$ (verification is left as an end-of-section problem).The gravitational force is given by Newton's law of gravitation as:

$$F_G = G \frac{mM}{r^2},$$

where $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$.

Here $m$ and $M$ represent the electron and proton masses, which can be found in the appendices. Entering values for the knowns yields

$$F_G = (6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2) \times \frac{(9.11 \times 10^{-31}\ \text{kg})(1.67 \times 10^{-27}\ \text{kg})}{(0.530 \times 10^{-10}\ \text{m})^2}$$
$$= 3.61 \times 10^{-47}\ \text{N}$$

This is also an attractive force, although it is traditionally shown as positive since gravitational force is always attractive. The ratio of the magnitude of the electrostatic force to gravitational force in this case is, thus,

$$\frac{F}{F_G} = 2.27 \times 10^{39}.$$

**Discussion**

This is a remarkably large ratio! Note that this will be the ratio of electrostatic force to gravitational force for an electron and a proton at any distance (taking the ratio before entering numerical values shows that the distance cancels). This ratio gives some indication of just how much larger the Coulomb force is than the gravitational force between two of the most common particles in nature.

As the example implies, gravitational force is completely negligible on a small scale, where the interactions of individual charged particles are important. On a large scale, such as between the Earth and a person, the reverse is true. Most objects are nearly electrically neutral, and so attractive and repulsive **Coulomb forces** nearly

cancel. Gravitational force on a large scale dominates interactions between large objects because it is always attractive, while Coulomb forces tend to cancel.

## Section Summary

- Frenchman Charles Coulomb was the first to publish the mathematical equation that describes the electrostatic force between two objects.
- Coulomb's law gives the magnitude of the force between point charges. It is

$$F = k\frac{|q_1 q_2|}{r2},$$

  where $q_1$ and $q_2$ are two point charges separated by a distance $r$, and

$$k \approx 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

- This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.
- The Coulomb force is extraordinarily strong compared with the gravitational force, another basic force—but unlike gravitational force it can cancel, since it can be either attractive or repulsive.
- The electrostatic force between two subatomic particles is far greater than the gravitational force between the same two particles.

## *Conceptual Questions*

**1:** Figure 3 shows the charge distribution in a water molecule, which is called a polar molecule because it has an inherent separation of charge. Given water's polar character, explain what effect humidity has on removing excess charge from objects.

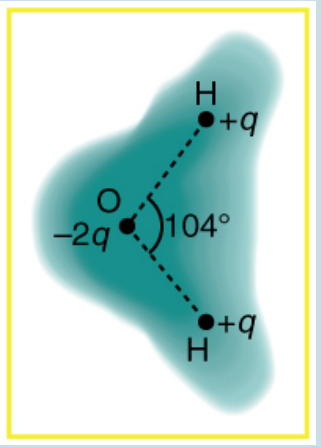



***Figure 3.*** *Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a polar molecule. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.*

**2:** Using Figure 3, explain, in terms of Coulomb's law, why a polar molecule (such as in Figure 3) is attracted by both positive and negative charges.

**3:** Given the polar character of water molecules, explain how ions in the air form nucleation centers for rain droplets.

## *Problems & Exercises*

**1:** What is the repulsive force between two pith balls that are 8.00 cm apart and have equal charges of − 30.0 nC?

**2:** (a) How strong is the attractive force between a glass rod with a 0.700μC charge and a silk cloth with a −0.600μC charge, which are 12.0 cm apart, using the approximation that they act like point charges? (b) Discuss how the answer to this problem might be affected if the charges are distributed over some area and do not act like point charges.

**3:** Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three?

**4:** Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?

**5:** How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them?

**6:** If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them? You will see that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.

**7:** A test charge of $+2\ \mu\text{C}$ is placed halfway between a charge of $+6\ \mu\text{C}$ and another of $+4\ \mu\text{C}$ separated by 10 cm. (a) What is the magnitude of the force on the test charge? (b) What is the direction of this force (away from or toward the $+6\ \mu\text{C}$ charge)?

**8:** Bare free charges do not remain stationary when close together. To illustrate this, calculate the acceleration of two isolated protons separated by 2.00 nm (a typical distance between gas atoms). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

**9:** (a) By what factor must you change the distance between two point charges to change the force between them by a factor of 10? (b) Explain how the distance can either increase or decrease by this factor and still cause a factor of 10 change in the force.

**10:** Suppose you have a total charge $q_{total}$ that you can split in any manner. Once split, the separation distance is fixed. How do you split the charge to achieve the greatest force?

**11:** (a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of

a 10.0 mg piece of tape held 1.00 cm above another. (b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.

**12:** (a) Find the ratio of the electrostatic to gravitational force between two electrons. (b) What is this ratio for two protons? (c) Why is the ratio different for electrons and protons?

**13:** At what distance is the electrostatic force between two protons equal to the weight of one proton?

**14:** A certain five cent coin contains 5.00 g of nickel. What fraction of the nickel atoms' electrons, removed and placed 1.00 m above it, would support the weight of this coin? The atomic mass of nickel is 58.7, and each nickel atom contains 28 electrons and 28 protons.

**15:** (a) Two point charges totaling $8.00\ \mu\text{C}$ exert a repulsive force of 0.150 N on one another when separated by 0.500 m. What is the charge on each? (b) What is the charge on each if the force is attractive?

**16:** Point charges of $5.00\ \mu\text{C}$ and $-3.00\ \mu\text{C}$ are placed 0.250 m apart. (a) Where can a third charge be placed so that the net force on it is zero? (b) What if both charges are positive?

**17:** Two point charges $q_1$ and $q_2$ are 3.00 m apart, and their total charge is $20\ \mu\text{C}$. (a) If the force of repulsion between them is 0.075N, what are magnitudes of the two charges? (b) If one charge attracts the other with a force of 0.525N, what are the magnitudes of the two charges? Note that you may need to solve a quadratic equation to reach your answer.

## Glossary

**Coulomb's law**
: the mathematical equation calculating the electrostatic force vector between two charged particles

**Coulomb force**
: another term for the electrostatic force

**electrostatic force**
: the amount and direction of attraction or repulsion between two charged bodies

## *Solutions*

**2:**

(a) 0.263 N

(b) If the charges are distributed over some area, there will be a concentration of charge along the side closest to the oppositely charged object. This effect will increase the net force.

**4:**

The separation decreased by a factor of 5.

$$F \quad k\frac{|q_1 q_2|}{r_2} = ma \Rightarrow a = \frac{kq^2}{mr^2}$$

$$\frac{(9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{m})^2}{(1.67\times10^{-27}\ \text{kg})(2.00\times10^{-9}\ \text{m})^2}$$

$$3.45 \times 10^{16}\ \text{m/s}^2$$

**9:**

(a) 3.2

(b) If the distance increases by 3.2, then the force will decrease by a factor of 10 ; if the distance decreases by 3.2, then the force will increase by a factor of 10. Either way, the force changes by a factor of 10.

**11:**

(a) $\mathbf{1.04 \times 10^{-9}\ C}$

(b) This charge is approximately 1 nC, which is consistent with the magnitude of charge typical for static electricity

**14:**

$\mathbf{1.02 \times 10^{-11}}$

**16:**

(a). 0.859 m beyond negative charge on line connecting two charges
(b). 0.109 m from lesser charge on line connecting two charges

# 70. 12.4 Electric Field: Concept of a Field Revisited

*Summary*

- Describe a force field and calculate the strength of an electric field due to a point charge.
- Calculate the force exerted on a test charge by an electric field.
- Explain the relationship between electrical force (F) on a test charge and electrical field strength (E).

Contact forces, such as between a baseball and a bat, are explained on the small scale by the interaction of the charges in atoms and molecules in close proximity. They interact through forces that include the **Coulomb force**. Action at a distance is a force between objects that are not close enough for their atoms to "touch." That is, they are separated by more than a few atomic diameters.

For example, a charged rubber comb attracts neutral bits of paper from a distance via the Coulomb force. It is very useful to think of an object being surrounded in space by a **force field**. The force field carries the force to another object (called a test object) some distance away.

## Concept of a Field

A field is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding the earth (and all other masses) represents the gravitational force that would be experienced if another mass were placed at a given point within the field.

In the same way, the Coulomb force field surrounding any charge extends throughout space. Using Coulomb's law, $F = k|q_1 q_2|/r^2$, its magnitude is given by the equation $F = k|qQ|/r^2$, for a **point charge** (a particle having a charge $Q$) acting on a **test charge** $q$ at a distance $r$ (see [link]). Both the magnitude and direction of the Coulomb force field depend on $Q$ and the test charge $q$.

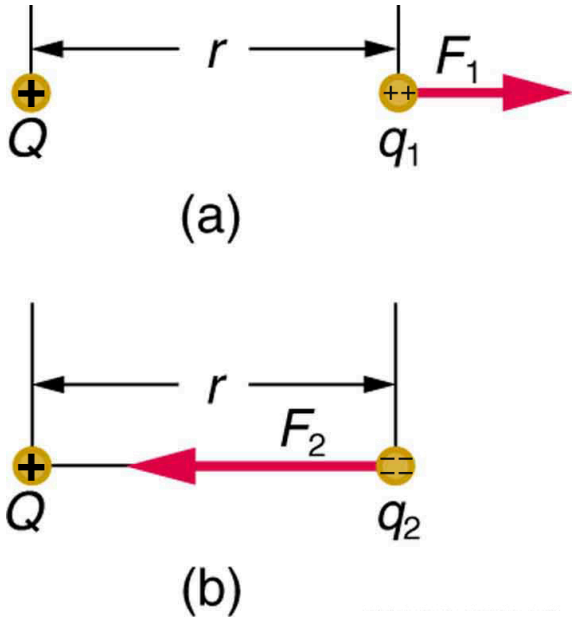



***Figure 1.*** *The Coulomb force field due to a positive charge* **Q** *is shown acting on two different charges. Both charges are the same distance from* **Q**. *(a) Since* $q_1$ *is positive, the force* **F1** *acting on it is repulsive. (b) The charge* $q_2$ *is negative and greater in magnitude than* $q_1$, *and so the force* $F_2$ *acting on it is attractive and stronger than* $F_1$. *The Coulomb force field is thus not unique at any point in space, because it depends on the test charges* $q_1$ *and* $q_2$ *as well as the charge* **Q**.

To simplify things, we would prefer to have a field that depends only on $Q$ and not on the test charge $q$. The electric field is defined in such a manner that it represents only the charge creating it and is unique at every point in space. Specifically, the electric field $E$ is defined to be the ratio of the Coulomb force to the test charge:

$$E = \frac{F}{q},$$

where $F$ is the electrostatic force (or Coulomb force) exerted on a

positive test charge $q$. It is understood that $E$ is in the same direction as $F$. It is also assumed that $q$ is so small that it does not alter the charge distribution creating the electric field. The units of electric field are newtons per coulomb (N/C). If the electric field is known, then the electrostatic force on any charge $q$ is simply obtained by multiplying charge times electric field, or $\mathbf{F} = q\mathbf{E}$. Consider the electric field due to a point charge $Q$. According to Coulomb's law, the force it exerts on a test charge $q$ is $F = k|qQ|/r^2$. Thus the magnitude of the electric field, $E$, for a point charge is

$$E = \left|\frac{F}{q}\right| = k\left|\frac{qQ}{qr^2}\right| = k\frac{|Q|}{r^2}.$$

Since the test charge cancels, we see that

$$E = k\frac{|Q|}{r^2}$$

The electric field is thus seen to depend only on the charge $Q$ and the distance $r$; it is completely independent of the test charge $q$.

## Example 1: Calculating the Electric Field of a Point Charge

Calculate the strength and direction of the electric field $E$ due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.

**Strategy**

We can find the electric field created by a point charge by using the equation $\boldsymbol{E = kQ/r^2}$.

**Solution**

Here $\boldsymbol{Q = 2.00 \times 10^{-9}\ \text{C}}$ and $\boldsymbol{r = 5.00 \times 10^{-3}\ \text{m}}$. Entering those values into the above equation gives

$$\boldsymbol{E} \quad \boldsymbol{k\frac{Q}{r^2}}$$

$$\boldsymbol{(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2) \times \frac{(2.00 \times 10^{-9}\ \text{C})}{(5.00 \times 10^{-3}\ \text{m})^2}}$$

$$\boldsymbol{7.19 \times 10^5\ \text{N/C}.}$$

**Discussion**

This **electric field strength** is the same at any point 5.00 mm away from the charge $\boldsymbol{Q}$ that creates the field. It is positive, meaning that it has a direction pointing away from the charge $\boldsymbol{Q}$.

## Example 2: Calculating the Force Exerted on a Point Charge by an Electric Field

What force does the electric field found in the previous example exert on a point charge of $-0.250\ \mu\text{C}$?

**Strategy**

Since we know the electric field strength and the charge in the field, the force on that charge can be calculated using the definition of electric field $\mathbf{E} = \mathbf{F}/q$ rearranged to $\mathbf{F} = q\mathbf{E}$.

**Solution**

The magnitude of the force on a charge $q = -0.250\ \mu\text{C}$ exerted by a field of strength $E = 7.20 \times 10^5$ N/C is thus,

$$F = qE$$

$$(0.250 \times 10^{-6}\ \text{C})(7.20 \times 10^5\ \text{N/C})$$

$$0.180\ \text{N}.$$

Because $q$ is negative, the force is directed opposite to the direction of the field.

**Discussion**

The force is attractive, as expected for unlike charges.

(The field was created by a positive charge and here acts on a negative charge.) The charges in this example are typical of common static electricity, and the modest attractive force obtained is similar to forces experienced in static cling and similar situations.

## PhET Explorations: Electric Field of Dreams

Play ball! Add charges to the Field of Dreams and see how they react to the electric field. Turn on a background electric field and adjust the direction and magnitude.

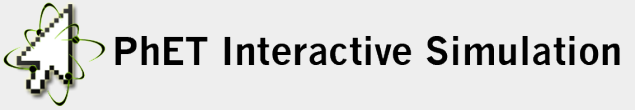



***Figure 2.*** *Electric Field of Dreams*

## Section Summary

- The electrostatic force field surrounding a charged object extends out into space in all directions.
- The electrostatic force exerted by a point charge on a test charge at a distance $r$ depends on the charge of both charges, as well as the distance between the two.
- The electric field $\mathbf{E}$ is defined to be

$$E = \frac{\mathbf{F}}{q},$$

where $\mathbf{F}$ is the Coulomb or electrostatic force exerted on a small positive test charge $q$. $\mathbf{E}$ has units of N/C.

- The magnitude of the electric field $\mathbf{E}$ created by a point charge $Q$ is

$$\mathbf{E} = k\frac{|Q|}{r^2}.$$

where $r$ is the distance from $Q$. The electric field $E$ is a vector and fields due to multiple charges add like vectors.

### *Conceptual Questions*

**1:** Why must the test charge $q$ in the definition of the electric field be vanishingly small?

**2:** Are the direction and magnitude of the Coulomb force unique at a given point in space? What about the electric field?

## *Problem Exercises*

**1:** What is the magnitude and direction of an electric field that exerts a $\mathbf{2.00 \times 10^{-5}\ N}$ upward force on a $\mathbf{-1.75\ \mu C}$ charge?

**2:** What is the magnitude and direction of the force exerted on a $\mathbf{3.50\ \mu C}$ charge by a 250 N/C electric field that points due east?

**3:** Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 mC (such as found on the terminal of a Van de Graaff).

**4:** (a) What magnitude point charge creates a 10,000 N/C electric field at a distance of 0.250 m? (b) How large is the field at 10.0 m?

**5:** Calculate the initial (from rest) acceleration of a proton in a $\mathbf{5.00 \times 10^{6}\ N/C}$ electric field (such as created by a research Van de Graaff). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

**6:** (a) Find the direction and magnitude of an electric field that exerts a $\mathbf{4.80 \times 10^{-17}\ N}$ westward force on an electron. (b) What magnitude and direction force does this field exert on a proton?

## Glossary

**field**
: a map of the amount and direction of a force acting on other objects, extending out into space

**point charge**
: A charged particle, designated $Q$, generating an electric field

**test charge**
: A particle (designated $q$) with either a positive or negative charge set down within an electric field generated by a point charge

### *Problem Exercises*

**2:** $\mathbf{8.75 \times 10{-}4\ N}$

**4:**

(a) $\mathbf{6.94 \times 10^{-8}\ C}$

(b) $\mathbf{6.25\ N/C}$

**6:**

(a) 300 N/C (east)

(b) $\mathbf{4.80 \times 10^{-17}\ N\ (east)}$

# 71. 12.5 Electric Field Lines: Multiple Charges

## *Summary*

- Calculate the total force (magnitude and direction) exerted on a test charge from more than one charge
- Describe an electric field diagram of a positive point charge; of a negative point charge with twice the magnitude of positive charge
- Draw the electric field lines between two points of the same charge; between two points of opposite charge.

Drawings using lines to represent electric fields around charged objects are very useful in visualizing field strength and direction. Since the electric field has both magnitude and direction, it is a vector. Like all vectors, the electric field can be represented by an arrow that has length proportional to its magnitude and that points in the correct direction. (We have used arrows extensively to represent force vectors, for example.)

Figure 1 shows two pictorial representations of the same electric field created by a positive point charge $Q$. Figure 1 (b) shows the standard representation using continuous lines. Figure 1 (b) shows numerous individual arrows with each arrow representing the force on a test charge $q$. Field lines are essentially a map of infinitesimal force vectors.

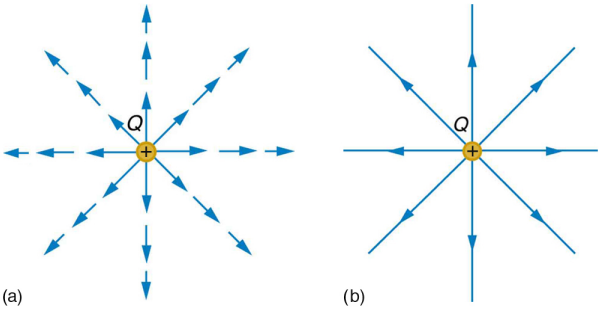



***Figure 1.*** *Two equivalent representations of the electric field due to a positive charge* **Q**. *(a) Arrows representing the electric field's magnitude and direction. (b) In the standard representation, the arrows are replaced by continuous field lines having the same direction at any point as the electric field. The closeness of the lines is directly related to the strength of the electric field. A test charge placed anywhere will feel a force in the direction of the field line; this force will have a strength proportional to the density of the lines (being greater near the charge, for example).*

Note that the electric field is defined for a positive test charge $q$, so that the field lines point away from a positive charge and toward a negative charge. (See Figure 2.) The electric field strength is exactly proportional to the number of field lines per unit area, since the magnitude of the electric field for a point charge is $E = k|Q|/r^2$ and area is proportional to $r^2$. This pictorial representation, in which field lines represent the direction and their closeness (that is, their areal density or the number of lines crossing a unit area) represents strength, is used for all fields: electrostatic, gravitational, magnetic, and others.

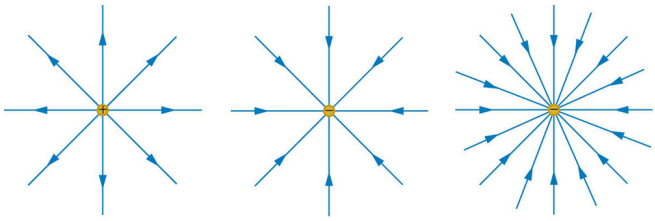



***Figure 2.*** *The electric field surrounding three different point charges. (a) A positive charge. (b) A negative charge of equal magnitude. (c) A larger negative charge.*

In many situations, there are multiple charges. The total electric field created by multiple charges is the vector sum of the individual fields created by each charge. The following example shows how to add electric field vectors.

## Example 1: Adding Electric Fields

Find the magnitude and direction of the total electric field due to the two point charges, $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$, at the origin of the coordinate system as shown in Figure 3.

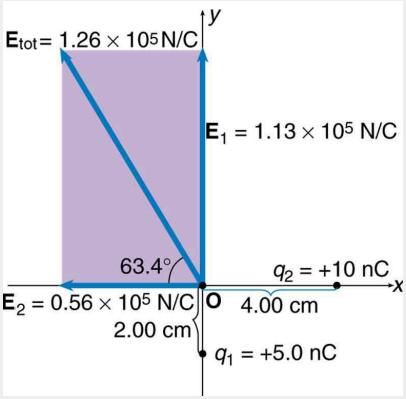



***Figure 3.*** *The electric fields* $\mathbf{E_1}$ *and* $\mathbf{E_2}$ *at the origin O add to* $\mathbf{E_{tot}}$.

**Strategy**

Since the electric field is a vector (having magnitude and direction), we add electric fields with the same vector techniques used for other types of vectors. We first must find the electric field due to each charge at the point of interest, which is the origin of the coordinate system (O) in this instance. We pretend that there is a positive test charge, $\boldsymbol{q}$, at point O, which allows us to determine the direction of the fields $\mathbf{E_1}$ and $\mathbf{E_2}$. Once those fields are found, the total field can be determined using vector addition.

**Solution**

The electric field strength at the origin due to $\boldsymbol{q_1}$ is labeled $\boldsymbol{E_1}$ and is calculated:

$$E_1 \quad k\frac{q_1}{r_1^2} = (8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\frac{5.00 \times 10^{-9}\ \text{C}}{2.00 \times 10^{-2}\ \text{m}^2}$$

$$E_1 \quad 1.124 \times 10^5\ \text{N/C}.$$

Similarly, $E_2$ is

$$E_2 \quad k\frac{q_2}{r_2^2} = (8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\frac{10.0 \times 10^{-9}\ \text{C}}{4.00 \times 10^{-2}\ \text{m}^2}$$

$$E_1 \quad 0.5619 \times 10^5\ \text{N/C}.$$

Four digits have been retained in this solution to illustrate that $E_1$ is exactly twice the magnitude of $E_2$. Now arrows are drawn to represent the magnitudes and directions of $E_1$ and $E_2$. (See Figure 3.) The direction of the electric field is that of the force on a positive charge so both arrows point directly away from the positive charges that create them. The arrow for $E_1$ is exactly twice the length of that for $E_2$. The arrows form a right triangle in this case and can be added using the Pythagorean theorem. The magnitude of the total field $E_{\text{tot}}$ is

$$E_{\text{tot}} \quad (E_1^2 + E_2^2)^{1/2}$$

$$\{(1.124 \times 10^5\ \text{N/C})^2 + (0.5619 \times 10^5\ \text{N/C})^2\}^{1/2}$$

$$1.26 \times 10^5\ \text{N/C}.$$

The direction is

$$\theta \quad \tan^{-1}\left(\frac{E_1}{E_2}\right)$$

$$\tan^{-1}\left(\frac{1.124\times10^{5}\ \text{N/C}}{0.5619\times10^{5}\ \text{N/C}}\right)$$

$$63.4^{\circ}$$

or $63.4^{\circ}$ above the $x$-axis.

**Discussion**

In cases where the electric field vectors to be added are not perpendicular, vector components or graphical techniques can be used. The total electric field found in this example is the total electric field at only one point in space. To find the total electric field due to these two charges over an entire region, the same technique must be repeated for each point in the region. This impossibly lengthy task (there are an infinite number of points in space) can be avoided by calculating the total field at representative points and using some of the unifying features noted next.

Figure 4 shows how the electric field from two point charges can be drawn by finding the total field at representative points and drawing electric field lines consistent with those points. While the electric fields from multiple charges are more complex than those of single charges, some simple features are easily noticed.

For example, the field is weaker between like charges, as shown by the lines being farther apart in that region. (This is because the fields from each charge exert opposing forces on any charge placed between them.) (See Figure 4 and Figure 5(a).) Furthermore, at a

great distance from two like charges, the field becomes identical to the field from a single, larger charge.

Figure 5(b) shows the electric field of two unlike charges. The field is stronger between the charges. In that region, the fields from each charge are in the same direction, and so their strengths add. The field of two unlike charges is weak at large distances, because the fields of the individual charges are in opposite directions and so their strengths subtract. At very large distances, the field of two unlike charges looks like that of a smaller single charge.

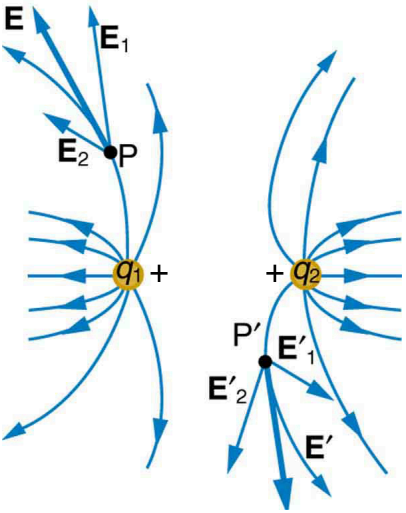



***Figure 4.*** *Two positive point charges* $q_1$ *and* $q_2$ *produce the resultant electric field shown. The field is calculated at representative points and then smooth field lines drawn following the rules outlined in the text.*

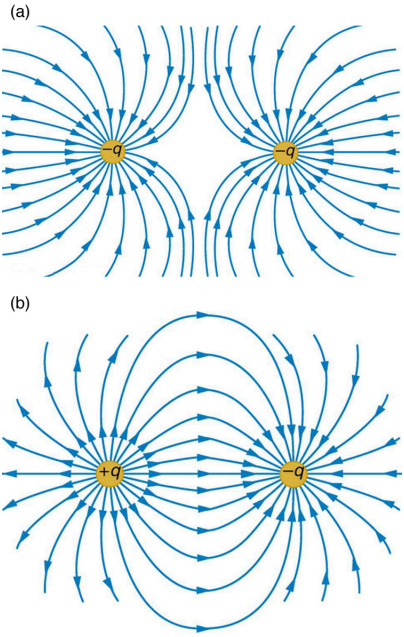



***Figure 5.*** *(a) Two negative charges produce the fields shown. It is very similar to the field produced by two positive charges, except that the directions are reversed. The field is clearly weaker between the charges. The individual forces on a test charge in that region are in opposite directions. (b) Two opposite charges produce the field shown, which is stronger in the region between the charges.*

We use electric field lines to visualize and analyze electric fields (the lines are a pictorial tool, not a physical entity in themselves). The properties of electric field lines for any charge distribution can be summarized as follows:

1. Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of

isolated charges.
2. The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
3. The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
4. The direction of the electric field is tangent to the field line at any point in space.
5. Field lines can never cross.

The last property means that the field is unique at any point. The field line represents the direction of the field; so if they crossed, the field would have two directions at that location (an impossibility if the field is unique).

PhET Explorations: Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.



PhET Interactive Simulation

***Figure 6.*** *Charges and Fields*

## Section Summary

- Drawings of electric field lines are useful visual tools. The properties of electric field lines for any charge distribution are that:
- Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- The direction of the electric field is tangent to the field line at any point in space.
- Field lines can never cross.

### *Conceptual Questions*

**1:** Compare and contrast the Coulomb force field and the electric field. To do this, make a list of five properties for the Coulomb force field analogous to the five properties listed for electric field lines. Compare each item in your list of Coulomb force field properties with those of the electric field—are they the same or different? (For example, electric field lines cannot cross. Is the same true for Coulomb field lines?)

**2:** Figure 7 shows an electric field extending over three regions, labeled I, II, and III. Answer the following

questions. (a) Are there any isolated charges? If so, in what region and what are their signs? (b) Where is the field strongest? (c) Where is it weakest? (d) Where is the field the most uniform?

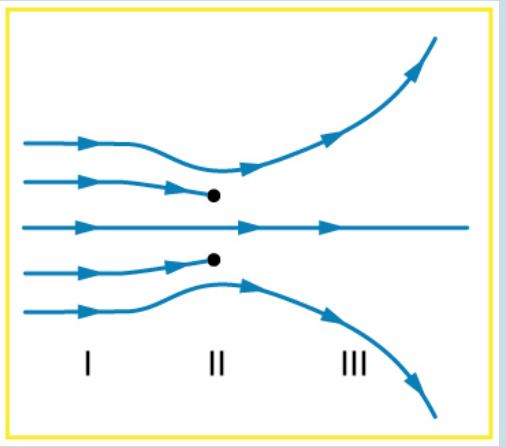



***Figure 7.***

## *Problem Exercises*

**1:** (a) Sketch the electric field lines near a point charge $+q$. (b) Do the same for a point charge $-3.00q$

**2:** Sketch the electric field lines a long distance from the charge distributions shown in Figure 5 (a) and (b)

**3:** Figure 8 shows the electric field lines near two charges

$q_1$ and $q_2$. What is the ratio of their magnitudes? (b) Sketch the electric field lines a long distance from the charges shown in the figure.

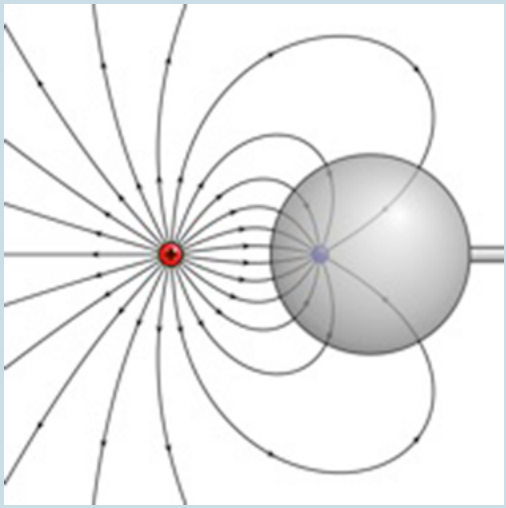


***Figure 8.*** *The electric field near two charges.*

**4:** Sketch the electric field lines in the vicinity of two opposite charges, where the negative charge is three times greater in magnitude than the positive. (See Figure 8 for a similar situation).

## Glossary

**electric field**
: a three-dimensional map of the electric force extended out into space from a point charge

**electric field lines**
: a series of lines drawn from a point charge representing the magnitude and direction of force exerted by that charge

**vector**
: a quantity with both magnitude and direction

**vector addition**
: mathematical combination of two or more vectors, including their magnitudes, directions, and positions

# 72. 12.6 Applications of Electrostatics

*Summary*

- Name several real-world applications of the study of electrostatics.

The study of electrostatics has proven useful in many areas. This module covers just a few of the many applications of electrostatics.

## The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. Figure 1 shows a schematic of a large research version. Van de Graaffs utilize both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere, because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess

charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.

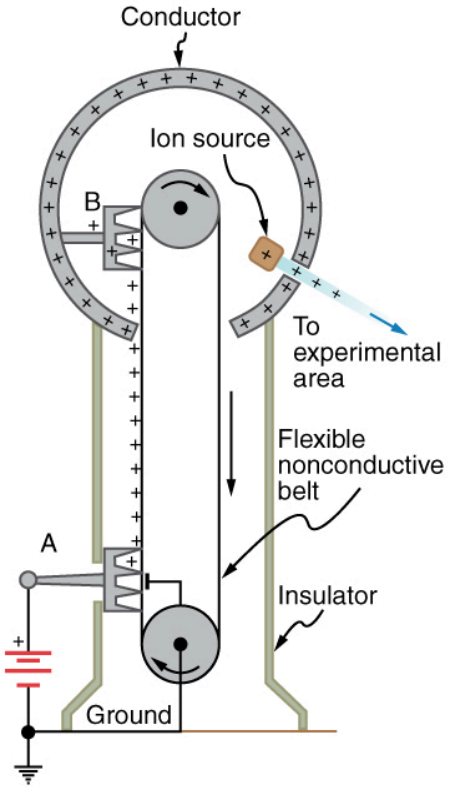



***Figure 1.*** *Schematic of Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.*

### Take-Home Experiment: Electrostatics and Humidity

Rub a comb through your hair and use it to lift pieces of paper. It may help to tear the pieces of paper rather than cut them neatly. Repeat the exercise in your bathroom after you have had a long shower and the air in the bathroom is moist. Is it easier to get electrostatic effects in dry or moist air? Why would torn paper be more attractive to the comb than cut paper? Explain your observations.

## Xerography

Most copy machines use an electrostatic process called xerography—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in Figure 2.

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a photoconductor. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is grounded so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the

second stage, the surface of the drum is exposed to the image of whatever is to be copied. Where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, and so the image has been transferred to the drum.

The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it will be attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner within the fibers of the paper.

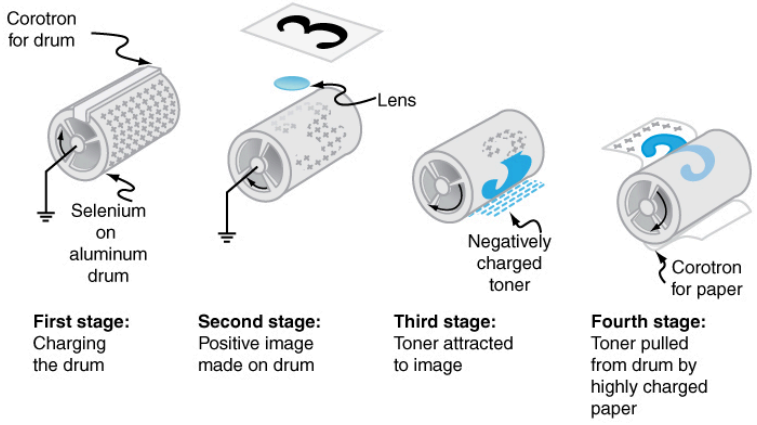



***Figure 2.*** *Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy.*

## Laser Printers

Laser printers use the xerographic process to make high-quality

images on paper, employing a laser to produce an image on the photoconducting drum as shown in Figure 3. In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and may contain a computer more powerful than the one giving them the raw data to be printed.

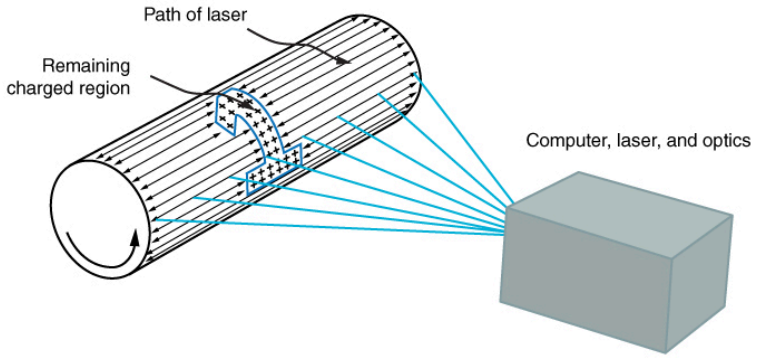



***Figure 3.*** *In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positive charge image. The other steps for charging the drum and transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images.*

## Ink Jet Printers and Electrostatic Painting

The ink jet printer, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge. (See Figure 4.)

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black

jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)

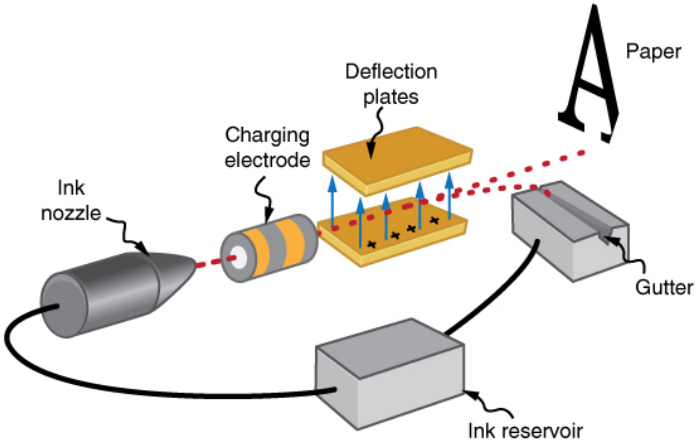



***Figure 4.*** *The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are then used to direct the droplets to the correct positions on a page.*

Electrostatic painting employs electrostatic charge to spray paint onto odd-shaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach those hard-to-get at places, applying an even coat in a controlled manner. If the object is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

## Smoke Precipitators and Electrostatic Air

# Cleaning

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles. (See Figure 5.)

Large electrostatic precipitators are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.

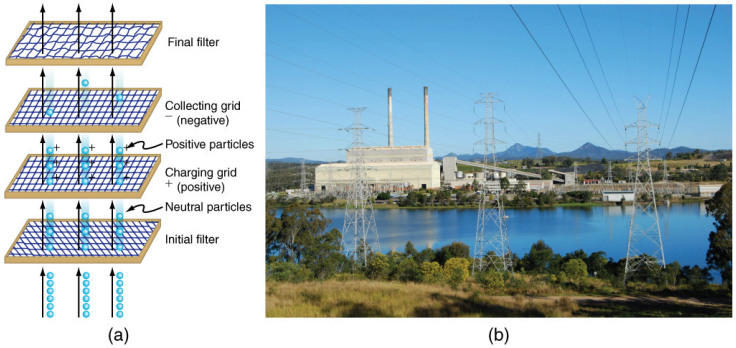



***Figure 5.*** *(a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of electrostatic precipitators is seen by the absence of smoke from this power plant. (credit: Cmdalgleish, Wikimedia Commons)*

## Problem-Solving Strategies for Electrostatics

1. Examine the situation to determine if static electricity is involved. This may concern separated stationary charges, the forces among them, and the electric fields they create.
2. Identify the system of interest. This includes noting the number, locations, and types of charges involved.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful. Determine whether the Coulomb force is to be considered directly—if so, it may be useful to draw a free-body diagram, using electric field lines.
4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is important to distinguish the Coulomb force $\boldsymbol{F}$ from the electric field $\boldsymbol{E}$, for example.
5. Solve the appropriate equation for the quantity to be determined (the unknown) or draw the field lines as requested.
6. Examine the answer to see if it is reasonable: Does it make sense? Are units correct and the numbers involved reasonable?

## Integrated Concepts

The Integrated Concepts exercises for this module involve concepts such as electric charges, electric fields, and several other topics. Physics is most interesting when applied to general situations involving more than a narrow set of physical principles. The electric field exerts force on charges, for example, so it can be included in force diagrams and net force calculations. The following worked example illustrates how this strategy is applied to an Integrated Concept problem:

### Example 1: Acceleration of a Charged Drop of Gasoline

If steps are not taken to ground a gasoline pump, static electricity can be placed on gasoline when filling your car's tank. Suppose a tiny drop of gasoline has a mass of $\mathbf{4.00 \times 10^{-15}\ kg}$ and is given a positive charge of $\mathbf{3.20 \times 10^{-19}\ C}$. (a) Find the weight of the drop. (b) Calculate the electric force on the drop if there is an upward electric field of strength $\mathbf{3.00 \times 10^{5}\ N/C}$ due to other static electricity in the vicinity. (c) Calculate the drop's acceleration.

**Strategy**

To solve an integrated concept problem, we must first identify the physical principles involved and identify the

chapters in which they are found. Part (a) of this example asks for weight. This is a topic of dynamics and is defined in Chapter 4 Dynamics: Force and Newton's Laws of Motion. Part (b) deals with electric force on a charge, a topic of Chapter 12 Electric Charge and Electric Field. Part (c) asks for acceleration, knowing forces and mass. These are part of Newton's laws, also found in Chapter 4 Dynamics: Force and Newton's Laws of Motion.

The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so on.

**Solution for (a)**

Weight is mass times the acceleration due to gravity, as first expressed in

$$w = mg.$$

Entering the given mass and the average acceleration due to gravity yields

$$w = (4.00 \times 10^{-15}\ \text{kg})(9.80\ \text{m/s}^2) = 3.92 \times 10^{-14}\ \text{N}.$$

**Discussion for (a)**

This is a small weight, consistent with the small mass of the drop.

**Solution for (b)**

The force an electric field exerts on a charge is given by rearranging the following equation:

$$\text{electric force} = F = q\,E$$

Here we are given the charge ($\mathbf{3.20 \times 10^{-19}\ C}$ is twice the fundamental unit of charge) and the electric field strength, and so the electric force is found to be

$$\boldsymbol{F = (3.20 \times 10^{-19}\ C)(3.00 \times 10^{5}\ N/C) = 9.60 \times 10^{-14}\ N.}$$

**Discussion for (b)**

While this is a small force, it is greater than the weight of the drop.

**Solution for (c)**

The acceleration can be found using Newton's second law, provided we can identify all of the external forces acting on the drop. We assume only the drop's weight and the electric force are significant. Since the drop has a positive charge and the electric field is given to be upward, the electric force is upward. We thus have a one-dimensional (vertical direction) problem, and we can state Newton's second law as

net force = ma = (mass) (acceleration)

where $\boldsymbol{F_{net} = F - w}$. Entering this and the known values into the expression for Newton's second law yields

$$\boldsymbol{a} \quad \frac{\boldsymbol{F - w}}{\boldsymbol{m}}$$

$$\frac{\mathbf{9.60 \times 10^{-14}\ N - 3.92 \times 10^{-14}\ N}}{\mathbf{4.00 \times 10^{-15}\ kg}}$$

$$\mathbf{14.2\ m/s^2.}$$

**Discussion for (c)**

This is an upward acceleration great enough to carry

the drop to places where you might not wish to have gasoline.

This worked example illustrates how to apply problem-solving strategies to situations that include topics in different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These are found throughout the text, and many worked examples show how to use them for single topics. In this integrated concepts example, you can see how to apply them across several topics. You will find these techniques useful in applications of physics outside a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

## Unreasonable Results

The Unreasonable Results exercises for this module have results that are unreasonable because some premise is unreasonable or because certain of the premises are inconsistent with one another. Physical principles applied correctly then produce unreasonable results. The purpose of these problems is to give

practice in assessing whether nature is being accurately described, and if it is not to trace the source of difficulty.

Problem-Solving Strategy

To determine if an answer is reasonable, and to determine the cause if it is not, do the following.

1. Solve the problem using strategies as outlined above. Use the format followed in the worked examples in the text to solve the problem as usual.
2. Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, and so on?
3. If the answer is unreasonable, look for what specifically could cause the identified difficulty. Usually, the manner in which the answer is unreasonable is an indication of the difficulty. For example, an extremely large Coulomb force could be due to the assumption of an excessively large separated charge.

## Section Summary

- Electrostatics is the study of electric fields in static equilibrium.
- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink-jet printers and electrostatic air filters.

## *Problems & Exercises*

**1:** (a) What is the electric field 5.00 m from the centre of the terminal of a Van de Graaff with a 3.00 mC charge, noting that the field is equivalent to that of a point charge at the centre of the terminal? (b) At this distance, what force does the field exert on a $\mathbf{2.00\ \mu C}$ charge on the Van de Graaff's belt?

**2:** (a) What is the direction and magnitude of an electric field that supports the weight of a free electron near the surface of Earth? (b) Discuss what the small value for this field implies regarding the relative strength of the gravitational and electrostatic forces.

**3:** A simple and common technique for accelerating electrons is shown in Figure 6, where there is a uniform electric field between two plates. Electrons are released, usually from a hot filament, near the negative plate, and there is a small hole in the positive plate that allows the electrons to continue moving. (a) Calculate the acceleration of the electron if the field strength is $\mathbf{2.50 \times 10^4\ N/C}$. (b) Explain why the electron will not be pulled back to the positive plate once it moves through the hole.

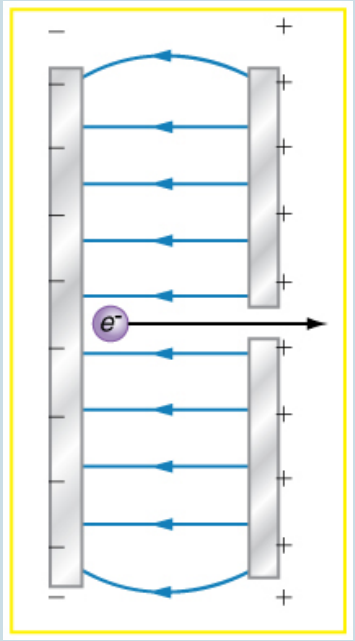



***Figure 6.*** *Parallel conducting plates with opposite charges on them create a relatively uniform electric field used to accelerate electrons to the right. Those that go through the hole can be used to make a TV or computer screen glow or to produce X-rays.*

**4:** Earth has a net charge that produces an electric field of approximately 150 N/C downward at its surface. (a) What is the magnitude and sign of the excess charge, noting the electric field of a conducting sphere is equivalent to a point charge at its centre? (b) What acceleration will the field produce on a free electron near Earth's surface? (c) What

mass object with a single extra electron will have its weight supported by this field?

**5:** Point charges of $\mathbf{25.0\ \mu C}$ and $\mathbf{45.0\ \mu C}$ are placed 0.500 m apart. (a) At what point along the line between them is the electric field zero? (b) What is the electric field halfway between them?

**6:** What can you say about two charges $q_1$ and $q_2$, if the electric field one-fourth of the way from $q_1$ to $q_2$ is zero?

## *Problems & Exercises*

**Integrated Concepts**

**1:** Calculate the angular velocity $\omega$ of an electron orbiting a proton in the hydrogen atom, given the radius of the orbit is $\mathbf{0.530 \times 10^{-10}\ m}$. You may assume that the proton is stationary and the centripetal force is supplied by Coulomb attraction.

**2:** An electron has an initial velocity of $\mathbf{5.00 \times 10^{6}\ m/s}$ in a uniform $\mathbf{2.00 \times 10^{5}\ N/C}$ strength electric field. The field accelerates the electron in the direction opposite to its initial velocity. (a) What is the direction of the electric field? (b) How far does the electron travel before coming to rest? (c) How long does it take the electron to come to rest? (d) What is the electron's velocity when it returns to its starting point?

**3:** The practical limit to an electric field in air is about $\mathbf{3.00 \times 10^6\ N/C}$. Above this strength, sparking takes place because air begins to ionize and charges flow, reducing the field. (a) Calculate the distance a free proton must travel in this field to reach 3.00% of the speed of light, starting from rest. (b) Is this practical in air, or must it occur in a vacuum? Remember that the speed of light = c= 3.00 x $10^8$ m/s

**4:** A 5.00 g charged insulating ball hangs on a 30.0 cm long string in a uniform horizontal electric field as shown in Figure 7. Given the charge on the ball is $\mathbf{1.00\ \mu C}$, find the strength of the field.

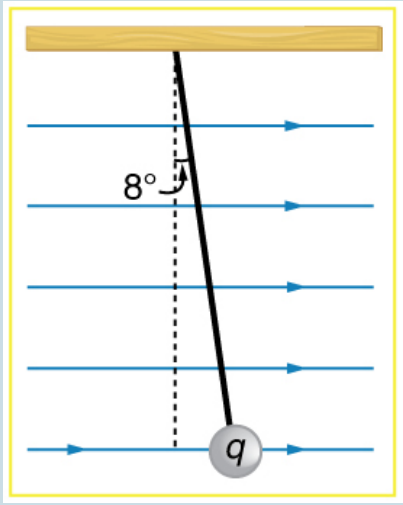



***Figure 7.*** *A horizontal electric field causes the charged ball to hang at an angle of 8.00°.*

**5:** Figure 8 shows an electron passing between two charged metal plates that create an 100 N/C vertical electric field perpendicular to the electron's original horizontal velocity. (These can be used to change the electron's direction, such as in an oscilloscope.) The initial speed of the electron is $\mathbf{3.00 \times 10^6\ m/s}$, and the horizontal distance it travels in the uniform field is 4.00 cm. (a) What is its vertical deflection? (b) What is the vertical component of its final velocity? (c) At what angle does it exit? Neglect any edge effects.

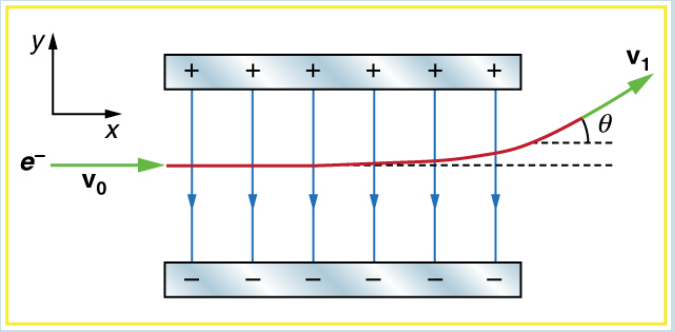



***Figure 8.***

**6:** The classic Millikan oil drop experiment was the first to obtain an accurate measurement of the charge on an electron. In it, oil drops were suspended against the gravitational force by a vertical electric field. (See Figure 9.) Given the oil drop to be $\mathbf{1.00\ \mu m}$ in radius and have a density of 920 kg/m$^3$: (a) Find the weight of the drop. (b) If the drop has a single excess electron, find the electric field strength needed to balance its weight. Millikan won the Nobel Prize in 1923 for this work.

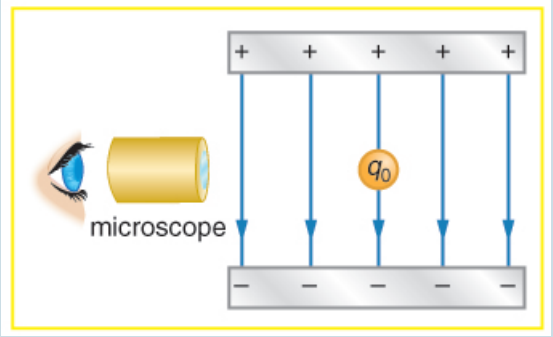



***Figure 9.*** *In the Millikan oil drop experiment, small drops can be suspended in an electric field by the force exerted on a single excess electron. Classically, this experiment was used to determine the electron charge* $\mathbf{q_e}$ *by measuring the electric field and mass of the drop.*

## *Unreasonable Results*

**1:** (a) Calculate the electric field strength near a 10.0 cm diameter conducting sphere that has 1.00 C of excess charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

**2:** (a) Two 0.500 g raindrops in a thunderhead are 1.00 cm apart when they each acquire 1.00 mC charges. Find their acceleration. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

**3:** A wrecking yard inventor wants to pick up cars by

charging a 0.400 m diameter ball and inducing an equal and opposite charge on the car. If a car has a 1000 kg mass and the ball is to be able to lift it from a distance of 1.00 m: (a) What minimum charge must be used? (b) What is the electric field near the surface of the ball? (c) Why are these results unreasonable? (d) Which premise or assumption is responsible?

## *Construct Your Own Problems*

**1:** Consider two insulating balls with evenly distributed equal and opposite charges on their surfaces, held with a certain distance between the centers of the balls. Construct a problem in which you calculate the electric field (magnitude and direction) due to the balls at various points along a line running through the centers of the balls and extending to infinity on either side. Choose interesting points and comment on the meaning of the field at those points. For example, at what points might the field be just that due to one ball and where does the field become negligibly small? Among the things to be considered are the magnitudes of the charges and the distance between the centers of the balls. Your instructor may wish for you to consider the electric field off axis or for a more complex array of charges, such as those in a water molecule.

## Glossary

**Van de Graaff generator**
: a machine that produces a large amount of excess charge, used for experiments with high voltage

**electrostatics**
: the study of electric forces that are static or slow-moving

**photoconductor**
: a substance that is an insulator until it is exposed to light, when it becomes a conductor

**xerography**
: a dry copying process based on electrostatics

**grounded**
: connected to the ground with a conductor, so that charge flows freely to and from the Earth to the grounded object

**laser printer**
: uses a laser to create a photoconductive image on a drum, which attracts dry ink particles that are then rolled onto a sheet of paper to print a high-quality copy of the image

**ink-jet printer**
: small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper

**electrostatic precipitators**
: filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

## *Solutions*

**Problems & Exercises**

2: (a) $\mathbf{5.58 \times 10^{-11}\ N/C}$

(b)the coulomb force is extraordinarily stronger than gravity

**4:** (a) $\mathbf{-6.76 \times 10^{5}\ C}$

(b) $\mathbf{2.63 \times 10^{13}\ m/s^{2}\ (upward)}$

(c) $\mathbf{2.45 \times 10^{-18}\ kg}$

**6:** The charge $q_2$ is 9 times greater than $q_1$.

# 73. 12.7 Optional: Electric Forces in Biology

## *Summary*

- Describe how a water molecule is polar.
- Explain electrostatic screening by a water molecule within a living cell.

Classical electrostatics has an important role to play in modern molecular biology. Large molecules such as proteins, nucleic acids, and so on—so important to life—are usually electrically charged. DNA itself is highly charged; it is the electrostatic force that not only holds the molecule together but gives the molecule structure and strength. Figure 1 is a schematic of the DNA double helix.

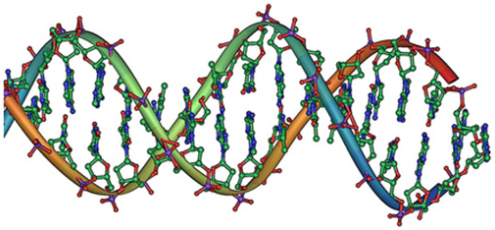


***Figure 1.*** *DNA is a highly charged molecule. The DNA double helix shows the two coiled strands each containing a row of nitrogenous bases, which "code" the genetic information needed by a living organism. The strands are connected by bonds between pairs of bases. While pairing combinations between certain bases are fixed (C-G and A-T), the sequence of nucleotides in the strand varies. (credit: Jerome Walker)*

The four nucleotide bases are given the symbols A (adenine), C (cytosine), G (guanine), and T (thymine). The order of the four bases varies in each strand, but the pairing between bases is always the same. C and G are always paired and A and T are always paired, which helps to preserve the order of bases in cell division (mitosis) so as to pass on the correct genetic information. Since the Coulomb force drops with distance $\left(F \propto 1/r^2\right)$, the distances between the base pairs must be small enough that the electrostatic force is sufficient to hold them together.

DNA is a highly charged molecule, with about $2q_e$ (fundamental charge) per $0.3 \times 10^{-9}\ \text{m}$. The distance separating the two strands that make up the DNA structure is about 1 nm, while the distance separating the individual atoms within each base is about 0.3 nm.

One might wonder why electrostatic forces do not play a larger role in biology than they do if we have so many charged molecules. The reason is that the electrostatic force is "diluted" due to **screening** between molecules. This is due to the presence of other charges in the cell.

## Polarity of Water Molecules

The best example of this charge screening is the water molecule, represented as H2O. Water is a strongly **polar molecule**. Its 10 electrons (8 from the oxygen atom and 2 from the two hydrogen atoms) tend to remain closer to the oxygen nucleus than the hydrogen nuclei. This creates two centers of equal and opposite charges—what is called a **dipole**, as illustrated in Figure 2. The magnitude of the dipole is called the dipole moment.

These two centers of charge will terminate some of the electric field lines coming from a free charge, as on a DNA molecule. This results in a reduction in the strength of the **Coulomb interaction**.

One might say that screening makes the Coulomb force a short range force rather than long range.

Other ions of importance in biology that can reduce or screen Coulomb interactions are $Na^+$, and $K^+$, and $Cl^-$. These ions are located both inside and outside of living cells. The movement of these ions through cell membranes is crucial to the motion of nerve impulses through nerve axons.

Recent studies of electrostatics in biology seem to show that electric fields in cells can be extended over larger distances, in spite of screening, by "microtubules" within the cell. These microtubules are hollow tubes composed of proteins that guide the movement of chromosomes when cells divide, the motion of other organisms within the cell, and provide mechanisms for motion of some cells (as motors).

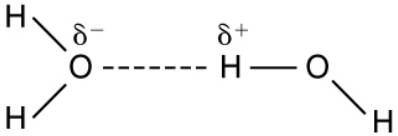



***Figure 2.*** *This schematic shows water ($H_2O$) as a polar molecule. Unequal sharing of electrons between the oxygen (O) and hydrogen (H) atoms leads to a net separation of positive and negative charge—forming a dipole. The symbols $\delta^-$ and $\delta^+$ indicate that the oxygen side of the $H_2O$ molecule tends to be more negative, while the hydrogen ends tend to be more positive. This leads to an attraction of opposite charges between molecules.*

## Section Summary

- Many molecules in living organisms, such as DNA, carry a charge.

- An uneven distribution of the positive and negative charges within a polar molecule produces a dipole.
- The effect of a Coulomb field generated by a charged object may be reduced or blocked by other nearby charged objects.
- Biological systems contain water, and because water molecules are polar, they have a strong effect on other molecules in living systems.

### *Conceptual Question*

**1:** A cell membrane is a thin layer enveloping a cell. The thickness of the membrane is much less than the size of the cell. In a static situation the membrane has a charge distribution of $\mathbf{-2.5 \times 10^{-6}\ C/m^2}$ on its inner surface and $\mathbf{+2.5 \times 10^{-6}\ C/m^2}$ on its outer surface. Draw a diagram of the cell and the surrounding cell membrane. Include on this diagram the charge distribution and the corresponding electric field. Is there any electric field inside the cell? Is there any electric field outside the cell?

## Glossary

**dipole**
a molecule's lack of symmetrical charge distribution, causing one side to be more positive and another to be more negative

**polar molecule**
a molecule with an asymmetrical distribution of positive and

negative charge

**screening**

the dilution or blocking of an electrostatic force on a charged object by the presence of other charges nearby

**Coulomb interaction**

the interaction between two charged particles generated by the Coulomb forces they exert on one another

# 74. 12.8 Optional: Conductors and Electric Fields in Static Equilibrium

*Summary*

- List the three properties of a conductor in electrostatic equilibrium.
- Explain the effect of an electric field on free charges in a conductor.
- Explain why no electric field may exist inside a conductor.
- Describe the electric field surrounding Earth.
- Explain what happens to an electric field applied to an irregular conductor.
- Describe how a lightning rod works.
- Explain how a metal car may protect passengers inside from the dangerous electric fields caused by a downed line touching the car.

**Conductors** contain **free charges** that move easily. When excess charge is placed on a conductor or the conductor is put into a static electric field, charges in the conductor quickly respond to reach a steady state called **electrostatic equilibrium**.

Figure 1 shows the effect of an electric field on free charges in a conductor. The free charges move until the field is perpendicular to the conductor's surface. There can be no component of the field

parallel to the surface in electrostatic equilibrium, since, if there were, it would produce further movement of charge. A positive free charge is shown, but free charges can be either positive or negative and are, in fact, negative in metals. The motion of a positive charge is equivalent to the motion of a negative charge in the opposite direction.

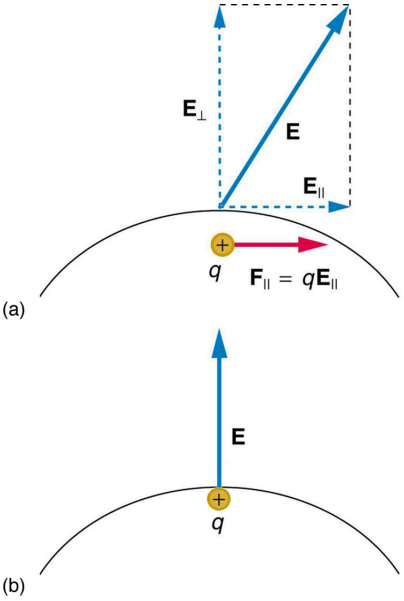



***Figure 1.*** *When an electric field* **E** *is applied to a conductor, free charges inside the conductor move until the field is perpendicular to the surface. (a) The electric field is a vector quantity, with both parallel and perpendicular components. The parallel component* ($\mathbf{E}_\parallel$) *exerts a force* ($\mathbf{F}_\parallel$) *on the free charge q, which moves the charge until* $\mathbf{F}_\parallel = \mathbf{0}$. *(b) The resulting field is perpendicular to the surface. The free charge has been brought to the conductor's surface, leaving electrostatic forces in equilibrium.*

A conductor placed in an **electric field** will be **polarized**. Figure 2 shows the result of placing a neutral conductor in an originally uniform electric field. The field becomes stronger near the conductor but entirely disappears inside it.

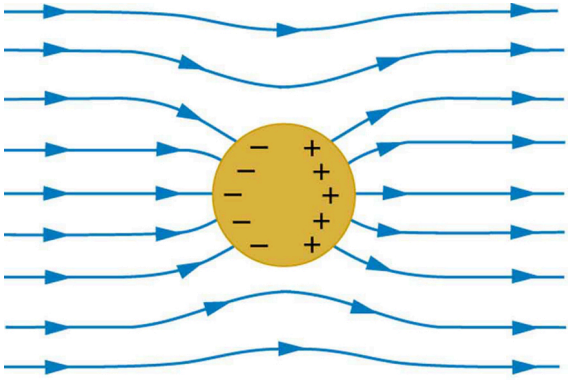


***Figure 2.*** *This illustration shows a spherical conductor in static equilibrium with an originally uniform electric field. Free charges move within the conductor, polarizing it, until the electric field lines are perpendicular to the surface. The field lines end on excess negative charge on one section of the surface and begin again on excess positive charge on the opposite side. No electric field exists inside the conductor, since free charges in the conductor would continue moving in response to any field until it was neutralized.*

## Misconception Alert: Electric Field inside a Conductor

Excess charges placed on a spherical conductor repel and move until they are evenly distributed, as shown in Figure 3. Excess charge is forced to the surface until the field inside the conductor is zero. Outside the conductor, the field is exactly the same as if the conductor were replaced by a point charge at its centre equal to the excess charge.

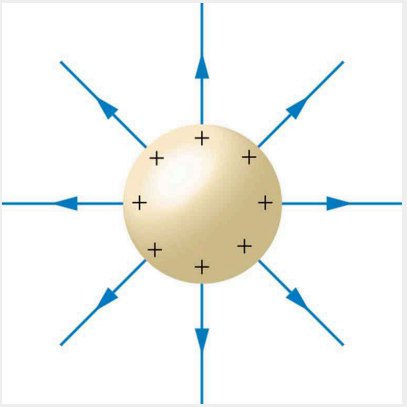


***Figure 3.*** *The mutual repulsion of excess positive charges on a spherical conductor distributes them uniformly on its surface. The resulting electric field is perpendicular to the surface and zero inside. Outside the conductor, the field is identical to that of a point charge at the center equal to the excess charge.*

## Properties of a Conductor in Electrostatic Equilibrium

1. The electric field is zero inside a conductor.

2. Just outside a conductor, the electric field lines are perpendicular to its surface, ending or beginning on charges on the surface.
3. Any excess charge resides entirely on the surface or surfaces of a conductor.

The properties of a conductor are consistent with the situations already discussed and can be used to analyze any conductor in electrostatic equilibrium. This can lead to some interesting new insights, such as described below.

How can a very uniform electric field be created? Consider a system of two metal plates with opposite charges on them, as shown in Figure 4. The properties of conductors in electrostatic equilibrium indicate that the electric field between the plates will be uniform in strength and direction. Except near the edges, the excess charges distribute themselves uniformly, producing field lines that are uniformly spaced (hence uniform in strength) and perpendicular to the surfaces (hence uniform in direction, since the plates are flat). The edge effects are less important when the plates are close together.

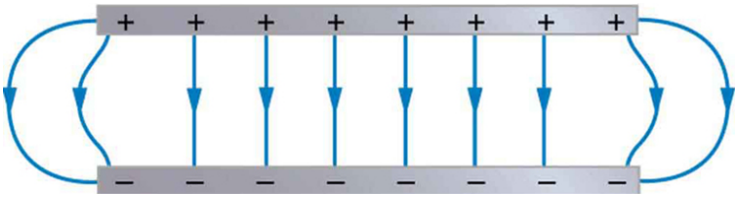


***Figure 4.*** *Two metal plates with equal, but opposite, excess charges. The field between them is uniform in strength and direction except near the edges. One use of such a field is to produce uniform acceleration of charges between the plates, such as in the electron gun of a cathode ray TV tube.*

## Earth's Electric Field

A near uniform electric field of approximately 150 N/C, directed downward, surrounds Earth, with the magnitude increasing slightly as we get closer to the surface. What causes the electric field? At around 100 km above the surface of Earth we have a layer of charged particles, called the **ionosphere**. The ionosphere is responsible for a range of phenomena including the electric field surrounding Earth. In fair weather the ionosphere is positive and the Earth largely negative, maintaining the electric field (Figure 5(a)).

In storm conditions clouds form and localized electric fields can be larger and reversed in direction (Figure 5(b)). The exact charge distributions depend on the local conditions, and variations of Figure 5(b) are possible.

If the electric field is sufficiently large, the insulating properties of the surrounding material break down and it becomes conducting. For air this occurs at around 3 million N/C or 3 million V/m. Air ionizes ions and electrons recombine, and we get discharge in the form of lightning sparks and corona discharge.



(a) (b)

***Figure 5.*** *Earth's electric field. (a) Fair weather field. Earth and the ionosphere (a layer of charged particles) are both conductors. They produce a uniform electric field of about 150 N/C. (credit: D. H. Parks) (b) Storm fields. In the presence of storm clouds, the local electric fields can be larger. At very high fields, the insulating properties of the air break down and lightning can occur. (credit: Jan-Joost Verhoef)*

## Electric Fields on Uneven Surfaces

So far we have considered excess charges on a smooth, symmetrical conductor surface. What happens if a conductor has sharp corners or is pointed? Excess charges on a nonuniform conductor become concentrated at the sharpest points. Additionally, excess charge may move on or off the conductor at the sharpest points.

To see how and why this happens, consider the charged conductor in Figure 6. The electrostatic repulsion of like charges is most effective in moving them apart on the flattest surface, and so they become least concentrated there. This is because the forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surfaces are different. The component parallel to the surface is

greatest on the flattest surface and, hence, more effective in moving the charge.

The same effect is produced on a conductor by an externally applied electric field, as seen in Figure 6 (c). Since the field lines must be perpendicular to the surface, more of them are concentrated on the most curved parts.

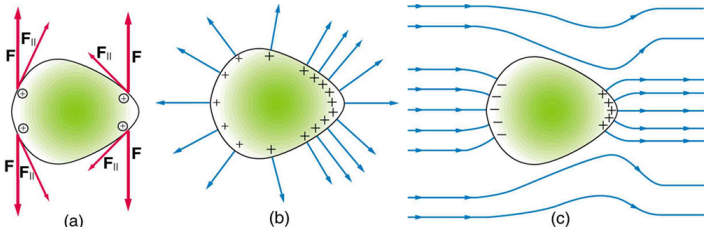



***Figure 6.*** *Excess charge on a nonuniform conductor becomes most concentrated at the location of greatest curvature. (a) The forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surface are different. It is* $\mathbf{F}_{\parallel}$ *that moves the charges apart once they have reached the surface. (b)* $\mathbf{F}_{\parallel}$ *is smallest at the more pointed end, the charges are left closer together, producing the electric field shown. (c) An uncharged conductor in an originally uniform electric field is polarized, with the most concentrated charge at its most pointed end.*

# Applications of Conductors

On a very sharply curved surface, such as shown in Figure 7, the charges are so concentrated at the point that the resulting electric field can be great enough to remove them from the surface. This can be useful.

Lightning rods work best when they are most pointed. The large charges created in storm clouds induce an opposite charge on a building that can result in a lightning bolt hitting the building. The induced charge is bled away continually by a lightning rod, preventing the more dramatic lightning strike.

Of course, we sometimes wish to prevent the transfer of charge

rather than to facilitate it. In that case, the conductor should be very smooth and have as large a radius of curvature as possible. (See Figure 8.) Smooth surfaces are used on high-voltage transmission lines, for example, to avoid leakage of charge into the air.

Another device that makes use of some of these principles is a **Faraday cage**. This is a metal shield that encloses a volume. All electrical charges will reside on the outside surface of this shield, and there will be no electrical field inside. A Faraday cage is used to prohibit stray electrical fields in the environment from interfering with sensitive measurements, such as the electrical signals inside a nerve cell.

During electrical storms if you are driving a car, it is best to stay inside the car as its metal body acts as a Faraday cage with zero electrical field inside. If in the vicinity of a lightning strike, its effect is felt on the outside of the car and the inside is unaffected, provided you remain totally inside. This is also true if an active ("hot") electrical wire was broken (in a storm or an accident) and fell on your car.

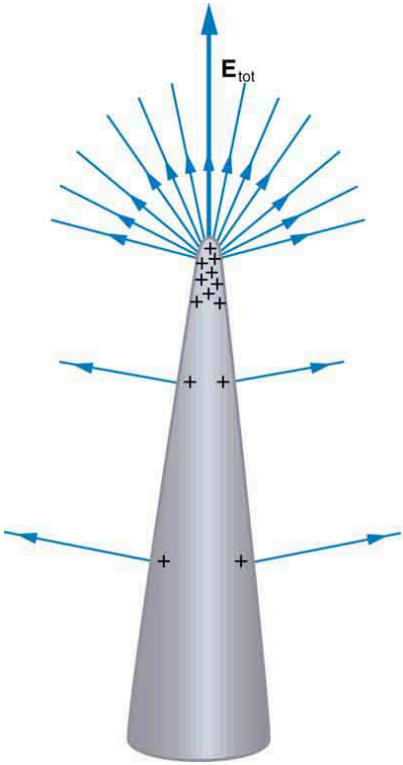



***Figure 7.*** *A very pointed conductor has a large charge concentration at the point. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor. Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed.*



(a) (b)

***Figure 8.*** *(a) A lightning rod is pointed to facilitate the transfer of charge. (credit: Romaine, Wikimedia Commons) (b) This Van de Graaff generator has a smooth surface with a large radius of curvature to prevent the transfer of charge and allow a large voltage to be generated. The mutual repulsion of like charges is evident in the person's hair while touching the metal sphere. (credit: Jon 'ShakataGaNai' Davis/Wikimedia Commons).*

## Section Summary

- A conductor allows free charges to move about within it.
- The electrical forces around a conductor will cause free charges to move around inside the conductor until static equilibrium is reached.
- Any excess charge will collect along the surface of a conductor.
- Conductors with sharp corners or points will collect more charge at those points.
- A lightning rod is a conductor with sharply pointed ends that collect excess charge on the building caused by an electrical storm and allow it to dissipate back into the air.
- Electrical storms result when the electrical field of Earth's surface in certain locations becomes more strongly charged, due to changes in the insulating effect of the air.
- A Faraday cage acts like a shield around an object, preventing electric charge from penetrating inside.

## *Conceptual Questions*

**1:** Is the object in Figure 9 a conductor or an insulator? Justify your answer.

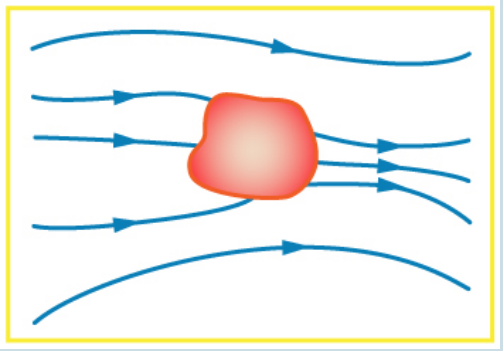


***Figure 9.***

**2:** If the electric field lines in the figure above were perpendicular to the object, would it necessarily be a conductor? Explain.

**3:** The discussion of the electric field between two parallel conducting plates, in this module states that edge effects are less important if the plates are close together. What does close mean? That is, is the actual plate separation crucial, or is the ratio of plate separation to plate area crucial?

**4:** Would the self-created electric field at the end of a pointed conductor, such as a lightning rod, remove positive or negative charge from the conductor? Would the same sign charge be removed from a neutral pointed conductor

by the application of a similar externally created electric field? (The answers to both questions have implications for charge transfer utilizing points.)

**5:** Why is a golfer with a metal club over her shoulder vulnerable to lightning in an open fairway? Would she be any safer under a tree?

**6:** Can the belt of a Van de Graaff accelerator be a conductor? Explain.

**7:** Are you relatively safe from lightning inside an automobile? Give two reasons.

**8:** Discuss pros and cons of a lightning rod being grounded versus simply being attached to a building.

**9:** Using the symmetry of the arrangement, show that the net Coulomb force on the charge $q$ at the center of the square below (Figure 10) is zero if the charges on the four corners are exactly equal.

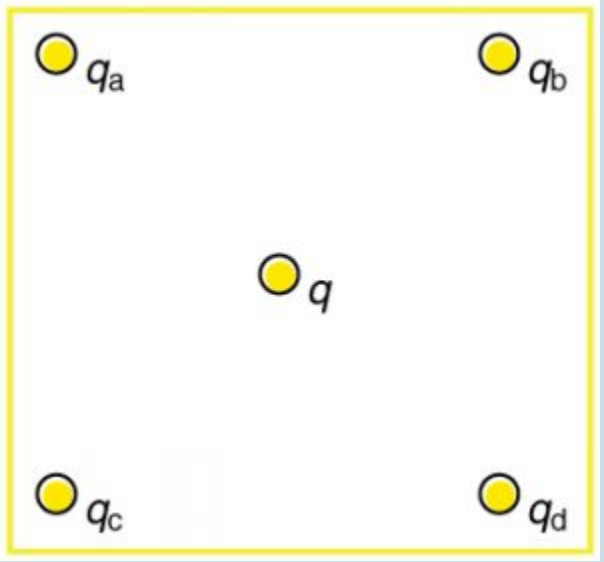



***Figure 10.*** *Four point charges $q_a$, $q_b$, $q_c$, and $q_d$ lie on the corners of a square and $q$ is located at its center.*

**10:** (a) Using the symmetry of the arrangement, show that the electric field at the center of the square in Figure 10 is zero if the charges on the four corners are exactly equal. (b) Show that this is also true for any combination of charges in which $q_a = q_d$ and $q_b = q_c$

**11:** (a) What is the direction of the total Coulomb force on $q$ in Figure 10 if $q$ is negative, $q_a = q_c$ and both are negative, and $q_b = q_c$ and both are positive? (b) What is the direction of the electric field at the center of the square in this situation?

**12:** Considering Figure 10, suppose that $q_a = q_d$ and $q_b = q_c$. First show that *qq size 12{q} {}* is in static equilibrium.

(You may neglect the gravitational force.) Then discuss whether the equilibrium is stable or unstable, noting that this may depend on the signs of the charges and the direction of displacement of $q$ from the center of the square.

**13:** If $q_a = 0$ in Figure 10, under what conditions will there be no net Coulomb force on $q$?

**14:** In regions of low humidity, one develops a special "grip" when opening car doors, or touching metal door knobs. This involves placing as much of the hand on the device as possible, not just the ends of one's fingers. Discuss the induced charge and explain why this is done.

**15:** Tollbooth stations on roadways and bridges usually have a piece of wire stuck in the pavement before them that will touch a car as it approaches. Why is this done?

**16:** Suppose a woman carries an excess charge. To maintain her charged status can she be standing on ground wearing just any pair of shoes? How would you discharge her? What are the consequences if she simply walks away?

## *Problems & Exercises*

**1:** Sketch the electric field lines in the vicinity of the conductor in Figure 11 given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?



***Figure 11.***

**2:** Sketch the electric field lines in the vicinity of the conductor in Figure 12 given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?



***Figure 12.***

**3:** Sketch the electric field between the two conducting plates shown in Figure 13, given the top plate is positive and an equal amount of negative charge is on the bottom plate. Be certain to indicate the distribution of charge on the plates.

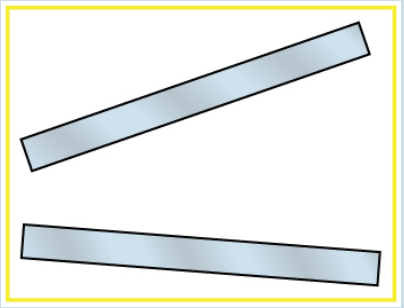


***Figure 13.***

**4:** Sketch the electric field lines in the vicinity of the charged insulator in Figure 14 noting its nonuniform charge distribution.

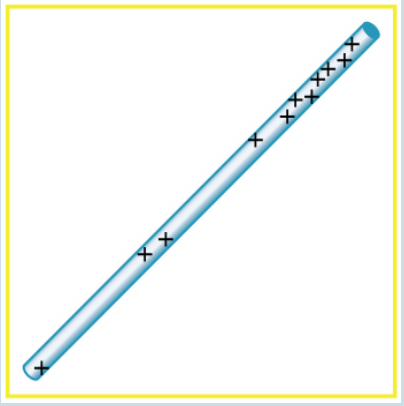


***Figure 14.*** *A charged insulating rod such as might be used in a classroom demonstration.*

**5:** What is the force on the charge located at x=8.00 cm in Figure 15(a) given that $q = 1.00\ \mu C$ ?

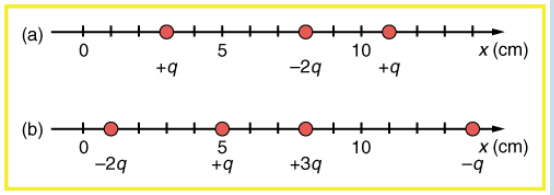



***Figure 15.*** *(a) Point charges located at 3.00, 8.00, and 11.0 cm along the x-axis. (b) Point charges located at 1.00, 5.00, 8.00, and 14.0 cm along the x-axis.*

**6:** (a) Find the total electric field at x=1.00 cm in Figure 15(b) given that q = 5.00 nC. (b) Find the total electric field at x = 11.00 cm in Figure 15(b). (c) If the charges are allowed to move and eventually be brought to rest by friction, what will the final charge configuration be? (That is, will there be a single charge, double charge, etc., and what will its value(s) be?)

**7:** (a) Find the electric field at $x = 5.00\ \text{cm}$ in Figure 15(a), given that $q = 1.00\ \mu C$. (b) At what position between 3.00 and 8.00 cm is the total electric field the same as that for $-2q$ alone? (c) Can the electric field be zero anywhere between 0.00 and 8.00 cm? (d) At very large positive or negative values of *x*, the electric field approaches zero in both (a) and (b). In which does it most rapidly approach zero and why? (e) At what position to the right of 11.0 cm is the total electric field zero, other than at infinity? (Hint: A graphing calculator can yield considerable insight in this problem.)

**8:** (a) Find the total Coulomb force on a charge of 2.00 nC located at x=4.00 cm in Figure 15 (b), given that $q = 1.00\ \mu\text{C}$. (b) Find the $x$-position at which the electric field is zero in Figure 15 (b).

**9:** Using the symmetry of the arrangement, determine the direction of the force on $q$ in the figure below, given that $q_a = q_b = +7.50\ \mu\text{C}$ and $q_c = q_d = -7.50\ \mu\text{C}$. (b) Calculate the magnitude of the force on the charge $q$, given that the square is 10.0 cm on a side and $q = 2.00\ \mu\text{C}$.

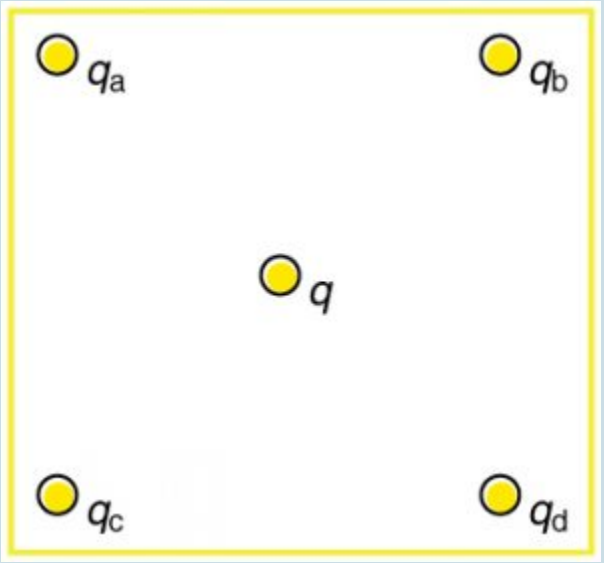



***Figure 16.***

**10:** (a) Using the symmetry of the arrangement, determine the direction of the electric field at the center of

the square in Figure 16, given that $q_a = q_b = -1.00\ \mu C$ and $q_c = q_d = +1.00\ \mu C$. (b) Calculate the magnitude of the electric field at the location of $q$, given that the square is 5.00 cm on a side.

**11:** Find the electric field at the location of $q_a$ in Figure 16 given that $q_b = q_c = q_d = +2.00\ nC$, $q = -1.00\ nC$, and the square is 20.0 cfm on a side.

**13:** (a) Find the electric field at the location of qaqa in Figure 17, given that $q_b = +10.00\ \mu C$ and $q_c = -5.00\ \mu C$. (b) What is the force on $q_a$, given that $q_a = +1.50\ nC$?

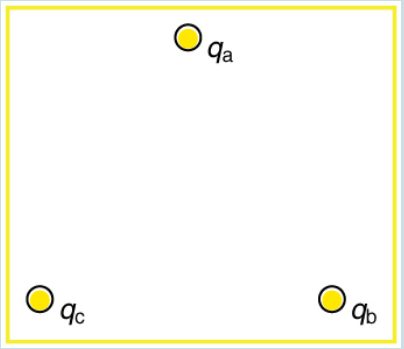



***Figure 17.*** *Point charges located at the corners of an equilateral triangle 25.0 cm on a side.*

## Glossary

**conductor**
: an object with properties that allow charges to move about freely within it

**free charge**
: an electrical charge (either positive or negative) which can move about separately from its base molecule

**electrostatic equilibrium**
: an electrostatically balanced state in which all free electrical charges have stopped moving about

**polarized**
: a state in which the positive and negative charges within an object have collected in separate locations

**ionosphere**
: a layer of charged particles located around 100 km above the surface of Earth, which is responsible for a range of phenomena including the electric field surrounding Earth

**Faraday cage**
: a metal shield which prevents electric charge from penetrating its surface

## *Solutions*

**Problems & Exercises**

**6:** (a) infinitely large as r = 0 cm.

(b) $\mathbf{2.12 \times 10^5\ N/C}$

(c) one charge of $+q$

**8:** (a) 0.252 N to the left (b) x = 6.07 cm

**10:** (a)The electric field at the centre of the square will be straight up, since $q_a$ and $q_b$ are positive and $q_c$ and $q_d$ are negative and all have the same magnitude.

(b) 2.04 x 10 $^{7}$ N} / C (upward)

PART XIV

# CHAPTER 13 ELECTRIC POTENTIAL AND ELECTRIC FIELD



***Figure 1.*** *Automated external defibrillator unit* (AED) (*credit: U.S. Defense Department photo/Tech. Sgt. Suzanne M. Day*)

In Chapter 12 Electric Charge and Electric Field, we just scratched the surface (or at least rubbed it) of electrical phenomena. Two of the most familiar aspects of electricity are its energy and *voltage*. We know, for example, that great amounts of electrical energy can be stored in batteries, are transmitted cross-country through power lines, and may jump from clouds to explode the sap of trees. In a similar manner, at molecular levels, *ions* cross cell membranes and transfer information. We also know about voltages associated with electricity. Batteries are typically a few volts, the outlets in your home produce 120 volts, and power lines can be as high as hundreds

of thousands of volts. But energy and voltage are not the same thing. A motorcycle battery, for example, is small and would not be very successful in replacing the much larger car battery, yet each has the same voltage. In this chapter, we shall examine the relationship between voltage and electrical energy and begin to explore some of the many applications of electricity.

# 75. 13.1 Electric Potential Energy: Potential Difference

*Summary*

- Define electric potential and electric potential energy.
- Describe the relationship between potential difference and electrical potential energy.
- Explain electron volt and its usage in submicroscopic process.
- Determine electric potential energy given potential difference and amount of charge.

When a free positive charge $\boldsymbol{q}$ is accelerated by an electric field, such as shown in Figure 1, it is given kinetic energy. The process is analogous to an object being accelerated by a gravitational field. It is as if the charge is going down an electrical hill where its electric potential energy is converted to kinetic energy. Let us explore the work done on a charge $\boldsymbol{q}$ by the electric field in this process, so that we may develop a definition of electric potential energy.

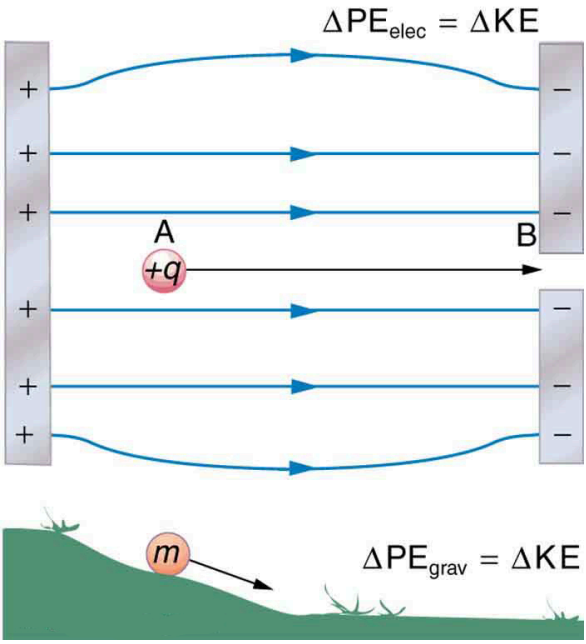



***Figure 1.*** *A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases potential energy is converted to another form. Work is done by a force, but since this force is conservative, we can write* $W = -\Delta PE$.

The electrostatic or Coulomb force is conservative, which means that the work done on $q$ is independent of the path taken. This is exactly analogous to the gravitational force in the absence of dissipative forces such as friction. When a force is conservative, it is possible to define a potential energy associated with the force, and it is usually easier to deal with the potential energy (because it depends only on position) than to calculate the work directly.

We use the letters PE to denote electric potential energy, which has units of joules (J). The change in potential energy, $\Delta PE$, is crucial, since the work done by a conservative force is the negative of the change in potential energy; that is, $W = -\Delta PE$. For

example, work $W$ done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative $\Delta PE$. There must be a minus sign in front of $\Delta PE$ to make $W$ positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.


## Potential Energy

$W = -\Delta PE$. For example, work $W$ done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative $\Delta PE$. There must be a minus sign in front of $\Delta PE$ to make $W$ positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.


Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of voltage (related to electric potential energy) than to deal with the Coulomb force directly.

Calculating the work directly is generally difficult, since $W = Fd\cos\theta$ and the direction and magnitude of $F$ can be complex for multiple charges, for odd-shaped objects, and along arbitrary paths. But we do know that, since $F = qE$, the work, and hence $\Delta PE$, is proportional to the test charge $q$. To have a physical quantity that is independent of test charge, we define

**electric potential** $V$ (or simply potential, since electric is understood) to be the potential energy per unit charge:

$$V = \frac{\mathbf{PE}}{q}.$$

Electric Potential

This is the electric potential energy per unit charge.

$$V = \frac{\mathbf{PE}}{q}$$

Since PE is proportional to $q$, the dependence on $q$ cancels. Thus $V$ does not depend on $q$. The change in potential energy $\Delta\mathbf{PE}$ is crucial, and so we are concerned with the difference in potential or potential difference $\Delta V$ between two points, where

$$\Delta V = V_{\mathbf{B}} - V_{\mathbf{A}} = \frac{\Delta\mathbf{PE}}{q}.$$

The **potential difference** between points A and B, $V_{\mathbf{B}} - V_{\mathbf{A}}$, is thus defined to be the change in potential energy of a charge $q$ moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$\mathbf{1V} = 1\,\frac{\mathbf{J}}{\mathbf{C}}$$

## Potential Difference

The potential difference between points A and B, $V_B - V_A$, is defined to be the change in potential energy of a charge $q$ moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1\text{V} = 1\,\frac{\text{J}}{\text{C}}$$

The familiar term **voltage** is the common name for potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor.

In summary, the relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = \text{q}\Delta\text{V}.$$

## Potential Difference and Electrical Potential Energy

The relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta \mathrm{PE}}{q} \text{ and } \Delta \mathrm{PE} = q\Delta V.$$

The second equation is equivalent to the first.

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other since $\Delta \mathrm{PE} = q\Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12 V batteries.

## Example 1: Calculating Energy

Suppose you have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each

deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

**Strategy**

To say we have a 12.0 V battery means that its terminals have a 12.0 V potential difference. When such a battery moves charge, it puts the charge through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\mathbf{\Delta PE} = \boldsymbol{q}\mathbf{\Delta}\boldsymbol{V}$.

So to find the energy output, we multiply the charge moved by the potential difference.

**Solution**

For the motorcycle battery, $\boldsymbol{q} = \mathbf{5000\ C}$ and $\mathbf{\Delta}\boldsymbol{V} = \mathbf{12.0\ V}$. The total energy delivered by the motorcycle battery is

$$\begin{aligned} \mathbf{\Delta PE_{cycle}} &= \mathbf{(5000\ C)(12.0\ V)} \\ &= \mathbf{(5000\ C)(12.0\ J/C)} \\ &= \mathbf{6.00 \times 10^4\ J}. \end{aligned}$$

Similarly, for the car battery, $\boldsymbol{q} = \mathbf{60,000\ C}$ and

$$\begin{aligned} \mathbf{\Delta PE_{cycle}} &= \mathbf{(60,000\ C)(12.0\ V)} \\ &= \mathbf{7.20 \times 10^5\ J} \end{aligned}$$

**Discussion**

While voltage and energy are related, they are not the

same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a low car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B) as shown in Figure 2. The change in potential is $\boldsymbol{\Delta V = V_B - V_A = +12\ V}$ and the charge $\boldsymbol{q}$ is negative, so that $\boldsymbol{\Delta PE = q\Delta V}$ is negative, meaning the potential energy of the battery has decreased when $\boldsymbol{q}$ has moved from A to B.

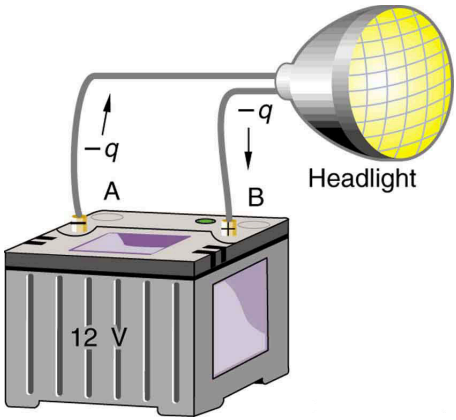



***Figure 2.*** *A battery moves negative charge from its negative terminal through a headlight to its positive terminal. Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative. Inside the battery, both positive and negative charges move.*

## Example 2: How Many Electrons Move through a Headlight Each Second?

When a 12.0 V car battery runs a single 30.0 W headlight, how many electrons pass through it each second?

**Strategy**

To find the number of electrons, we must first find the charge that moved in 1.00 s. The charge moved is related to voltage and energy through the equation $\mathbf{\Delta PE} = \boldsymbol{q}\mathbf{\Delta}\boldsymbol{V}$. A 30.0 W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\mathbf{\Delta PE} = \mathbf{-30.0\ J}$ and, since the electrons are going from the negative terminal to the positive, we see that $\mathbf{\Delta}\boldsymbol{V} = \mathbf{+12.0}\ \boldsymbol{V}$.

**Solution**

To find the charge $\boldsymbol{q}$ moved, we solve the equation $\mathbf{\Delta PE} = \boldsymbol{q}\mathbf{\Delta}\boldsymbol{V}$:

$$\boldsymbol{q} = \frac{\mathbf{\Delta PE}}{\mathbf{\Delta}\boldsymbol{V}}.$$

Entering the values for $\mathbf{\Delta PE}$ and $\mathbf{\Delta V}$, we get

$$\boldsymbol{q} = \frac{\mathbf{-30.0\ J}}{\mathbf{+12.0\ V}} = \frac{\mathbf{-30.0\ J}}{\mathbf{+12.0\ J/C}}$$
$$\mathbf{= -2.50\ C}.$$

The number of electrons $\mathbf{n_e}$ is the total charge divided by the charge per electron. That is,

$$\mathbf{ne} = \frac{\mathbf{-2.50\ C}}{\mathbf{-1.60 \times 10^{-19}\ C/e^-}}$$
$$\mathbf{= 1.56 \times 10^{19}\ electrons.}$$

**Discussion**

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary systems. In fact, electricity

had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

## The Electron Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful x rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects. Figure 3 shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates as it might be in an old-model television tube or oscilloscope. The electron is given kinetic energy that is later converted to another form—light in the television tube, for example. (Note that downhill for the electron is uphill for a positive charge.) Since energy is related to voltage by $\boldsymbol{\Delta PE = q\Delta V}$, we can think of the joule as a coulomb-volt.

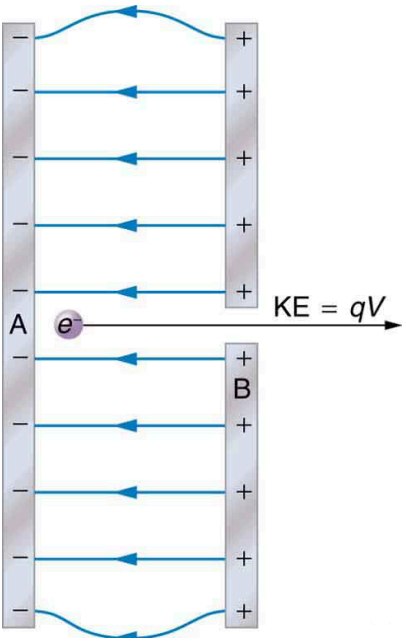



***Figure 3.*** *A typical electron gun accelerates electrons using a potential difference between two metal plates. The energy of the electron in electron volts is numerically the same as the voltage between the plates. For example, a 5000 V potential difference produces 5000 eV electrons.*

On the submicroscopic scale, it is more convenient to define an energy unit called the **electron volt** (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\mathbf{1eV} \quad \mathbf{(1.60 \times 10^{-19}\ C)\ (1\ V) = (1.60 \times 10^{-19}\ C)\ (1\ J/C)}$$

$$\mathbf{1.60 \times 10^{-19}\ J.}$$

## Electron Volt

On the submicroscopic scale, it is more convenient to define an energy unit called the electron volt (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\mathbf{1eV} \quad \mathbf{(1.60 \times 10^{-19}\ C)\ (1\ V) = (1.60 \times 10^{-19}\ C)\ (1\ J/C)}$$

$$\mathbf{1.60 \times 10^{-19}\ J.}$$

An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V is given 50 eV. A potential difference of 100,000 V (100 kV) will give an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V will be given 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron volt a simple and convenient energy unit in such circumstances.

## Connections: Energy Units

The electron volt (eV) is the most common energy unit for submicroscopic processes. This will be

particularly noticeable in the chapters on modern physics. Energy is so important to so many subjects that there is a tendency to define a special energy unit for each major topic. There are, for example, calories for food energy, kilowatt-hours for electrical energy, and therms for natural gas energy.

The electron volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it is given an energy of 30 keV (30,000 eV) and it can break up as many as 6000 of these molecules $\mathbf{(30,000\ eV \div 5\ eV\ per\ molecule = 6000\ molecules)}$. Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can, thus, produce significant biological damage.

## Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) of work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

**Mechanical energy** is the sum of the kinetic energy and potential energy of a system; that is, $\mathbf{KE + PE = constant}$. A loss of PE of a charged particle becomes an increase in its KE. Here PE

is the electric potential energy. Conservation of energy is stated in equation form as

$$\mathbf{KE + PE = constant}$$

or

$$\mathbf{KE_i + PE_i = KE_f + PE_f},$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

## Electrical Potential Energy Converted to Kinetic Energy

Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)

**Strategy**

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be $\mathbf{KE_i = 0}$, $\mathbf{KE_f = \frac{1}{2}mv^2}$, $\mathbf{PE_i = qV}$, and $\mathbf{PE_f = 0}$.

**Solution**

Conservation of energy states that

$$\mathbf{KE_i + PE_i = KE_f + PE_f}.$$

Entering the forms identified above, we obtain

$$\boldsymbol{qV = \frac{mv^2}{2}}.$$

We solve this for $\boldsymbol{v}$:

$$\boldsymbol{v = \sqrt{\frac{2qV}{m}}}.$$

Entering values for $\boldsymbol{q}$, $\boldsymbol{V}$, and $\boldsymbol{m}$ gives

$$\boldsymbol{v} \quad \sqrt{\frac{\mathbf{2(-1.60\times10^{-19}\ C)(-100\ J/C)}}{\mathbf{9.11\times10^{-31}\ kg}}}.$$

$$\mathbf{5.93 \times 10^6\ m/s}$$

**Discussion**

Note that both the charge and the initial voltage are negative, as in Figure 3. From the discussions in Chapter 18 Electric Charge and Electric Field, we know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns. Those higher voltages produce electron speeds so great that relativistic effects must be taken

into account. That is why a low voltage is considered (accurately) in this example.

## Section Summary

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $\boldsymbol{V_B - V_A}$, defined to be the change in potential energy of a charge $\boldsymbol{q}$ moved from A to B, is equal to the change in potential energy divided by the charge, Potential difference is commonly called voltage, represented by the symbol $\boldsymbol{\Delta V}$.

$$\boldsymbol{\Delta V = \frac{\Delta PE}{q} \text{ and } \Delta PE = q\Delta V.}$$

- An electron volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\boldsymbol{1eV \quad (1.60 \times 10^{-19}\ C)(1\ V) = 1.60 \times 10^{-19}\ C)(1\ J/C)}$$

$$\boldsymbol{1.60 \times 10^{-19}\ J.}$$

- Mechanical energy is the sum of the kinetic energy and potential energy of a system, that is, $\boldsymbol{KE + PE}$. This sum is a constant.

## *Conceptual Questions*

**1:** Voltage is the common word for potential difference. Which term is more descriptive, voltage or potential difference?

**2:** If the voltage between two points is zero, can a test charge be moved between them with zero net work being done? Can this necessarily be done without exerting a force? Explain.

**3:** What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?

**4:** Voltages are always measured between two points. Why?

**5:** How are units of volts and electron volts related? How do they differ?

## *Problems & Exercises*

**1:** Find the ratio of speeds of an electron and a negative hydrogen ion (one having an extra electron) accelerated through the same voltage, assuming non-relativistic final speeds. Take the mass of the hydrogen ion to be $\mathbf{1.67 \times 10^{-27}\ kg}$.

**2:** An evacuated tube uses an accelerating voltage of 40

kV to accelerate electrons to hit a copper plate and produce x rays. Non-relativistically, what would be the maximum speed of these electrons?

**3:** A bare helium nucleus has two positive charges and a mass of $\mathbf{6.64 \times 10^{-27}\ kg}$. (a) Calculate its kinetic energy in joules at 2.00% of the speed of light. (b) What is this in electron volts? (c) What voltage would be needed to obtain this energy?

**4: Integrated Concepts**
Singly charged gas ions are accelerated from rest through a voltage of 13.0 V. At what temperature will the average kinetic energy of gas molecules be the same as that given these ions?

**5: Integrated Concepts**
The temperature near the center of the Sun is thought to be 15 million degrees Celsius $\mathbf{(1.5 \times 10^{7}\ ^{\circ}C)}$. Through what voltage must a singly charged ion be accelerated to have the same energy as the average kinetic energy of ions at this temperature?

**6: Integrated Concepts**
(a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms? (b) Considering the high-power output, why doesn't the defibrillator produce serious burns?

**7: Integrated Concepts**
A lightning bolt strikes a tree, moving 20.0 C of charge through a potential difference of $\mathbf{1.00 \times 10^{2}\ MV}$. (a) What energy was dissipated? (b) What mass of water could be raised from $\mathbf{15\ ^{\circ}C}$ to the boiling point and then

boiled by this energy? (c) Discuss the damage that could be caused to the tree by the expansion of the boiling steam.

**8: Integrated Concepts**

A 12.0 V battery-operated bottle warmer heats 50.0 g of glass, $\mathbf{2.50 \times 10^2\ g}$ of baby formula, and $\mathbf{2.00 \times 10^2\ g}$ of aluminum from $\mathbf{20.0^\circ C}$ to $\mathbf{90.0^\circ C}$. (a) How much charge is moved by the battery? (b) How many electrons per second flow if it takes 5.00 min to warm the formula? (Hint: Assume that the specific heat of baby formula is about the same as the specific heat of water.)

**9: Integrated Concepts**

A battery-operated car utilizes a 12.0 V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car from rest to 25.0 m/s, make it climb a $\mathbf{2.00 \times 10^2\ m}$ high hill, and then cause it to travel at a constant 25.0 m/s by exerting a $\mathbf{5.00 \times 10^2\ N}$ force for an hour.

**10: Integrated Concepts**

Fusion probability is greatly enhanced when appropriate nuclei are brought close together, but mutual Coulomb repulsion must be overcome. This can be done using the kinetic energy of high-temperature gas ions or by accelerating the nuclei toward one another. (a) Calculate the potential energy of two singly charged nuclei separated by $\mathbf{1.00 \times 10^{-12}\ m}$ by finding the voltage of one at that distance and multiplying by the charge of the other. (b) At what temperature will atoms of a gas have an average kinetic energy equal to this needed electrical potential energy?

**11: Unreasonable Results**

(a) Find the voltage near a 10.0 cm diameter metal sphere that has 8.00 C of excess positive charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

**12: Construct Your Own Problem**

Consider a battery used to supply energy to a cellular phone. Construct a problem in which you determine the energy that must be supplied by the battery, and then calculate the amount of charge it must be able to move in order to supply this energy. Among the things to be considered are the energy needs and battery voltage. You may need to look ahead to interpret manufacturer's battery ratings in ampere-hours as energy in joules.

## Glossary

**electric potential**
: potential energy per unit charge

**potential difference (or voltage)**
: change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

**electron volt**
: the energy given to a fundamental charge accelerated through a potential difference of one volt

**mechanical energy**
: sum of the kinetic energy and potential energy of a system; this sum is a constant

## *Solutions*

**Problems & Exercises**

**1:** 42.8

**4:** $1.00 \times 10^5$ K

**6:** (a) $4 \times 10^4$ W (b) A defibrillator does not cause serious burns because the skin conducts electricity well at high voltages, like those used in defibrillators. The gel used aids in the transfer of energy to the body, and the skin doesn't absorb the energy, but rather lets it pass through to the heart.

**8:** (a) $7.40 \times 10^3$ C (b) $1.54 \times 10^{20}$ electrons per second

**9:** $3.89 \times 10^6$ C

**11:** (a) $1.44 \times 10^{12}$ V

(b) This voltage is very high. A 10.0 cm diameter sphere could never maintain this voltage; it would discharge.

(c) An 8.00 C charge is more charge than can reasonably be accumulated on a sphere of that size.

# 76. 13.2 Electric Potential in a Uniform Electric Field

*Learning Objectives*

- Describe the relationship between voltage and electric field.
- Derive an expression for the electric potential and electric field.
- Calculate electric field strength given distance and voltage.

In the previous section, we explored the relationship between voltage and energy. In this section, we will explore the relationship between voltage and electric field. For example, a uniform electric field $\mathbf{E}$ is produced by placing a potential difference (or voltage) $\boldsymbol{\Delta V}$ across two parallel metal plates, labeled A and B. (See Figure 1.) Examining this will tell us what voltage is needed to produce a certain electric field strength; it will also reveal a more fundamental relationship between electric potential and electric field. From a physicist's point of view, either $\boldsymbol{\Delta V}$ or $\mathbf{E}$ can be used to describe any charge distribution. $\boldsymbol{\Delta V}$ is most closely tied to energy, whereas $\mathbf{E}$ is most closely related to force. $\boldsymbol{\Delta V}$ is a **scalar** quantity and has no direction, while $\mathbf{E}$ is a **vector** quantity, having both magnitude and direction. (Note that the magnitude of the electric field strength, a scalar quantity, is represented by $\mathbf{E}$ below.) The relationship between $\boldsymbol{\Delta V}$ and $\mathbf{E}$ is revealed by calculating the work done by the force in moving a charge from point A to point

B. But, as noted in Chapter 13.1 Electric Potential Energy: Potential Difference, this is complex for arbitrary charge distributions, requiring calculus. We therefore look at a uniform electric field as an interesting special case.

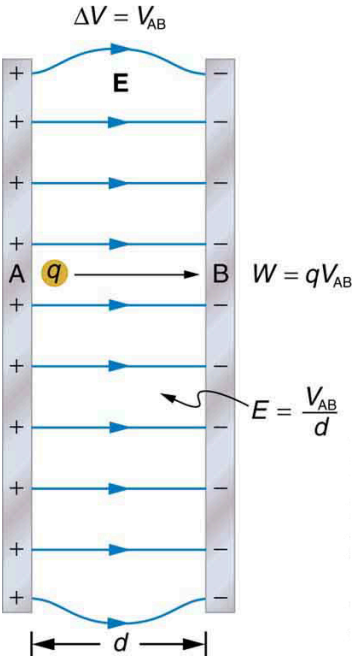



***Figure 1.*** *The relationship between* V *and* E *for parallel conducting plates is* E = V/*d. (Note that* ΔV = $V_{AB}$ *in magnitude. For a charge that is moved from plate* A *at higher potential to plate* B *at lower potential, a minus sign needs to be included as follows:* $-\Delta V = V_A - V_B = V_{AB}$. *See the text for details.)*

The work done by the electric field in Figure 1 to move a positive charge $\boldsymbol{q}$ from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

$$\boldsymbol{W = -\Delta PE = -q\Delta V.}$$

The potential difference between points A and B is

$$-\Delta V = -(V_B - V_A) = V_{A-V_B} = V_{AB}.$$

Entering this into the expression for work yields

$$W = qV_{AB}.$$

Work is $W = Fd\cos\theta$, since the path is parallel to the field, and so $W = Fd$. Since $F = qE$, we see that $W = qEd$. Substituting this expression for work into the previous equation gives

$$qEd = qV_{AB}.$$

The charge cancels, and so the voltage between points A and B is seen to be

$$\left.\begin{array}{l} V_{AB} = Ed \\ E = \frac{V_{AB}}{d} \end{array}\right\} \textbf{(uniform } E \textbf{ - field only)},$$

where $d$ is the distance from A to B, or the distance between the plates in Figure 1. Note that the above equation implies the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus the following relation among units is valid:

$$1\ \text{N/C} = 1\ \text{V/m}.$$

### Voltage between Points A and B

$$\left.\begin{aligned} V_{\mathrm{AB}} &= Ed \\ E &= \frac{V_{\mathrm{AB}}}{d} \end{aligned}\right\}$$

**(uniform $E$ - field only),**

where $d$ is the distance from A to B, or the distance between the plates.

### Example 1: What Is the Highest Voltage Possible between Two Plates?

Dry air will support a maximum electric field strength of about $3.0 \times 10^6\ \mathrm{V/m}$. Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?

**Strategy**

We are given the maximum electric field $E$ between

the plates and the distance $\boldsymbol{d}$ between them. The equation $\boldsymbol{V_{AB} = Ed}$ can thus be used to calculate the maximum voltage.

**Solution**

The potential difference or voltage between the plates is

$$\boldsymbol{V_{AB} = Ed.}$$

Entering the given values for $\boldsymbol{E}$ and $\boldsymbol{d}$ gives

$$\boldsymbol{V_{AB} = (3.0 \times 10^6\ \text{V/m})(0.025\ \text{m}) = 7.5 \times 10^4\ \text{V}}$$

or

$$\boldsymbol{V_{AB} = 75\ \text{kV}.}$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

**Discussion**

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5 cm (1 in.) gap, or 150 kV for a 5 cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage will cause a spark if there are points on the surface, since points create greater fields than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up, say with static electricity, on dry days.




***Figure 2.*** *A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays). (credit: Daderot, Wikimedia Commons)*

## Example 2: Field and Force inside an

## Electron Gun

(a) An electron gun has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. What is the electric field strength between the plates? (b) What force would this field exert on a piece of plastic with a $\mathbf{0.500\ \mu C}$ charge that gets between the plates?

**Strategy**

Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression $\boldsymbol{E = \frac{V_{AB}}{d}}$. Once the electric field strength is known, the force on a charge is found using $\mathbf{F = qE}$. Since the electric field is in only one direction, we can write this equation in terms of the magnitudes, $\boldsymbol{F = q\ E}$.

**Solution for (a)**

The expression for the magnitude of the electric field between two uniform metal plates is

$$\boldsymbol{E = \frac{V_{AB}}{d}}.$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for $\boldsymbol{V_{AB}}$ and the plate separation of 0.0400 m, we obtain

$$E = \frac{25.0\ \text{kV}}{0.0400\ \text{m}} = 6.25 \times 10^5\ \text{V/m}.$$

**Solution for (b)**

The magnitude of the force on a charge in an electric field is obtained from the equation

$$F = qE.$$

Substituting known values gives

$$F = (0.500 \times 10^{-6}\ \text{C})(6.25 \times 10^5\ \text{V/m}) = 0.313\ \text{N}.$$

**Discussion**

Note that the units are newtons, since $1\ \text{V/m} = 1\ \text{N/C}$. The force on the charge is the same no matter where the charge is located between the plates. This is because the electric field is uniform between the plates.

In more general situations, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of $\mathbf{E}$ and also in the direction of lower potential $V$. Furthermore, the magnitude of $\mathbf{E}$ equals the rate of decrease of $V$ with distance. The faster $V$ decreases over distance, the greater the electric field. In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s},$$

where $\Delta s$ is the distance over which the change in potential, $\Delta V$, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

### Relationship between Voltage and Electric Field

In equation form, the general relationship between voltage and electric field is

$$\boldsymbol{E = -\frac{\Delta V}{\Delta s}},$$

where $\boldsymbol{\Delta s}$ is the distance over which the change in potential, $\boldsymbol{\Delta V}$, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

For continually changing potentials, $\boldsymbol{\Delta V}$ and $\boldsymbol{\Delta s}$ become infinitesimals and differential calculus must be employed to determine the electric field.

## Section Summary

- The voltage between points A and B is

$$\left.\begin{aligned} \boldsymbol{V_{\mathrm{AB}} &= Ed} \\ \boldsymbol{E &= \frac{V_{\mathrm{AB}}}{d}} \end{aligned}\right\} \textbf{(uniform } \boldsymbol{E} \textbf{ - field only)},$$

- where $\boldsymbol{d}$ is the distance from A to B, or the distance between the plates.
- In equation form, the general relationship between voltage and electric field is

$$\boldsymbol{E = -\frac{\Delta V}{\Delta s}},$$

- where $\boldsymbol{\Delta s}$ is the distance over which the change in potential, $\boldsymbol{\Delta V}$, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential.) The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

## *Conceptual Questions*

**1:** Discuss how potential difference and electric field strength are related. Give an example.

**2:** What is the strength of the electric field in a region where the electric potential is constant?

**3:** Will a negative charge, initially at rest, move toward higher or lower potential? Explain why.

## *Problems & Exercises*

**1:** Show that units of V/m and N/C for electric field strength are indeed equivalent.

**2:** What is the strength of the electric field between two parallel conducting plates separated by 1.00 cm and having a potential difference (voltage) between them of $\mathbf{1.50 \times 10^4\ V}$?

**3:** The electric field strength between two parallel conducting plates separated by 4.00 cm is $\mathbf{7.50 \times 10^4\ V/m}$. (a) What is the potential difference between the plates? (b) The plate with the lowest potential is taken to be at zero volts. What is the potential 1.00 cm from that plate (and 3.00 cm from the other)?

**4:** How far apart are two conducting plates that have an electric field strength of $\mathbf{4.50 \times 10^3\ V/m}$ between them, if their potential difference is 15.0 kV?

**5:** (a) Will the electric field strength between two parallel conducting plates exceed the breakdown strength for air $\mathbf{(3.0 \times 10^6\ V/m)}$ if the plates are separated by 2.00 mm and a potential difference of $\mathbf{5.0 \times 10^3\ V}$ is applied? (b) How close together can the plates be with this applied voltage?

**6:** The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength? (The value is surprisingly large, but correct. Membranes are discussed in Chapter 19.5 Capacitors and Dielectrics and Chapter 20.7 Nerve Conduction–Electrocardiograms.) You may assume a uniform electric field.

**7:** Membrane walls of living cells have surprisingly large electric fields across them due to separation of ions. (Membranes are discussed in some detail in Chapter 20.7

Nerve Conduction–Electrocardiograms.) What is the voltage across an 8.00 nm–thick membrane if the electric field strength across it is 5.50 MV/m? You may assume a uniform electric field.

**8:** Two parallel conducting plates are separated by 10.0 cm, and one of them is taken to be at zero volts. (a) What is the electric field strength between them, if the potential 8.00 cm from the zero volt plate (and 2.00 cm from the other) is 450 V? (b) What is the voltage between the plates?

**9:** Find the maximum potential difference between two parallel conducting plates separated by 0.500 cm of air, given the maximum sustainable electric field strength in air to be $\mathbf{3.0 \times 10^6\ V/m}$.

**10:** A doubly charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

**11:** An electron is to be accelerated in a uniform electric field having a strength of $\mathbf{2.00 \times 10^6\ V/m}$. (a) What energy in keV is given to the electron if it is accelerated through 0.400 m? (b) Over what distance would it have to be accelerated to increase its energy by 50.0 GeV?

## Glossary

**scalar**
physical quantity with magnitude but no direction

**vector**

physical quantity with both magnitude and direction

## *Solutions*

**Problems & Exercises**

**3:** (a) 3.00 kV

(b) 750 V

**5:** (a) No. The electric field strength between the plates is $\mathbf{2.5 \times 10^6\ V/m}$, which is lower than the breakdown strength for air $\mathbf{(3.0 \times 10^6\ V/m)}$.

(b) 1.7 mm

**7:** 44.0 mV

**9:** 15 kV

**11:** (a) 800 KeV

(b) 25.0 km

# 77. 13.3 Electrical Potential Due to a Point Charge

*Summary*

- Explain point charges and express the equation for electric potential of a point charge.
- Distinguish between electric potential and electric field.
- Determine the electric potential of a point charge given charge and distance.

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider. Using calculus to find the work needed to move a test charge $q$ from a large distance away to a distance of $r$ from a point charge $Q$, and noting the connection between work and potential $(W = -q\Delta V)$, it can be shown that the *electric potential* $V$ *of a point charge* is

$$V = \frac{kQ}{r} \textbf{ (Point Charge),}$$

where $k$ is a constant equal to $9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

### Electric Potential $V$ of a Point Charge

The electric potential $\boldsymbol{V}$ of a point charge is given by

$$\boldsymbol{V = \frac{kQ}{r}\ \textbf{(Point Charge)},}$$

The potential at infinity is chosen to be zero. Thus $\boldsymbol{V}$ for a point charge decreases with distance, whereas $\boldsymbol{E}$ for a point charge decreases with distance squared:

$$\boldsymbol{E = \frac{F}{q} = \frac{kQ}{r^2}.}$$

Recall that the electric potential $\boldsymbol{V}$ is a scalar and has no direction, whereas the electric field $\mathbf{E}$ is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as *vectors*, taking magnitude and direction into account. This is consistent with the fact that $\boldsymbol{V}$ is closely associated with energy, a scalar, whereas $\mathbf{E}$ is closely associated with force, a vector.

### Example 1: What Voltage Is Produced by

## a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb $(\mu\mathrm{C})$ range. What is the voltage 5.00 cm away from the center of a 1-cm diameter metal sphere that has a –3.00nC static charge?

**Strategy**

As we have discussed in Chapter 12 Electric Charge and Electric Field, charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus we can find the voltage using the equation $V = kQ/r$.

**Solution**

Entering known values into the expression for the potential of a point charge, we obtain

$$V = k\frac{Q}{r}$$

$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\left(\frac{-3.00 \times 10^9\ \mathrm{C}}{5.00 \times 10^2\ \mathrm{m}}\right)$$

$$= -539\ \mathrm{V}.$$

**Discussion**

The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower (more negative) than at larger

distances. Conversely, a negative charge would be repelled, as expected.

## Example 2: What Is the Excess Charge on a Van de Graaff Generator

A demonstration Van de Graaff generator has a 25.0 cm diameter metal sphere that produces a voltage of 100 kV near its surface. (See Figure 1.) What excess charge resides on the sphere? (Assume that each numerical value here is shown with three significant figures.)

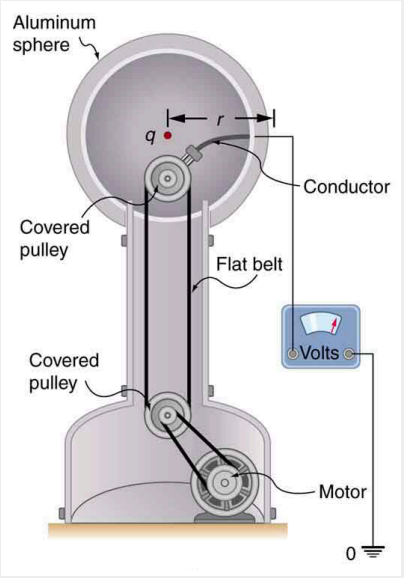



***Figure 1.*** *The voltage of this demonstration Van de Graaff generator is measured between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.*

**Strategy**

The potential on the surface will be the same as that of a point charge at the center of the sphere, 12.5 cm away. (The radius of the sphere is 12.5 cm.) We can thus determine the excess charge using the equation

$$V = \frac{kQ}{r}.$$

**Solution**

Solving for $Q$ and entering known values gives

$$Q \quad \frac{rV}{k}$$

$$\frac{(0.125\ \text{m})(100 \times 10^3\ \text{V})}{8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2}$$

$$1.39 \times 10^{-6}\ \text{C} = 1.39\ \mu\text{C}.$$

**Discussion**

This is a relatively small charge, but it produces a rather large voltage. We have another indication here that it is difficult to store isolated charges.

The voltages in both of these examples could be measured with a meter that compares the measured potential with ground potential. Ground potential is often taken to be zero (instead of taking the potential at infinity to be zero). It is the potential difference between two points that is of importance, and very often there is a tacit assumption that some reference point, such as Earth or a very distant point, is at zero potential. As noted in Chapter 13.1 Electric Potential Energy: Potential Difference, this is analogous to taking sea level as $h = 0$ when considering gravitational potential energy, $\text{PE}_g = mgh$.

## Section Summary

- Electric potential of a point charge is $V = kQ/r$.
- Electric potential is a scalar, and electric field is a vector. Addition of voltages as numbers gives the voltage due to a combination of point charges, whereas addition of individual fields as vectors gives the total electric field.

### *Conceptual Questions*

**1:** In what region of space is the potential due to a uniformly charged sphere the same as that of a point charge? In what region does it differ from that of a point charge?

**2:** Can the potential of a non-uniformly charged sphere be the same as that of a point charge? Explain.

### *Problems & Exercises*

**1:** A 0.500 cm diameter plastic sphere, used in a static electricity demonstration, has a uniformly distributed 40.0 pC charge on its surface. What is the potential near its surface?

**2:** What is the potential $0.530 \times 10^{-10}\ \text{m}$ from a

proton (the average distance between the proton and electron in a hydrogen atom)?

**3:** (a) A sphere has a surface uniformly charged with 1.00 C. At what distance from its center is the potential 5.00 MV? (b) What does your answer imply about the practical aspect of isolating such a large charge?

**4:** How far from a $\mathbf{1.00 \mu C}$ point charge will the potential be 100 V? At what distance will it be $\mathbf{2.00 \times 10^2\ V}$?

**5:** What are the sign and magnitude of a point charge that produces a potential of $\mathbf{-2.00\ V}$ at a distance of 1.00 mm?

**6:** If the potential due to a point charge is $\mathbf{5.00 \times 10^2\ V}$ at a distance of 15.0 m, what are the sign and magnitude of the charge?

**7:** In nuclear fission, a nucleus splits roughly in half. (a) What is the potential $\mathbf{2.00 \times 10^{-14}\ m}$ from a fragment that has 46 protons in it? (b) What is the potential energy in MeV of a similarly charged fragment at this distance?

**8:** A research Van de Graaff generator has a 2.00-m-diameter metal sphere with a charge of 5.00 mC on it. (a) What is the potential near its surface? (b) At what distance from its center is the potential 1.00 MV? (c) An oxygen atom with three missing electrons is released near the Van de Graaff generator. What is its energy in MeV at this distance?

**9:** An electrostatic paint sprayer has a 0.200-m-diameter metal sphere at a potential of 25.0 kV that repels paint droplets onto a grounded object. (a) What charge is on the

sphere? (b) What charge must a 0.100-mg drop of paint have to arrive at the object with a speed of 10.0 m/s?

**10:** In one of the classic nuclear physics experiments at the beginning of the 20th century, an alpha particle was accelerated toward a gold nucleus, and its path was substantially deflected by the Coulomb interaction. If the energy of the doubly charged alpha nucleus was 5.00 MeV, how close to the gold nucleus (79 protons) could it come before being deflected?

**11:** (a) What is the potential between two points situated 10 cm and 20 cm from a $3.0\mu C$ point charge? (b) To what location should the point at 20 cm be moved to increase this potential difference by a factor of two?

**12: Unreasonable Results**

(a) What is the final speed of an electron accelerated from rest through a voltage of 25.0 MV by a negatively charged Van de Graaff terminal?

(b) What is unreasonable about this result?

(c) Which assumptions are responsible?

## *Solutions*

**Problems & Exercises**

**1:** 144 V

**3:** (a) 1.80 km

(b) A charge of 1 C is a very large amount of charge; a sphere of radius 1.80 km is not practical.

**5:** $\mathbf{-2.22 \times 10^{-13}\ C}$

**7:** (a) $\mathbf{3.31 \times 10^{6}\ V}$

(b) 152 MeV

**9:** (a) $\mathbf{2.78 \times 10^{-7}\ C}$

(b) $\mathbf{2.00 \times 10^{-10}\ C}$

**12:** (a) $\mathbf{2.96 \times 10^{9}\ m/s}$

(b) This velocity is far too great. It is faster than the speed of light.

(c) The assumption that the speed of the electron is far less than that of light and that the problem does not require a relativistic treatment produces an answer greater than the speed of light.

PART XV

# CHAPTER 14 ELECTRIC CURRENT, RESISTANCE, AND OHM'S LAW



***Figure 1.*** *Electric energy in massive quantities is transmitted from this hydroelectric facility, the Srisailam power station located along the Krishna River in India, by the movement of charge—that is, by electric current. (credit: Chintohere, Wikimedia Commons)*

The flicker of numbers on a handheld calculator, nerve impulses carrying signals of vision to the brain, an ultrasound device sending a signal to a computer screen, the brain sending a message for a baby to twitch its toes, an electric train pulling its load over a mountain pass, a hydroelectric plant sending energy to metropolitan and rural users—these and many other examples of electricity involve *electric current, the movement of charge.* Humankind has indeed harnessed electricity, the basis of technology, to improve our quality of life. Whereas the previous two chapters concentrated on static electricity and the fundamental

force underlying its behavior, the next few chapters will be devoted to electric and magnetic phenomena involving current. In addition to exploring applications of electricity, we shall gain new insights into nature—in particular, the fact that all magnetism results from electric current.

# 78. 14.1 Current

*Summary*

- Define electric current, ampere, and drift velocity
- Describe the direction of charge flow in conventional current.
- Use drift velocity to calculate current and vice versa.

## Electric Current

Electric current is defined to be the rate at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, **electric current** $I$ is defined to be

$$I = \frac{\Delta Q}{\Delta t}$$

where $\Delta Q$ is the amount of charge passing through a given area in time $\Delta t$. (As in previous chapters, initial time is often taken to be zero, in which case $\Delta t = t$.) (See Figure 1.) The SI unit for current is the **ampere** (A), named for the French physicist André-

Marie Ampère (1775–1836). Since $I = \Delta Q/\Delta t$, we see that an ampere is one coulomb per second:

$$1\ \text{A} = 1\ \text{C/s}$$

Not only are fuses and circuit breakers rated in amperes (or amps), so are many electrical appliances.

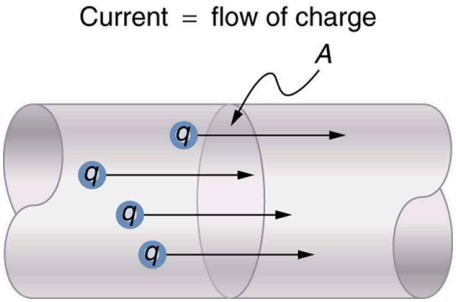



***Figure 1.*** *The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second.*

## Example 1: Calculating Currents: Current in a Truck Battery and a Handheld Calculator

(a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to

flow through a handheld calculator if a 0.300-mA current is flowing?

**Strategy**

We can use the definition of current in the equation $I = \Delta Q/\Delta t$ to find the current in part (a), since charge and time are given. In part (b), we rearrange the definition of current and use the given values of charge and current to find the time required.

**Solution for (a)**

Entering the given values for charge and time into the definition of current gives

$$I \quad \frac{\Delta Q}{\Delta t} = \frac{720\ \text{C}}{4.00\ \text{s}} = 180\ \text{C/s}$$

$$180\ \text{A}.$$

**Discussion for (a)**

This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these "starter motors" are fairly large because large frictional forces need to be overcome when setting something in motion.

**Solution for (b)**

Solving the relationship $I = \Delta Q/\Delta t$ for time $\Delta t$, and entering the known values for charge and current gives

$$\Delta t \quad \frac{\Delta Q}{I} = \frac{1.00\ \text{C}}{0.300 \times 10^{-3}\ \text{C/s}}$$

$$3.33 \times 10^{3}\ \text{s}.$$

**Discussion for (b)**

This time is slightly less than an hour. The small current used by the hand-held calculator takes a much longer time to move a smaller charge than the large current of the truck starter. So why can we operate our calculators only seconds after turning them on? It's because calculators require very little energy. Such small current and energy demands allow handheld calculators to operate from solar cells or to get many hours of use out of small batteries. Remember, calculators do not have moving parts in the same way that a truck engine has with cylinders and pistons, so the technology requires smaller currents.

Figure 2 shows a simple circuit and the standard schematic representation of a battery, conducting path, and load (a resistor). Schematics are very useful in visualizing the main features of a circuit. A single schematic can represent a wide variety of situations. The schematic in Figure 2 (b), for example, can represent anything from a truck battery connected to a headlight lighting the street in front of the truck to a small battery connected to a penlight lighting a keyhole in a door. Such schematics are useful because the analysis is the same for a wide variety of situations. We need to understand a few schematics to apply the concepts and analysis to many more situations.

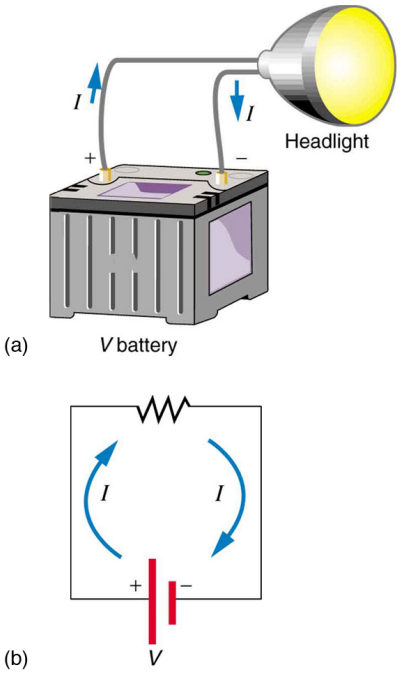



***Figure 2.*** *(a) A simple electric circuit. A closed path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by the two parallel red lines, conducting wires are shown as straight lines, and the zigzag represents the load. The schematic represents a wide variety of similar circuits.*

Note that the direction of current flow in Figure 2 is from positive to negative. *The direction of conventional current is the direction that positive charge would flow*. Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de

Graaff generator used for nuclear research can produce a current of pure positive charges, such as protons. Figure 3 illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American politician and scientist Benjamin Franklin in the 1700s. He named the type of charge associated with electrons negative, long before they were known to carry current in so many situations. Franklin, in fact, was totally unaware of the small-scale structure of electricity.

It is important to realize that there is an electric field in conductors responsible for producing the current, as illustrated in Figure 3. Unlike static electricity, where a conductor in equilibrium cannot have an electric field in it, conductors carrying a current have an electric field and are not in static equilibrium. An electric field is needed to supply energy to move the charges.

### Making Connections: Take-Home Investigation—Electric Current Illustration

Find a straw and little peas that can move freely in the straw. Place the straw flat on a table and fill the straw with peas. When you pop one pea in at one end, a different pea should pop out the other end. This demonstration is an analogy for an electric current. Identify what compares to the electrons and what compares to the supply of energy. What other analogies can you find for an electric current?

Note that the flow of peas is based on the peas physically bumping into each other; electrons flow due to mutually repulsive electrostatic forces.

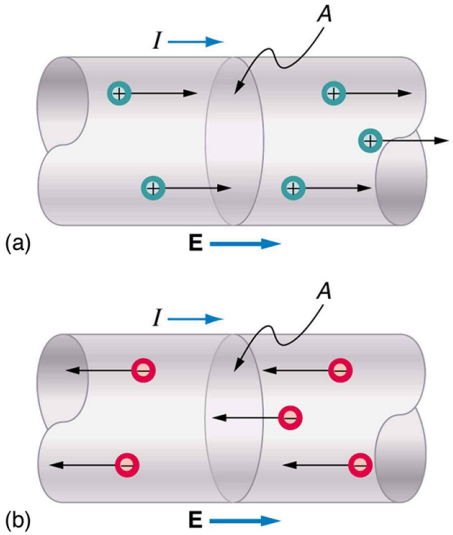



***Figure 3.*** *Current* **I** *is the rate at which charge moves through an area* **A**, *such as the cross-section of a wire. Conventional current is defined to move in the direction of the electric field. (a) Positive charges move in the direction of the electric field and the same direction as conventional current. (b) Negative charges move in the direction opposite to the electric field. Conventional current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.*

## Example 2: Calculating the Number of Electrons that Move through a Calculator

If the 0.300-mA current through the calculator mentioned in the Example 1 example is carried by electrons, how many electrons per second pass through it?

**Strategy**

The current calculated in the previous example was defined for the flow of positive charge. For electrons, the magnitude is the same, but the sign is opposite, $I_{\text{electrons}} = -0.300 \times 10^{-3}\ \text{C/s}$. Since each electron ($e^-$) has a charge of $-1.60 \times 10^{-19}\ \text{C}$, we can convert the current in coulombs per second to electrons per second.

**Solution**

Starting with the definition of current, we have

$$I_{\text{electrons}} = \frac{\Delta Q_{\text{electrons}}}{\Delta t} = \frac{-0.300 \times 10^{-3}\ \text{C}}{\text{s}}.$$

We divide this by the charge per electron, so that

$$\frac{e^-}{\text{s}} \quad \frac{-0.300\times10^{-3}\ \text{C}}{\text{s}} \times \frac{1\ e^-}{-1.60\times10^{-19}\ \text{C}}$$

$$1.88 \times 10^{15}\ \frac{e^-}{\text{s}}$$

**Discussion**

There are so many charged particles moving, even in small currents, that individual charges are not noticed, just as individual water molecules are not noticed in water flow. Even more amazing is that they do not always keep moving forward like soldiers in a parade. Rather they are like a crowd of people with movement in different directions but a general trend to move forward. There are lots of collisions with atoms in the metal wire and, of course, with other electrons.

## Drift Velocity

Electrical signals are known to move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a switch is flicked. Most electrical signals carried by currents travel at speeds on the order of $10^8\ \text{m/s}$, a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move *much* more slowly on average, typically drifting at speeds on the order of $10^{-4}\ \text{m/s}$. How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that

the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in Figure 4, the incoming charge pushes other charges ahead of it, which in turn push on charges farther down the line. The density of charge in a system cannot easily be increased, and so the signal is passed on rapidly. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this rapidly moving signal or shock wave is a rapidly propagating change in electric field.s

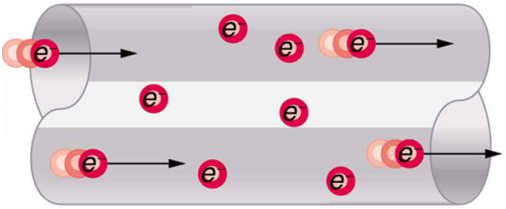


***Figure 4.*** *When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.*

Good conductors have large numbers of free charges in them. In metals, the free charges are free electrons. Figure 5 shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electric field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The **drift velocity** $v_d$ is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges. If we have an estimate of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.

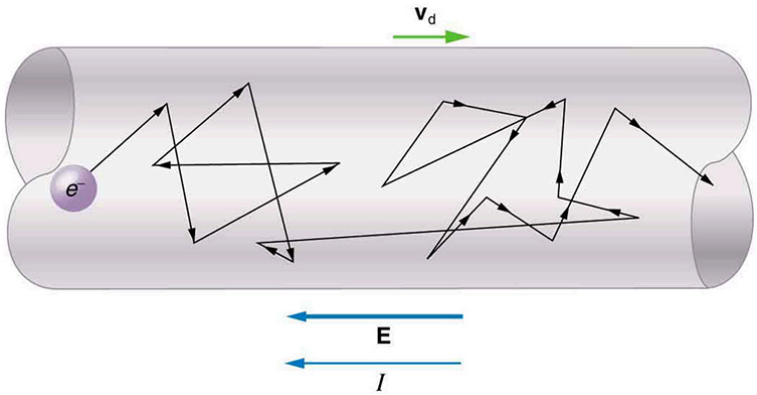



***Figure 5.*** *Free electrons moving in a conductor make many collisions with other electrons and atoms. The path of one electron is shown. The average velocity of the free charges is called the drift velocity,* $\mathbf{v_d}$*, and it is in the direction opposite to the electric field for electrons. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.*

## Conduction of Electricity and Heat

Good electrical conductors are often good heat conductors, too. This is because large numbers of free electrons can carry electrical current and can transport thermal energy.

The free-electron collisions transfer energy to the atoms of the conductor. The electric field does work in moving the electrons through a distance, but that work does not increase the kinetic energy (nor speed, therefore) of the electrons. The work is

transferred to the conductor's atoms, possibly increasing temperature. Thus a continuous power input is required to keep a current flowing. An exception, of course, is found in superconductors, for reasons we shall explore in a later chapter. Superconductors can have a steady current without a continual supply of energy—a great energy savings. In contrast, the supply of energy can be useful, such as in a lightbulb filament. The supply of energy is necessary to increase the temperature of the tungsten filament, so that the filament glows.

### Making Connections: Take-Home Investigation—Filament Observations

Find a lightbulb with a filament. Look carefully at the filament and describe its structure. To what points is the filament connected?

We can obtain an expression for the relationship between current and drift velocity by considering the number of free charges in a segment of wire, as illustrated in Figure 6. *The number of free charges per unit volume* is given the symbol $n$ and depends on the material. The shaded segment has a volume $Ax$, so that the number of free charges in it is $nAx$. The charge $\Delta Q$ in this segment is thus $qnAx$, where $q$ is the amount of charge on each carrier. (Recall that for electrons, $q$ is $-1.60 \times 10^{-19}\ \text{C}$.) Current is charge moved per unit time; thus, if all the original charges move out of this segment in time $\Delta t$, the current is

$$I = \frac{\Delta Q}{\Delta t} = \frac{qnAx}{\Delta t}.$$

Note that $x/\Delta t$ is the magnitude of the drift velocity, $v_d$, since the charges move an average distance $x$ in a time $\Delta t$. Rearranging terms gives

$$I = nqAv_d,$$

where $I$ is the current through a wire of cross-sectional area $A$ made of a material with a free charge density $n$. The carriers of the current each have charge $q$ and move with a drift velocity of magnitude $v_d$.

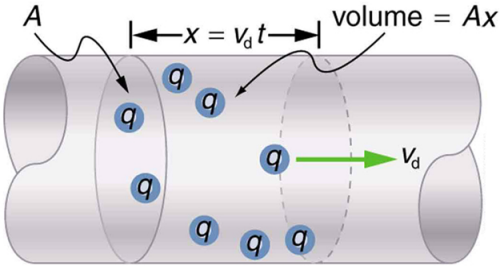



***Figure 6.*** *All the charges in the shaded volume of this wire move out in a time* **t**, *having a drift velocity of magnitude* $v_d = x/t$. *See text for further discussion.*

Note that simple drift velocity is not the entire story. The speed of an electron is much greater than its drift velocity. In addition, not all of the electrons in a conductor can move freely, and those that do might move somewhat faster or slower than the drift velocity. So what do we mean by free electrons? Atoms in a metallic conductor are packed in the form of a lattice structure. Some electrons are far enough away from the atomic nuclei that they do not experience the attraction of the nuclei as much as the inner electrons do. These are the free electrons. They are not bound to a single atom but

can instead move freely among the atoms in a "sea" of electrons. These free electrons respond by accelerating when an electric field is applied. Of course as they move they collide with the atoms in the lattice and other electrons, generating thermal energy, and the conductor gets warmer. In an insulator, the organization of the atoms and the structure do not allow for such free electrons.

## Example 3: Calculating Drift Velocity in a Common Wire

Calculate the drift velocity of electrons in a 12-gauge copper wire (which has a diameter of 2.053 mm) carrying a 20.0-A current, given that there is one free electron per copper atom. (Household wiring often contains 12-gauge copper wire, and the maximum current allowed in such wire is usually 20 A.) The density of copper is $\mathbf{8.80 \times 10^3\ kg/m^3}$.

**Strategy**

We can calculate the drift velocity using the equation $\boldsymbol{I = nqAv_d}$. The current $\boldsymbol{I = 20.0\ A}$ is given, and $\boldsymbol{q = -1.60 \times 10^{-19}\ C}$ is the charge of an electron. We can calculate the area of a cross-section of the wire using the formula $\boldsymbol{A = \pi r^2}$, where $\boldsymbol{r}$ is one-half the given diameter, 2.053 mm. We are given the density of copper, $\mathbf{8.80 \times 10^3\ kg/m^3}$, and the periodic table shows that the atomic mass of copper is 63.54 g/mol.

We can use these two quantities along with Avogadro's number, $\mathbf{6.02 \times 10^{23}\ atoms/mol}$, to determine $\boldsymbol{n}$, the number of free electrons per cubic meter.

**Solution**

First, calculate the density of free electrons in copper. There is one free electron per copper atom. Therefore, is the same as the number of copper atoms per $\mathbf{m^3}$. We can now find $\boldsymbol{n}$ as follows:

$$n \quad \frac{1\ e^-}{\text{atom}} \times \frac{6.02 \times 10^{23}\ \text{atoms}}{\text{mol}} \times \frac{1\ \text{mol}}{63.54\ \text{g}} \times \frac{1000\ \text{g}}{\text{kg}} \times \frac{8.80 \times 10^3\ \text{kg}}{1\ \text{m}^3}$$

$$8.342 \times 10^{28}\ e^-/\text{m}^3$$

The cross-sectional area of the wire is

$$A \quad \pi r^2$$

$$\pi\left(\frac{2.053 \times 10^{-3}\ \text{m}}{2}\right)^2$$

$$3.310 \times 10^{-6}\ \text{m}^2$$

Rearranging $I = nqAv_\text{d}$ to isolate drift velocity gives

$$v_\text{d} = \frac{I}{nqA}$$

$$= \frac{20.0\ \text{A}}{(8.342 \times 10^{28}/\text{m}^3)(-1.60 \times 10^{-19}\ \text{C})(3.310 \times 10^{-6}\ \text{m}^2)}$$

$$= -4.53 \times 10^{-4}\ \text{m/s}$$

**Discussion**

The minus sign indicates that the negative charges are moving in the direction opposite to conventional current. The small value for drift velocity (on the order of $10^{-4}\ \mathrm{m/s}$) confirms that the signal moves on the order of $10^{12}$ times faster (about $10^{8}\ \mathrm{m/s}$) than the charges that carry it.

## Section Summary

- Electric current $I$ is the rate at which charge flows, given by
$$I = \frac{\Delta Q}{\Delta t},$$
where $\Delta Q$ is the amount of charge passing through an area in time $\Delta t$.
- The direction of conventional current is taken as the direction in which positive charge moves.
- The SI unit for current is the ampere (A), where $1\ \mathrm{A} = 1\ \mathrm{C/s}$.
- Current is the flow of free charges, such as electrons and ions.
- Drift velocity $v_{\mathrm{d}}$ is the average speed at which these charges move.
- Current $I$ is proportional to drift velocity $v_{\mathrm{d}}$, as expressed in the relationship $I = nqAv_{\mathrm{d}}$. Here, $I$ is the current through a wire of cross-sectional area $A$. The wire's material has a free-charge density $n$, and each carrier has charge $q$

and a drift velocity $v_d$.

- Electrical signals travel at speeds about $10^{12}$ times greater than the drift velocity of free electrons.

## *Conceptual Questions*

**1:** Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

**2:** Car batteries are rated in ampere-hours ($A \cdot h$). To what physical quantity do ampere-hours correspond (voltage, charge, . . .), and what relationship do ampere-hours have to energy content?

**3:** If two different wires having identical cross-sectional areas carry the same current, will the drift velocity be higher or lower in the better conductor? Explain in terms of the equation $v_d = \frac{I}{nqA}$, by considering how the density of charge carriers $n$ relates to whether or not a material is a good conductor.

**4:** Why are two conducting paths from a voltage source to an electrical device needed to operate the device?

**5:** In cars, one battery terminal is connected to the metal body. How does this allow a single wire to supply current to electrical devices rather than two wires?

**6:** Why isn't a bird sitting on a high-voltage power line electrocuted? Contrast this with the situation in which a large bird hits two wires simultaneously with its wings.

## *Problems & Exercises*

**1:** What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?

**2:** A total of 600 C of charge passes through a flashlight in 0.500 h. What is the average current?

**3:** What is the current when a typical static charge of $\mathbf{0.250\ \mu C}$ moves from your finger to a metal doorknob in $\mathbf{1.00\ \mu s}$?

**4:** Find the current when 2.00 nC jumps between your comb and hair over a 0.500 – $\boldsymbol{\mu}\mathbf{s}$ time interval.

**5:** A large lightning bolt had a 20,000-A current and moved 30.0 C of charge. What was its duration?

**6:** The 200-A current through a spark plug moves 0.300 mC of charge. How long does the spark last?

**7:** (a) A defibrillator sends a 6.00-A current through the chest of a patient by applying a 10,000-V potential as in the figure below. What is the resistance of the path? (b) The defibrillator paddles make contact with the patient through a conducting gel that greatly reduces the path resistance. Discuss the difficulties that would ensue if a larger voltage were used to produce the same current through the patient, but with the path having perhaps 50 times the resistance. (Hint: The current must be about the same, so a higher voltage would imply greater power. Use this equation for power: $\boldsymbol{P = I^2 R}$.)

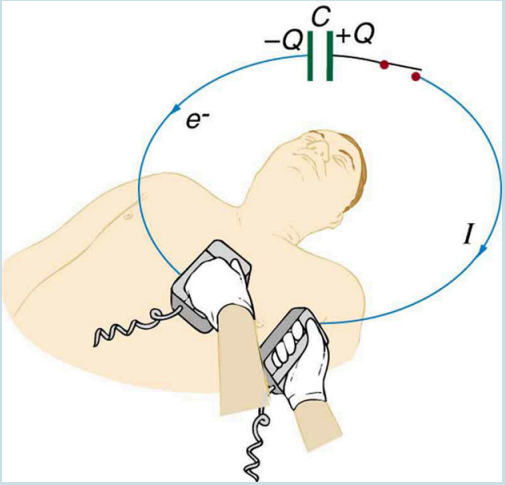



***Figure 7.*** *The capacitor in a defibrillation unit drives a current through the heart of a patient*

**8:** During open-heart surgery, a defibrillator can be used to bring a patient out of cardiac arrest. The resistance of the path is $500\ \Omega$ and a 10.0-mA current is needed. What voltage should be applied?

**9:** (a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves? (b) How many electrons pass through the wires connected to the patient? (See figure two problems earlier.)

**10:** A clock battery wears out after moving 10,000 C of charge through the clock at a rate of 0.500 mA. (a) How long did the clock run? (b) How many electrons per second flowed?

**11:** The batteries of a submerged non-nuclear submarine

supply 1000 A at full speed ahead. How long does it take to move Avogadro's number ($\mathbf{6.02 \times 10^{23}}$) of electrons at this rate?

**12:** Electron guns are used in X-ray tubes. The electrons are accelerated through a relatively large voltage and directed onto a metal target, producing X-rays. (a) How many electrons per second strike the target if the current is 0.500 mA? (b) What charge strikes the target in 0.750 s?

**13:** A large cyclotron directs a beam of $\mathbf{He}^{++}$ nuclei onto a target with a beam current of 0.250 mA. (a) How many $\mathbf{He}^{++}$ nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of $\mathbf{He}^{++}$ nuclei strike the target?

**14:** Repeat the above example on Example 3, but for a wire made of silver and given there is one free electron per silver atom.

**15:** Using the results of the above example on Example 3, find the drift velocity in a copper wire of twice the diameter and carrying 20.0 A.

**16:** A 14-gauge copper wire has a diameter of 1.628 mm. What magnitude current flows when the drift velocity is 1.00 mm/s? (See above example on Example 3 for useful information.)

**17:** SPEAR, a storage ring about 72.0 m in diameter at the Stanford Linear Accelerator (closed in 2009), has a 20.0-A circulating beam of electrons that are moving at nearly the speed of light. (See Figure 8.) How many electrons are in the beam?

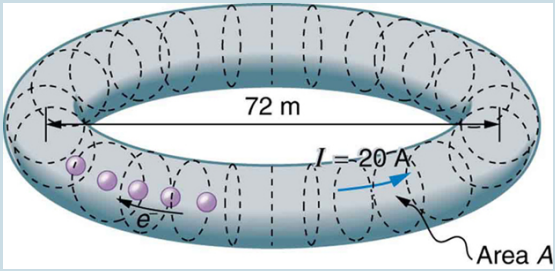



***Figure 8.*** *Electrons circulating in the storage ring called SPEAR constitute a 20.0-A current. Because they travel close to the speed of light, each electron completes many orbits in each second.*

## Glossary

**electric current**
: the rate at which charge flows, $I = \Delta Q/\Delta t$

**ampere**
: (amp) the SI unit for current; 1 A = 1 C/s

**drift velocity**
: the average velocity at which free charges flow in response to an electric field

## *Solutions*

**Problems & Exercises**

**1:** 0.278 mA

**3:** 0.250 A

**5:** 1.50ms

**7:** (a) $\mathbf{1.67\ k\Omega}$

(b) If a 50 times larger resistance existed, keeping the current about the same, the power would be increased by a factor of about 50 (based on the equation $\boldsymbol{P = I^2 R}$), causing much more energy to be transferred to the skin, which could cause serious burns. The gel used reduces the resistance, and therefore reduces the power transferred to the skin.

**9:** (a) 0.120 C

(b) $\mathbf{7.50 \times 10^{17}}$ electrons

**11:** 96.3 s

**13:** (a)

$$\mathbf{7.81 \times 10^{14}\ He^{++}\ nuclei/s}$$

(b)

$$\mathbf{4.00 \times 10^{3}\ s}$$

(c)

$$\mathbf{7.71 \times 10^{8}\ s}$$

**15:** $\mathbf{-1.13 \times 10^{-4}\ m/s}$

**17:** $\mathbf{9.42 \times 10^{13}\ electrons}$

# 79. 14.2 Ohm's Law: Resistance and Simple Circuits

*Summary*

- Explain the origin of Ohm's law.
- Calculate voltages, currents, or resistances with Ohm's law.
- Explain what an ohmic material is.
- Describe a simple circuit.

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference $\boldsymbol{V}$ that creates an electric field. The electric field in turn exerts force on charges, causing current.

## Ohm's Law

The current that flows through most substances is directly proportional to the voltage $\boldsymbol{V}$ applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate

experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

$$I \propto V.$$

This important relationship is known as Ohm's law. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn't always occur.

## Resistance and Simple Circuits

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called resistance **R**. Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

$$I \propto \frac{1}{R}.$$

Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives

$$I = \frac{V}{R}.$$

This relationship is also called Ohm's law. Ohm's law in this form really defines resistance for certain materials. Ohm's law (like Hooke's law) is not universally valid. The many substances for which Ohm's law holds are called ohmic. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance $R$ that is

independent of voltage $V$ and current $I$. An object that has simple resistance is called a *resistor*, even if its resistance is small. The unit for resistance is an ohm and is given the symbol $\Omega$ (upper case Greek omega). Rearranging $I = V/R$ gives $R = V/I$, and so the units of resistance are 1 ohm = 1 volt per ampere:

$$1\ \Omega = 1\ \frac{V}{A}$$

Figure 1 shows the schematic for a simple circuit. A simple circuit has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in $R$.

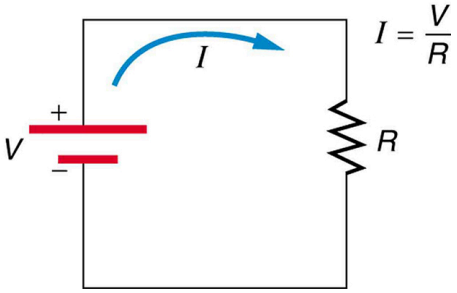



***Figure 1.*** *A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines. The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source.*

## Example 1: Calculating Resistance: An Automobile Headlight

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

**Strategy**

We can rearrange Ohm's law as stated by $I = V/R$ and use it to find the resistance.

**Solution**

Rearranging $I = V/R$ and substituting known values gives

$$R = \frac{V}{I} = \frac{12.0\ \text{V}}{2.50\ \text{A}} = 4.80\ \Omega$$

**Discussion**

This is a relatively small resistance, but it is larger than the cold resistance of the headlight.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^1 2\ \Omega$ or more. A dry person may have a hand-to-foot resistance of $10^5\ \Omega$, whereas the resistance of the human heart is about $10^3\ \Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5}\ \Omega$, and superconductors have no resistance at all (they are non-ohmic).

Additional insight is gained by solving $I = V/R$ yielding

$$V = IR.$$

This expression for $V$ can be interpreted as the *voltage drop across a resistor produced by the flow of current* $I$. The phrase $IR$ *drop* is often used for this voltage. For instance, the headlight in Example 1 has an $IR$ drop of 12.0 V. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since $PE = q\Delta V$, and the same $q$ flows through each. Thus the energy supplied by the voltage source and the energy converted by the resistor are equal. (See Figure 2.)

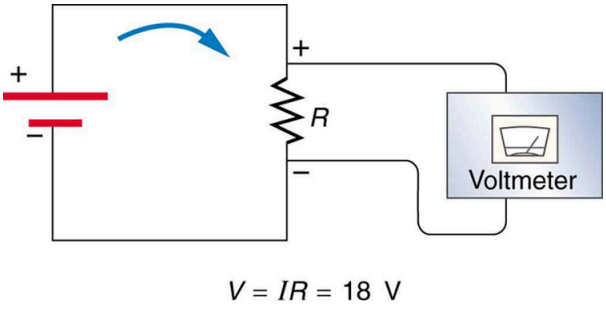



***Figure 2.*** *The voltage drop across a resistor in a simple circuit equals the voltage output of the battery.*

## Making Connections: Conservation of Energy

In a simple electrical circuit, the sole resistor converts energy supplied by the source into another form. Conservation of energy is evidenced here by the fact that all of the energy supplied by the source is converted to another form by the resistor alone. We will find that conservation of energy has other important applications in circuits and is a powerful tool in circuit analysis.

## PhET Explorations: Ohm's Law

See how the equation form of Ohm's law relates to a simple circuit. Adjust the voltage and resistance, and see the current change according to Ohm's law. The sizes of the symbols in the equation change to match the circuit diagram.

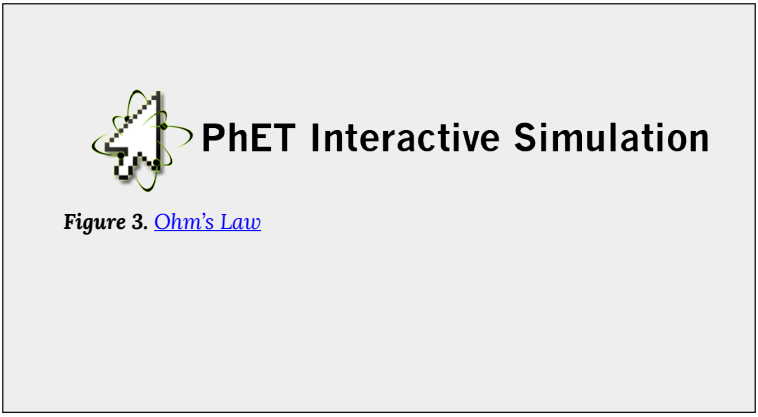



***Figure 3.*** *Ohm's Law*

## Section Summary

- A simple circuit is one in which there is a single voltage source and a single resistance.
- One statement of Ohm's law gives the relationship between current $\boldsymbol{I}$, voltage $\boldsymbol{V}$, and resistance $\boldsymbol{R}$ in a simple circuit to be $\boldsymbol{I = \frac{V}{R}}$.
- Resistance has units of ohms ($\boldsymbol{\Omega}$), related to volts and amperes by $\mathbf{1\ \Omega = 1\ V/A}$.
- There is a voltage or $\boldsymbol{IR}$ drop across a resistor, caused by the current flowing through it, given by $\boldsymbol{V = IR}$.

### *Conceptual Questions*

**1:** The $\boldsymbol{IR}$ drop across a resistor means that there is a

change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

**2:** How is the $IR$ drop in a resistor similar to the pressure drop in a fluid flowing through a pipe?

## *Problems & Exercises*

**1:** What current flows through the bulb of a 3.00-V flashlight when its hot resistance is $3.60\ \Omega$?

**2:** Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

**3:** What is the effective resistance of a car's starter motor when 150 A flows through it as the car battery applies 11.0 V to the motor?

**4:** How many volts are supplied to operate an indicator light on a DVD player that has a resistance of $140\ \Omega$, given that 25.0 mA passes through it?

**5:** (a) Find the voltage drop in an extension cord having a $0.0600\ \Omega$ resistance and through which 5.00 A is flowing. (b) A cheaper cord utilizes thinner wire and has a resistance of $0.300\ \Omega$. What is the voltage drop in it when 5.00 A flows? (c) Why is the voltage to whatever appliance is being used reduced by this amount? What is the effect on the appliance?

**6:** A power transmission line is hung from metal towers with glass insulators having a resistance of $\mathbf{1.00 \times 10^9\ \Omega}$. What current flows through the insulator if the voltage is 200 kV? (Some high-voltage lines are DC.)

## Glossary

**Ohm's law**
: an empirical relation stating that the current $I$ is proportional to the potential difference V, $\propto$ V; it is often written as $I = V/R$, where $R$ is the resistance

**resistance**
: the electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, $R = V/I$

**ohm**
: the unit of resistance, given by $1\Omega = 1\ V/A$

**ohmic**
: a type of a material for which Ohm's law is valid

**simple circuit**
: a circuit with a single voltage source and a single resistor

## *Solutions*

**Problems & Exercises**

**1:** 0.833 A

**3:** $\mathbf{7.33 \times 10^{-2}\ \Omega}$

**5:** (a) 0.300 V

(b) 1.50 V

(c) The voltage supplied to whatever appliance is being used is reduced because the total voltage drop from the wall to the final output of the appliance is fixed. Thus, if the voltage drop across the extension cord is large, the voltage drop across the appliance is significantly decreased, so the power output by the appliance can be significantly decreased, reducing the ability of the appliance to work properly.

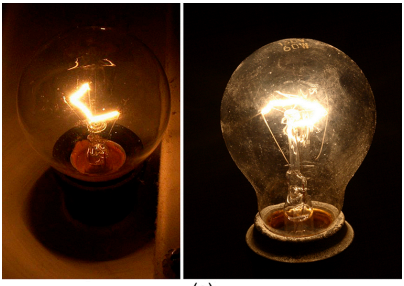
# 80. 14.3 Electric Power and Energy

*Summary*

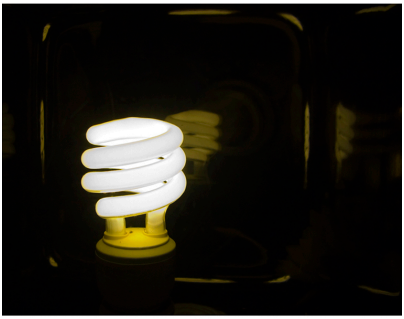
- Calculate the power dissipated by a resistor and power supplied by a power supply.
- Calculate the cost of electricity under various circumstances.

## Power in Electric Circuits

Power is associated by many people with electricity. Knowing that power is the rate of energy use or energy conversion, what is the expression for electric power? Power transmission lines might come to mind. We also think of lightbulbs in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb. (See Figure 1(a).) Since both operate on the same voltage, the 60-W bulb must draw more current to have a greater power rating. Thus the 60-W bulb's resistance must be lower than that of a 25-W bulb. If we increase voltage, we also increase power. For example, when a 25-W bulb that is designed to operate on 120 V is connected to 240 V, it briefly glows very brightly and then burns out. Precisely how are voltage, current, and resistance related to electric power?



(a)



(b)

***Figure 1.*** *(a) Which of these lightbulbs, the 25-W bulb (upper left) or the 60-W bulb (upper right), has the higher resistance? Which draws more current? Which uses the most energy? Can you tell from the color that the 25-W filament is cooler? Is the brighter bulb a different color and if so why? (credits: Dickbauch, Wikimedia Commons; Greg Westfall, Flickr) (b) This compact fluorescent light (CFL) puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit: dbgg1979, Flickr)*

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as $\mathbf{PE} = \boldsymbol{qV}$, where $\boldsymbol{q}$ is the charge moved and $\boldsymbol{V}$ is the voltage (or more precisely, the potential difference the charge moves through). Power is the rate at which energy is moved, and so electric power is

$$P = \frac{PE}{t} = \frac{qV}{t}.$$

Recognizing that current is $I = q/t$ (note that $\Delta t = t$ here), the expression for power becomes

$$P = IV.$$

Electric power ($P$) is simply the product of current times voltage. Power has familiar units of watts. Since the SI unit for potential energy (PE) is the joule, power has units of joules per second, or watts. Thus, $1\ \text{A} \cdot\ \text{V} = 1\ \text{W}$. For example, cars often have one or more auxiliary power outlets with which you can charge a cell phone or other electronic devices. These outlets may be rated at 20 A, so that the circuit can deliver a maximum power $P = IV = (20\ \text{A})(12\ \text{V}) = 240\ \text{W}$. In some applications, electric power may be expressed as volt-amperes or even kilovolt-amperes ($1\ \text{kA} \cdot\ \text{V} = 1\ \text{kW}$).

To see the relationship of power to resistance, we combine Ohm's law with $P = IV$. Substituting $I = V/R$ gives $P = (V/R)V = V^2/R$. Similarly, substituting $V = IR$ gives $P = I(IR) = I^2R$. Three expressions for electric power are listed together here for convenience:

$$P = IV$$
$$P = \frac{V^2}{R}$$
$$P = I^2R.$$

Note that the first equation is always valid, whereas the other two can be used only for resistors. In a simple circuit, with one voltage source and a single resistor, the power supplied by the voltage source and that dissipated by the resistor are identical. (In more complicated circuits, $P$ can be the power dissipated by a single device and not the total power in the circuit.)

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature its resistance is higher, too.

## Example 1: Calculating Power Dissipation and Current: Hot and Cold Power

(a) Consider the examples given in Chapter 14.2 Ohm's Law: Resistance and Simple Circuits. Then find the power dissipated by the car headlight in these examples, both when it is hot and when it is cold. (b) What current does it draw when cold?

**Strategy for (a)**

For the hot headlight, we know voltage and current, so we can use $P = IV$ to find the power. For the cold headlight, we know the voltage and resistance, so we can use $P = V^2/R$ to find the power.

**Solution for (a)**

Entering the known values of current and voltage for the hot headlight, we obtain

$$P = IV = (2.50\ \text{A})(12.0\ \text{V}) = 30.0\ \text{W}.$$

The cold resistance was $0.350\ \Omega$, and so the power it uses when first switched on is

$$P = \frac{V^2}{R} = \frac{(12.0\ \text{V})^2}{0.350\ \Omega} = 411\ \text{W}.$$

**Discussion for (a)**

The 30 W dissipated by the hot headlight is typical. But the 411 W when cold is surprisingly higher. The initial power quickly decreases as the bulb's temperature increases and its resistance increases.

**Strategy and Solution for (b)**

The current when the bulb is cold can be found several different ways. We rearrange one of the power equations, $P = I^2R$, and enter known values, obtaining

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{411\ \text{W}}{0.350\ \Omega}} = 34.3\ \text{A}$$

**Discussion for (b)**

The cold current is remarkably higher than the steady-state value of 2.50 A, but the current will quickly decline to that value as the bulb's temperature increases. Most fuses and circuit breakers (used to limit the current in a circuit) are designed to tolerate very high currents briefly as a device comes on. In some cases, such as with electric motors, the current remains

high for several seconds, necessitating special “slow blow” fuses.

## The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = E/t$, we see that

$$E = Pt$$

is the energy used by a device using power $P$ for a time interval $t$. For example, the more lightbulbs burning, the greater $P$ used; the longer they are on, the greater $t$ is. The energy unit on electric bills is the kilowatt-hour ($\mathbf{kW \cdot h}$), consistent with the relationship $E = Pt$. It is easy to estimate the cost of operating electric appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted to joules. You can prove to yourself that $\mathbf{1\ kW \cdot h = 3.6 \times 10^6\ J}$.

The electrical energy ($E$) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home’s use of energy goes to lighting, while the number for commercial

establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). (See Figure 1(b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last 5 times longer than CFLs. However, their cost is still high.

### Making Connections: Energy, Power, and Time

The relationship $\boldsymbol{E = Pt}$ is one that you will find useful in many different contexts. The energy your body uses in exercise is related to the power level and duration of your activity, for example. The amount of heating by a power source is related to the power level and time it is applied. Even the radiation dose of an X-ray image is related to the power and time of exposure.

## Example 2: Calculating the Cost Effectiveness of Compact Fluorescent Lights (CFL)

If the cost of electricity in your area is 12 cents per kWh, what is the total cost (capital plus operation) of using a 60-W incandescent bulb for 1000 hours (the lifetime of that bulb) if the bulb cost 25 cents? (b) If we replace this bulb with a compact fluorescent light that provides the same light output, but at one-quarter the wattage, and which costs $1.50 but lasts 10 times longer (10,000 hours), what will that total cost be?

**Strategy**

To find the operating cost, we first find the energy used in kilowatt-hours and then multiply by the cost per kilowatt-hour.

**Solution for (a)**

The energy used in kilowatt-hours is found by entering the power and time into the expression for energy:

$$\boldsymbol{E = Pt = (60\ \mathrm{W})(1000\ \mathrm{h}) = 60{,}000\ \mathrm{W \cdot\ h}.}$$

In kilowatt-hours, this is

$$\boldsymbol{E = 60.0\ \mathrm{kW \cdot\ h}.}$$

Now the electricity cost is

$$\mathbf{cost = (60.0\ kW \cdot h)(\$0.12/\ kW \cdot h) = \$7.20}.$$

The total cost will be $7.20 for 1000 hours (about one-half year at 5 hours per day).

**Solution for (b)**

Since the CFL uses only 15 W and not 60 W, the electricity cost will be $7.20/4 = $1.80. The CFL will last 10 times longer than the incandescent, so that the investment cost will be 1/10 of the bulb cost for that time period of use, or 0.1($1.50) = $0.15. Therefore, the total cost will be $1.95 for 1000 hours.

**Discussion**

Therefore, it is much cheaper to use the CFLs, even though the initial investment is higher. The increased cost of labor that a business must include for replacing the incandescent bulbs more often has not been figured in here.

## Making Connections: Take-Home Experiment—Electrical Energy Use Inventory

1) Make a list of the power ratings on a range of appliances in your home or room. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these

appliances in an average day (by estimating their time of use). Some appliances might only state the operating current. If the household voltage is 120 V, then use $P = IV$. 2) Check out the total wattage used in the rest rooms of your school's floor or building. (You might need to assume the long fluorescent lights in use are rated at 32 W.) Suppose that the building was closed all weekend and that these lights were left on from 6 p.m. Friday until 8 a.m. Monday. What would this oversight cost? How about for an entire year of weekends?

## Section Summary

- Electric power $P$ is the rate (in watts) that energy is supplied by a source or dissipated by a device.
- Three expressions for electrical power are

$$P = IV,$$

$$P = \frac{V^2}{R},$$

and

$$P = I^2 R.$$

- The energy used by a device with a power $P$ over a time $t$ is $E = Pt$.

## *Conceptual Questions*

**1:** Why do incandescent lightbulbs grow dim late in their lives, particularly just before their filaments break?

**2:** The power dissipated in a resistor is given by $P = V^2/R$, which means power decreases if resistance increases. Yet this power is also given by $P = I^2R$, which means power increases if resistance increases. Explain why there is no contradiction here.

## *Probles & Exercises*

**1:** What is the power of a $1.00 \times 10^2$ MV lightning bolt having a current of $2.00 \times 10^4$ A?

**2:** What power is supplied to the starter motor of a large truck that draws 250 A of current from a 24.0-V battery hookup?

**3:** A charge of 4.00 C of charge passes through a pocket calculator's solar cells in 4.00 h. What is the power output, given the calculator's voltage output is 3.00 V? (See Figure 2.)




***Figure 2.*** *The strip of solar cells just above the keys of this calculator convert light to electricity to supply its energy needs. (credit: Evan-Amos, Wikimedia Commons)*

**4:** How many watts does a flashlight that has $\mathbf{6.00 \times 10^2\ C}$ pass through it in 0.500 h use if its voltage is 3.00 V?

**5:** Find the power dissipated in each of these extension cords: (a) an extension cord having a $\mathbf{0.0600 -\ \Omega}$ resistance and through which 5.00 A is flowing; (b) a cheaper cord utilizing thinner wire and with a resistance of $\mathbf{0.300\ \Omega}$

**6:** Verify that the units of a volt-ampere are watts, as implied by the equation $\boldsymbol{P = IV}$.

**7:** Show that the units $\mathbf{1\ V^2/\Omega = 1\ W}$, as implied by the equation $\boldsymbol{P = V^2/R}$.

**8:** Show that the units $\mathbf{1\ A^2 \cdot \Omega = 1\ W}$, as implied by the equation $\boldsymbol{P = I^2 R}$.

**9:** Verify the energy unit equivalence that $\mathbf{1\ kW \cdot h = 3.60 \times 10^6\ J}$.

**10:** Electrons in an X-ray tube are accelerated through $\mathbf{1.00 \times 10^2\ kV}$ and directed toward a target to produce X-rays. Calculate the power of the electron beam in this tube if it has a current of 15.0 mA.

**11:** An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs $\mathbf{12.0\ cents/kW \cdot h}$? See Figure 3 below for an example of this type of heater.




***Figure 3.*** *On-demand electric hot water heater. Heat is supplied to water only when needed. (credit: aviddavid, Flickr)*

**12:** With a 1200-W toaster, how much electrical energy is needed to make a slice of toast (cooking time = 1 minute)? At $\mathbf{9.0\ cents/kW \cdot h}$, how much does this cost?

**13:** What would be the maximum cost of a CFL such that

the total cost (investment plus operating) would be the same for both CFL and incandescent 60-W bulbs? Assume the cost of the incandescent bulb is 25 cents and that electricity costs $\mathbf{10\ cents/kWh}$. Calculate the cost for 1000 hours, as in the cost effectiveness of CFL example.

**14:** Some makes of older cars have 6.00-V electrical systems. (a) What is the hot resistance of a 30.0-W headlight in such a car? (b) What current flows through it?

**15:** Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at 1.00 Ah and 1.58 V keep a 1.00-W flashlight bulb burning?

**16:** A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?

**17:** The average television is said to be on 6 hours per day. Estimate the yearly cost of electricity to operate 100 million TVs, assuming their power consumption averages 150 W and the cost of electricity averages $\mathbf{12.0\ cents/kW \cdot h}$.

**18:** An old lightbulb draws only 50.0 W, rather than its original 60.0 W, due to evaporative thinning of its filament. By what factor is its diameter reduced, assuming uniform thinning along its length? Neglect any effects caused by temperature differences.

**19:** 00-gauge copper wire has a diameter of 9.266 mm. Calculate the power loss in a kilometer of such wire when it carries $\mathbf{1.00 \times 10^2\ A}$.

**20: Integrated Concepts**

Cold vaporizers pass a current through water, evaporating it with only a small increase in temperature. One such home device is rated at 3.50 A and utilizes 120 V AC with 95.0% efficiency. (a) What is the vaporization rate in grams per minute? (b) How much water must you put into the vaporizer for 8.00 h of overnight operation? (See Figure 4.)

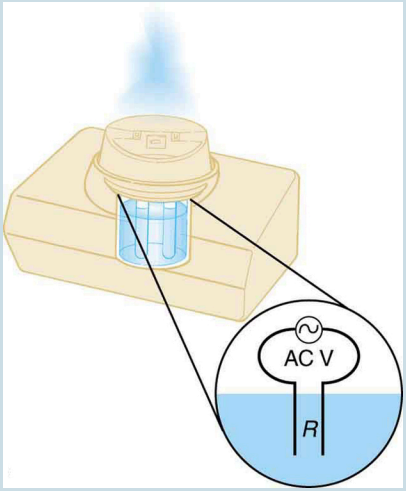



***Figure 4.*** *This cold vaporizer passes current directly through water, vaporizing it directly with relatively little temperature increase.*

**21: Integrated Concepts**

(a) What energy is dissipated by a lightning bolt having a 20,000-A current, a voltage of $\mathbf{1.00 \times 10^2\ MV}$, and a length of 1.00 ms? (b) What mass of tree sap could be raised from 18.0°C to its boiling point and then evaporated

by this energy, assuming sap has the same thermal characteristics as water?

**22: Integrated Concepts**

What current must be produced by a 12.0-V battery-operated bottle warmer in order to heat 75.0 g of glass, 250 g of baby formula, and $\mathbf{3.00 \times 10^2\ g}$ of aluminum from 20.0°C to 90.0°C in 5.00 min?

**23: Integrated Concepts**

How much time is needed for a surgical cauterizer to raise the temperature of 1.00 g of tissue from 37.0°C to 100°C and then boil away 0.500 g of water, if it puts out 2.00 mA at 15.0 kV? Ignore heat transfer to the surroundings.

**24: Integrated Concepts**

**24:** Hydroelectric generators (see Figure 5) at Hoover Dam produce a maximum current of $\mathbf{8.00 \times 10^3\ A}$ at 250 kV. (a) What is the power output? (b) The water that powers the generators enters and leaves the system at low speed (thus its kinetic energy does not change) but loses 160 m in altitude. How many cubic meters per second are needed, assuming 85.0% efficiency?



***Figure 5.*** *Hydroelectric generators at the Hoover dam. (credit: Jon Sullivan)*

**25: Integrated Concepts**

(a) Assuming 95.0% efficiency for the conversion of electrical power by the motor, what current must the 12.0-V batteries of a 750-kg electric car be able to supply: (a) To accelerate from rest to 25.0 m/s in 1.00 min? (b) To climb a $\mathbf{2.00 \times 10^2\ -m}$ -high hill in 2.00 min at a constant 25.0-m/s speed while exerting $\mathbf{5.00 \times 10^2\ N}$ of force to overcome air resistance and friction? (c) To travel at a constant 25.0-m/s speed, exerting a $\mathbf{5.00 \times 10^2\ N}$ force to overcome air resistance and friction? See Figure 6.



***Figure 6.*** *This REVAi, an electric car, gets recharged on a street in London. (credit: Frank Hebbert)*

**26: Integrated Concepts**

A light-rail commuter train draws 630 A of 650-V DC electricity when accelerating. (a) What is its power consumption rate in kilowatts? (b) How long does it take to reach 20.0 m/s starting from rest if its loaded mass is $\mathbf{5.30 \times 10^4\ kg}$, assuming 95.0% efficiency and constant power? (c) Find its average acceleration. (d) Discuss how the acceleration you found for the light-rail train compares to what might be typical for an automobile.

**27: Integrated Concepts**

(a) An aluminum power transmission line has a resistance of $\mathbf{0.0580\ \Omega/km}$. What is its mass per kilometer? (b) What is the mass per kilometer of a copper line having the same resistance? A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

**28: Integrated Concepts**

(a) An immersion heater utilizing 120 V can raise the temperature of a $\mathbf{1.00 \times 10^2\text{-g}}$ aluminum cup containing 350 g of water from 20.0ºC to 95.0ºC in 2.00 min. Find its resistance, assuming it is constant during the process. (b) A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

**29: Integrated Concepts**

(a) What is the cost of heating a hot tub containing 1500 kg of water from 10.0ºC to 40.0ºC, assuming 75.0% efficiency to account for heat transfer to the surroundings? The cost of electricity is 9.0 cents/kW h. (b) What current was used by the 220-V AC electric heater, if this took 4.00 h?

**30: Unreasonable Results**

(a) What current is needed to transmit $\mathbf{1.00 \times 10^2\ MW}$ of power at 480 V? (b) What power is dissipated by the transmission lines if they have a $\mathbf{1.00\ —\ \Omega}$ resistance? (c) What is unreasonable about this result? (d) Which assumptions are unreasonable, or which premises are inconsistent?

**31: Unreasonable Results**

(a) What current is needed to transmit $\mathbf{1.00 \times 10^2\ MW}$ of power at 10.0 kV? (b) Find the resistance of 1.00 km of wire that would cause a 0.0100% power loss. (c) What is the diameter of a 1.00-km-long copper wire having this resistance? (d) What is unreasonable about these results? (e) Which assumptions are unreasonable, or which premises are inconsistent?

**32: Construct Your Own Problem**

Consider an electric immersion heater used to heat a cup of water to make tea. Construct a problem in which you calculate the needed resistance of the heater so that it increases the temperature of the water and cup in a reasonable amount of time. Also calculate the cost of the electrical energy used in your process. Among the things to be considered are the voltage used, the masses and heat capacities involved, heat losses, and the time over which the heating takes place. Your instructor may wish for you to consider a thermal safety switch (perhaps bimetallic) that will halt the process before damaging temperatures are reached in the immersion unit.

## Glossary

**electric power**
the rate at which electrical energy is supplied by a source or dissipated by a device; it is the product of current times voltage

## *Solutions*

**Problems & Exercise**

**1:** $2.00 \times 10^{12}$ W

**5:** (a) 1.50 W (b) 7.50 W

**7:** $\mathbf{\frac{V^2}{\Omega} = \frac{V^2}{V/A} = AV = (\frac{C}{s})(\frac{J}{C})}$

**9:** $\mathbf{1\ kW \cdot h = (\frac{1 \times 10^3}{1\ s})(1\ h)(\frac{3600\ s}{1\ h}) = 3.60 \times 10^6\ J}$

**11:** $438/y

**13:** $6.25

**15:** 1.58 h

**17:** $3.94 billion/year

**19:** 25.5 W

**21:** (a) 2.00 x $10^9$ J (b) 769 kg

**23:** 45.0 s

**25:** (a) 343 A (b) $2.17 \times 10^3$ A (c) 1.10 x $10^3$ A

**27:** (a) 1.23 x $10^3$ kg b) 2.64 x $10^3$ kg

**29:** heat energy needed = 1.88 x $10^8$ J electrical energy required = ( 1.88 x $10^8$ J ) / 0.75 = 2.51 x $10^8$ J = 69.8 kWh = 623 cents = $6 to 1 sig. fig.

**30:** (a) 2.08 x $10^5$ A b) 4.33 x $10^4$ MW (c) The transmission lines dissipate more power than they are supposed to transmit. (d) A voltage of 480 V is unreasonably low for a transmission voltage. Long-distance transmission lines are kept at much higher voltages (often hundreds of kilovolts) to reduce power losses.

# 81. 14.4 Resistors in Series and Parallel

*Summary*

- Draw a circuit with resistors in parallel and in series.
- Calculate the voltage drop of a current across a resistor using Ohm's law.
- Contrast the way total resistance is calculated for resistors in series and in parallel.
- Explain why total resistance of a parallel circuit is less than the smallest resistance of any of the resistors in that circuit.
- Calculate total resistance of a circuit that contains a mixture of resistors connected in series and in parallel.

Electric circuits are commonplace. Some are simple, such as those in flashlights. Others, such as those used in supercomputers, are extremely complex. Most circuits have more than one component, called a **resistor** that limits the flow of charge in the circuit. A measure of this limit on charge flow is called **resistance**. The simplest combinations of resistors are the series and parallel connections illustrated in Figure 1. The total resistance of a combination of resistors depends on both their individual values and how they are connected.

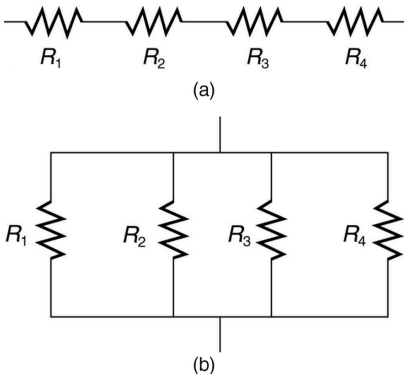



***Figure 1.*** *(a) A series connection of resistors. (b) A parallel connection of resistors.*

## Resistors in Series

When are resistors in **series**? Resistors are in series whenever the flow of charge, called the **current**, must flow through devices sequentially. For example, if current flows through a person holding a screwdriver and into the Earth, then $\mathbf{R_1}$ in Figure 1(a) could be the resistance of the screwdriver's shaft, $\mathbf{R_2}$ the resistance of its handle, $\mathbf{R_3}$ the person's body resistance, and $R_4$ the resistance of her shoes.

Figure 2 shows resistors in series connected to a **voltage** source. It seems reasonable that the total resistance is the sum of the individual resistances, considering that the current has to pass through each resistor in sequence. (This fact would be an advantage to a person wishing to avoid an electrical shock, who could reduce the current by wearing high-resistance rubber-soled shoes. It could be a disadvantage if one of the resistances were a faulty high-resistance cord to an appliance that would reduce the operating current.)

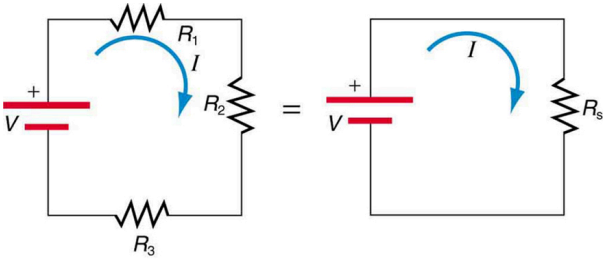



***Figure 2.*** *Three resistors connected in series to a battery (left) and the equivalent single or series resistance (right).*

To verify that resistances in series do indeed add, let us consider the loss of electrical power, called a **voltage drop**, in each resistor in Figure 2.

According to **Ohm's law**, the voltage drop, $V$, across a resistor when a current flows through it is calculated using the equation $V = IR$, where $I$ equals the current in amps (A) and $R$ is the resistance in ohms $(\Omega)$. Another way to think of this is that $V$ is the voltage necessary to make a current $I$ flow through a resistance $R$.

So the voltage drop across $R_1$ is $V_1 = IR_1$, that across $R_2$ is $V_2 = IR_2$, and that across $R_3$ is $V_3 = IR_3$. The sum of these voltages equals the voltage output of the source; that is,

$$V = V_1 + V_2 + V_3.$$

This equation is based on the conservation of energy and conservation of charge. Electrical potential energy can be described by the equation **PE = qV** where **q** is the electric charge and **V** is the voltage. Thus the energy supplied by the source is **PE = q V** while that dissipated by the resistors is

$$qV_1 + qV_2 + qV_3.$$

## Connections: Conservation Laws

The derivations of the expressions for series and parallel resistance are based on the laws of conservation of energy and conservation of charge, which state that total charge and total energy are constant in any process. These two laws are directly involved in all electrical phenomena and will be invoked repeatedly to explain both specific effects and the general behavior of electricity.

These energies must be equal, because there is no other source and no other destination for energy in the circuit. Thus, $\boldsymbol{qV = qV_1 + qV_2 + qV_3}$. The charge $\mathbf{q}$ cancels, yielding $\boldsymbol{V = V_1 + V_2 + V_3}$, as stated. (Note that the same amount of charge passes through the battery and each resistor in a given amount of time, since there is no capacitance to store charge, there is no place for charge to leak, and charge is conserved.)

Now substituting the values for the individual voltages gives

$$\boldsymbol{V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3).}$$

Note that for the equivalent single series resistance $\boldsymbol{R_s}$, we have

$$\boldsymbol{V = IR_s.}$$

This implies that the total or equivalent series resistance $\boldsymbol{R_s}$ of three resistors is

$$\boldsymbol{R_s = R_1 + R_2 + R_3.}$$

This logic is valid in general for any number of resistors in series; thus, the total resistance $\boldsymbol{R_s}$ of a series connection is

$$\$ R_s = R_1 + R_2 + R_3 + \ldots$$

as proposed. Since all of the current must pass through each resistor, it experiences the resistance of each, and resistances in series simply add up.

## Example 1: Calculating Resistance, Current, Voltage Drop, and Power Dissipation: Analysis of a Series Circuit

Suppose the voltage output of the battery in Figure 2 is $\mathbf{12.0\ V}$, and the resistances are $\boldsymbol{R_1 = 1.00\ \Omega}$, $\boldsymbol{R_2 = 6.00\ \Omega}$, and $\boldsymbol{R_3 = 13.0\ \Omega}$. (a) What is the total resistance? (b) Find the current. (c) Calculate the voltage drop in each resistor, and show these add to equal the voltage output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

**Strategy and Solution for (a)**

The total resistance is simply the sum of the individual resistances, as given by this equation:

$$R_s \quad R_1 + R_2 + R_3$$

$$1.00\ \Omega + 6.00\ \Omega + 13.0\ \Omega$$

$$20.0\ \Omega.$$

**Strategy and Solution for (b)**

The current is found using Ohm's law, $V = IR$. Entering the value of the applied voltage and the total resistance yields the current for the circuit:

$$I = \frac{V}{R_s} = \frac{12.0\ \text{V}}{20.0\ \Omega} = 0.600\ \text{A}.$$

**Strategy and Solution for (c)**

The voltage—or $IR$ drop—in a resistor is given by Ohm's law. Entering the current and the value of the first resistance yields

$$V_1 = IR_1 = (0.600\ \text{A})(1.0\ \Omega) = 0.600\ \text{V}.$$

Similarly,

$$V_2 = IR_2 = (0.600\ \text{A})(6.0\ \Omega) = 3.60\ \text{V}$$

and

$$V_3 = IR_3 = (0.600\ \text{A})(13.0\ \Omega) = 7.80\ \text{V}.$$

**Discussion for (c)**

The three $IR$ drops add to $12.0\ \text{V}$, as predicted:

$$V_1 + V_2 + V_3 = (0.600 + 3.60 + 7.80)\ \text{V} = 12.0\ \text{V}.$$

**Strategy and Solution for (d)**

The easiest way to calculate power in watts (W) dissipated by a resistor in a DC circuit is to use **Joule's law**, $P = IV$, where $P$ is electric power. In this

case, each resistor has the same full current flowing through it. By substituting Ohm's law $V = IR$ into Joule's law, we get the power dissipated by the first resistor as

$$P_1 = I^2 R_1 = (0.600\ \text{A})^2 (1.00\ \Omega) = 0.360\ \text{W}.$$

Similarly,

$$P_2 = I^2 R_2 = (0.600\ \text{A})^2 (6.00\ \Omega) = 2.16\ \text{W}$$

and

$$P_3 = I^2 R_3 = (0.600\ \text{A})^2 (13.0\ \Omega) = 4.68\ \text{W}.$$

**Discussion for (d)**

Power can also be calculated using either $P = IV$ or $P = \frac{V^2}{R}$, where $V$ is the voltage drop across the resistor (not the full voltage of the source). The same values will be obtained.

**Strategy and Solution for (e)**

The easiest way to calculate power output of the source is to use $P = IV$, where $V$ is the source voltage. This gives

$$P = (0.600\ \text{A})(12.0\ \text{V}) = 7.20\ \text{W}.$$

**Discussion for (e)**

Note, coincidentally, that the total power dissipated by the resistors is also 7.20 W, the same as the power put out by the source. That is,

$$P_1 + P_2 + P_3 = (0.360 + 2.16 + 4.68)\ \text{W} = 7.20\ \text{W}.$$

Power is energy per unit time (watts), and so conservation of energy requires the power output of the

source to be equal to the total power dissipated by the resistors.

### Major Features of Resistors in Series

1. Series resistances add:
$$R_s = R_1 + R_2 + R_3 + \ldots.$$
2. The same current flows through each resistor in series.
3. Individual resistors in series do not get the total source voltage, but divide it.

## Resistors in Parallel

Figure 3 shows resistors in **parallel**, wired to a voltage source. Resistors are in parallel when each resistor is connected directly to the voltage source by connecting wires having negligible resistance. Each resistor thus has the full voltage of the source applied to it.

Each resistor draws the same current it would if it alone were connected to the voltage source (provided the voltage source is not overloaded). For example, an automobile's headlights, radio, and so on, are wired in parallel, so that they utilize the full voltage of the

source and can operate completely independently. The same is true in your house, or any building. (See Figure 3(b).)

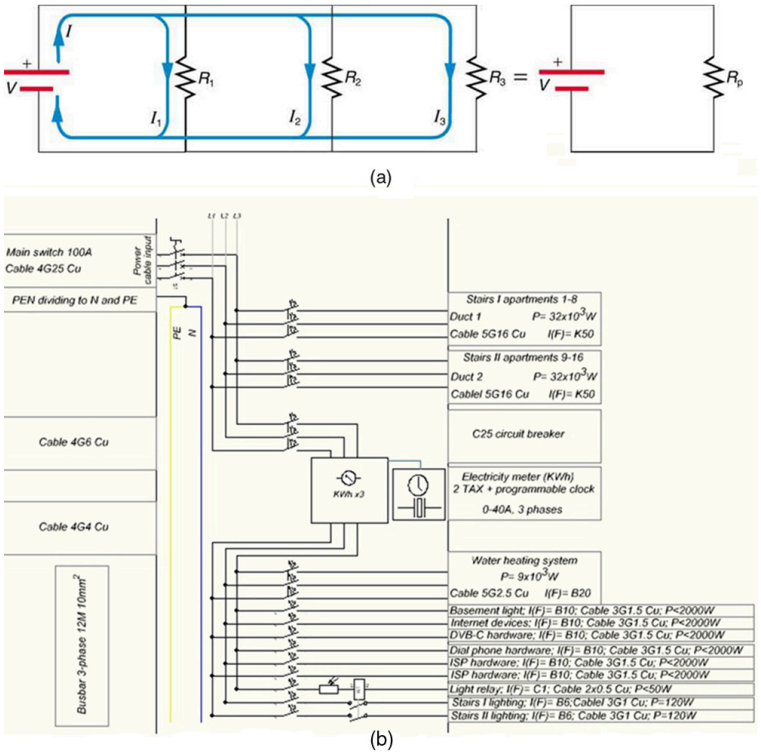



***Figure 3.*** *(a) Three resistors connected in parallel to a battery and the equivalent single or parallel resistance. (b) Electrical power setup in a house. (credit: Dmitry G, Wikimedia Commons)*

To find an expression for the equivalent parallel resistance $\boldsymbol{R_p}$, let us consider the currents that flow and how they are related to resistance. Since each resistor in the circuit has the full voltage, the currents flowing through the individual resistors are $\boldsymbol{I_1 = \frac{V}{R_1}}$

, $I_2 = \frac{V}{R_2}$, and $I_3 = \frac{V}{R_3}$. Conservation of charge implies that the total current $I$ produced by the source is the sum of these currents:

$$I = I_1 + I_2 + I_3 .$$

Substituting the expressions for the individual currents gives

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

Note that Ohm's law for the equivalent single resistance gives

$$I = \frac{V}{R_p} = V\left(\frac{1}{R_p}\right).$$

The terms inside the parentheses in the last two equations must be equal. Generalizing to any number of resistors, the total resistance $R_p$ of a parallel connection is related to the individual resistances by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

This relationship results in a total resistance $R_p$ that is less than the smallest of the individual resistances. (This is seen in the next example.) When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, and so the total resistance is lower.

## Example 2: Calculating Resistance, Current, Power Dissipation, and Power Output: Analysis of a Parallel Circuit

Let the voltage output of the battery and resistances in the parallel connection in Figure 3 be the same as the previously considered series connection: $V = 12.0\ \text{V}$, $R_1 = 1.00\ \Omega$, $R_2 = 6.00\ \Omega$, and $R_3 = 13.0\ \Omega$. (a) What is the total resistance? (b) Find the total current. (c) Calculate the currents in each resistor, and show these add to equal the total current output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

**Strategy and Solution for (a)**

The total resistance for a parallel combination of resistors is found using the equation below. Entering known values gives

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.00\ \Omega} + \frac{1}{6.00\ \Omega} + \frac{1}{13.0\ \Omega}.$$

Thus,

$$\frac{1}{R_p} = \frac{1.00}{\Omega} + \frac{0.1667}{\Omega} + \frac{0.07692}{\Omega}$$

$$= \frac{1.2436}{\Omega}$$

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

We must invert this to find the total resistance $R_p$. This yields

$$R_p = \frac{1}{1.2436}\,\Omega = 0.8041\ \Omega.$$

The total resistance with the correct number of significant digits is $R_p = 0.804\ \Omega$

**Discussion for (a)**

$R_p$ is, as predicted, less than the smallest individual resistance.

**Strategy and Solution for (b)**

The total current can be found from Ohm's law, substituting $R_p$ for the total resistance. This gives

$$I = \frac{V}{R_p} = \frac{12.0\ \text{V}}{0.8041\ \Omega} = 14.92\ \text{A}$$

**Discussion for (b)**

Current $I$ for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.

**Strategy and Solution for (c)**

The individual currents are easily calculated from

Ohm's law, since each resistor gets the full voltage. Thus,

$$I_1 = \frac{V}{R_1} = \frac{12.0\ \text{V}}{1.00\ \Omega} = 12.0\ \text{A}.$$

Similarly,

$$I_2 = \frac{V}{R_2} = \frac{12.0\ \text{V}}{6.00\ \Omega} = 2.00\ \text{A}$$

and

$$I_3 = \frac{V}{R_3} = \frac{12.0\ \text{V}}{13.0\ \Omega} = 0.92\ \text{A}.$$

**Discussion for (c)**

The total current is the sum of the individual currents:

$$I_1 + I_2 + I_3 = 14.92\ \text{A}.$$

This is consistent with conservation of charge.

**Strategy and Solution for (d)**

The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three are known. Let us use $P = \frac{V^2}{R}$, since each resistor gets full voltage. Thus,

$$P_1 = \frac{V^2}{R^1} = \frac{(12.0\ \text{V})^2}{1.00\ \Omega} = 144\ \text{W}.$$

Similarly,

$$P_2 = \frac{V^2}{R^2} = \frac{(12.0\ \text{V})^2}{6.00\ \Omega} = 24.0\ \text{W}$$

and

$$P_3 = \frac{V^2}{R^3} = \frac{(12.0\ \text{V})^2}{13.0\ \Omega} = 11.1\ \text{W}.$$

**Discussion for (d)**

The power dissipated by each resistor is considerably higher in parallel than when connected in series to the same voltage source.

**Strategy and Solution for (e)**

The total power can also be calculated in several ways. Choosing $P = IV$, and entering the total current, yields

$$P = IV = (14.92\ \text{A})(12.0\ \text{V}) = 179\ \text{W}.$$

**Discussion for (e)**

Total power dissipated by the resistors is also 179 W:

$$P_1 + P_2 + P_3 = 144\ \text{W} + 24.0\ \text{W} + 11.1\ \text{W} = 179\ \text{W}.$$

This is consistent with the law of conservation of energy.

**Overall Discussion**

Note that both the currents and powers in parallel connections are greater than for the same devices in series.

### Major Features of Resistors in Parallel

1. Parallel resistance is found from $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$, and it is smaller than any individual resistance in the combination.
2. Each resistor in parallel has the same full voltage of the source applied to it. (Power distribution systems most often use parallel connections to supply the myriad devices served with the same voltage and to allow them to operate independently.)
3. Parallel resistors do not each get the total current; they divide it.

## Combinations of Series and Parallel

More complex connections of resistors are sometimes just combinations of series and parallel. These are commonly encountered, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in Figure 4.

Various parts are identified as either series or parallel, reduced to their equivalents, and further reduced until a single resistance is left. The process is more time consuming than difficult.

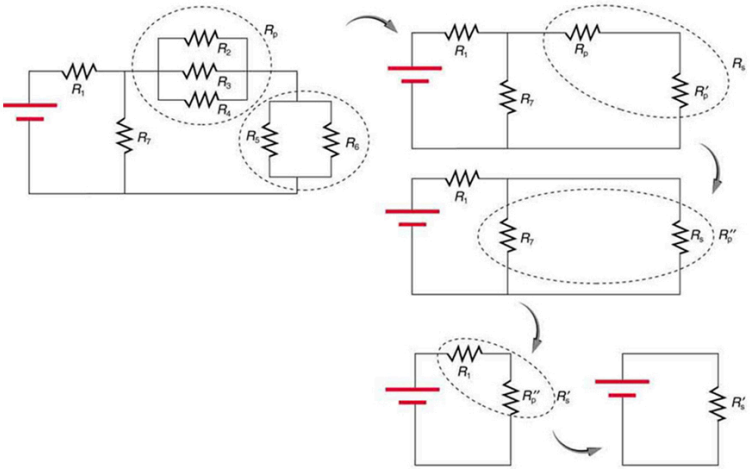



***Figure 4.*** *This combination of seven resistors has both series and parallel parts. Each is identified and reduced to an equivalent resistance, and these are further reduced until a single equivalent resistance is reached.*

The simplest combination of series and parallel resistance, shown in Figure 5, is also the most instructive, since it is found in many applications. For example, $\boldsymbol{R_1}$ could be the resistance of wires from a car battery to its electrical devices, which are in parallel. $\boldsymbol{R_2}$ and $\boldsymbol{R_3}$ could be the starter motor and a passenger compartment light. We have previously assumed that wire resistance is negligible, but, when it is not, it has important effects, as the next example indicates.

## Example 3: Calculating Resistance, $IR$ Drop, Current, and Power Dissipation: Combining Series and Parallel Circuits

Figure 5 shows the resistors from the previous two examples wired in a different way—a combination of series and parallel. We can consider $R_1$ to be the resistance of wires leading to $R_2$ and $R_3$. (a) Find the total resistance. (b) What is the $IR$ drop in $R_1$? (c) Find the current $I_2$ through $R_2$. (d) What power is dissipated by $R_2$?

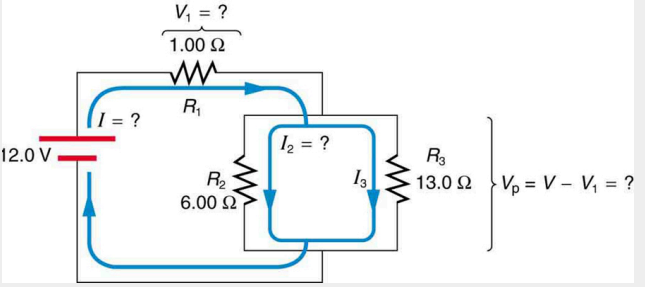



***Figure 5.*** *These three resistors are connected to a voltage source so that* $R_2$ *and* $R_3$ *are in parallel with one another and that combination is in series with* $R_1$.

**Strategy and Solution for (a)**

To find the total resistance, we note that $R_2$ and $R_3$ are in parallel and their combination $R_p$ is in

series with $R_1$. Thus the total (equivalent) resistance of this combination is

$$R_{\text{tot}} = R_1 + R_p.$$

First, we find $R_p$ using the equation for resistors in parallel and entering known values:

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6.00\ \Omega} + \frac{1}{13.0\ \Omega} = \frac{0.2436}{\Omega}.$$

Inverting gives

$$R_p = \frac{1}{0.2436}\ \Omega = 4.11\ \Omega.$$

So the total resistance is

$$R_{\text{tot}} = R_1 + R_p = 1.00\ \Omega + 4.11\ \Omega = 5.11\ \Omega.$$

**Discussion for (a)**

The total resistance of this combination is intermediate between the pure series and pure parallel values ($20.0\ \Omega$ and $0.804\ \Omega$, respectively) found for the same resistors in the two previous examples.

**Strategy and Solution for (b)**

To find the $IR$ drop in $R_1$, we note that the full current $I$ flows through $R_1$. Thus its $IR$ drop is

$$V_1 = IR_1.$$

We must find $I$ before we can calculate $V_1$. The total current $I$ is found using Ohm's law for the circuit. That is,

$$I = \frac{V}{R_{\text{tot}}} = \frac{12.0\ \text{V}}{5.11\ \Omega} = 2.35\ \text{A}.$$

Entering this into the expression above, we get

$$V_1 = IR_1 = (2.35\ \text{A})(1.00\ \Omega) = 2.35\ \text{V}.$$

**Discussion for (b)**

The voltage applied to $R_2$ and $R_3$ is less than the total voltage by an amount $V_1$. When wire resistance is large, it can significantly affect the operation of the devices represented by $R_2$ and $R_3$.

**Strategy and Solution for (c)**

To find the current through $R_2$, we must first find the voltage applied to it. We call this voltage $V_{\text{p}}$, because it is applied to a parallel combination of resistors. The voltage applied to both $R_2$ and $R_3$ is reduced by the amount $V_1$, and so it is

$$V_p = V - V_1 = 12.0\ \text{V} - 2.35\ \text{V} = 9.65\ \text{V}.$$

Now the current $I_2$ through resistance $R_2$ is found using Ohm's law:

$$I_2 = \frac{V_p}{R_2} = \frac{9.65\ \text{V}}{6.00\ \Omega} = 1.61\ \text{A}$$

**Discussion for (c)**

The current is less than the 2.00 A that flowed through $R_2$ when it was connected in parallel to the battery in the previous parallel circuit example.

**Strategy and Solution for (d)**

The power dissipated by $R_2$ is given by

$$P_2 = (I_2)^2 R_2 = (1.61\ \text{A})^2 (6.00\ \Omega) = 15.5\ \text{W}.$$

**Discussion for (d)**

The power is less than the 24.0 W this resistor dissipated when connected in parallel to the 12.0-V source.

## Practical Implications

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the $IR$ drop in the wires can also be significant.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to resistance inside the battery itself).

What is happening in these high-current situations is illustrated in Figure 6. The device represented by $R_3$ has a very low resistance, and so when it is switched on, a large current flows. This increased current causes a larger $IR$ drop in the wires represented by $R_1$, reducing the voltage across the light bulb (which is $R_2$), which then dims noticeably.

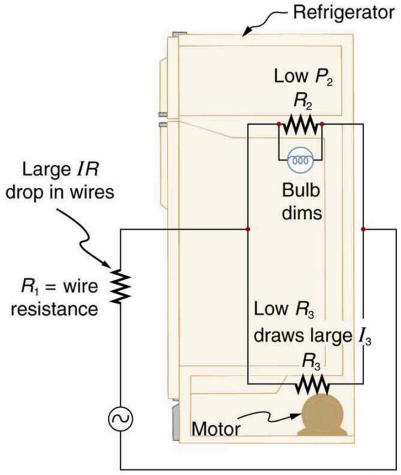



***Figure 6.*** *Why do lights dim when a large appliance is switched on? The answer is that the large current the appliance motor draws causes a significant IR drop in the wires and reduces the voltage across the light.*

## *Check Your Understanding*

**1:** Can any arbitrary combination of resistors be broken down into series and parallel combinations? See if you can draw a circuit diagram of resistors that cannot be broken down into combinations of series and parallel.

## Problem-Solving Strategies for Series and Parallel Resistors

1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the knowns for the problem, since they are labeled in your circuit diagram.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel. If your problem has a combination of series and parallel, reduce it in steps by considering individual groups of series or parallel connections, as done in this module and the examples. Special note: When finding $R_p$ or the equivalent resistance when the resistors are in parallel , the reciprocal must be taken with care.
5. Check to see whether the answers are reasonable and consistent. Units and numerical results must be reasonable. Total series resistance

should be greater, whereas total parallel resistance should be smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

## Section Summary

- The total resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances:
  $$R_s = R_1 + R_2 + R_3 + \cdots$$
- Each resistor in a series circuit has the same amount of current flowing through it.
- The voltage drop, or power dissipation, across each individual resistor in a series is different, and their combined total adds up to the power source input.
- The total resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula:
  $$\frac{1}{R_\text{p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$
- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single

resistance is eventually reached.

## *Conceptual Questions*

**1:** A switch has a variable resistance that is nearly zero when closed and extremely large when open, and it is placed in series with the device it controls. Explain the effect the switch in Figure 7 below has on current when open and when closed.

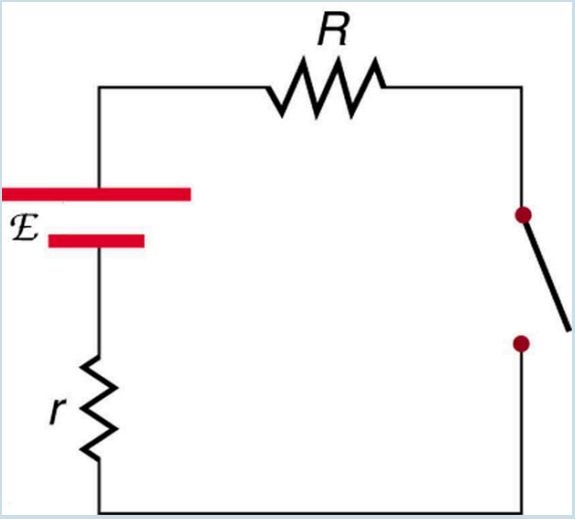



***Figure 7.*** *A switch is ordinarily in series with a resistance and voltage source. Ideally, the switch has nearly zero resistance when closed but has an extremely large resistance when open. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)*

**2:** What is the voltage across the open switch in Figure 7 shown above?

**3:** There is a voltage across an open switch, such as in Figure 7 shown above? Why, then, is the power dissipated by the open switch small?

**4:** Why is the power dissipated by a closed switch, such as in Figure 7 shown above small?

**5:** A student in a physics lab mistakenly wired a light bulb, battery, and switch as shown in Figure 8 below. Explain why the bulb is on when the switch is open, and off when the switch is closed. (Do not try this—it is hard on the battery!)

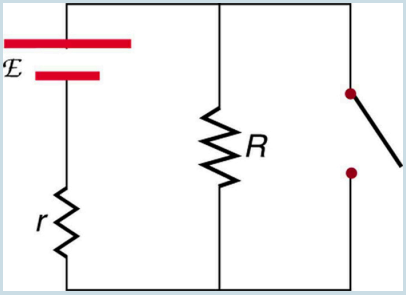



***Figure 8.*** *A wiring mistake put this switch in parallel with the device represented by* ***R****. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)*

**6:** Knowing that the severity of a shock depends on the magnitude of the current through your body, would you prefer to be in series or parallel with a resistance, such as the heating element of a toaster, if shocked by it? Explain.

**7:** Would your headlights dim when you start your car's engine if the wires in your automobile were superconductors? (Do not neglect the battery's internal resistance.) Explain.

**8:** Some strings of holiday lights are wired in series to save wiring costs. An old version utilized bulbs that break the electrical connection, like an open switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 40 identical bulbs, what is the normal operating voltage of each? Newer versions use bulbs that short circuit, like a closed switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 39 remaining identical bulbs, what is then the operating voltage of each?

**9:** If two household lightbulbs rated 60 W and 100 W are connected in series to household power, which will be brighter? Explain.

**10:** Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

**11:** Before World War II, some radios got power through a "resistance cord" that had a significant resistance. Such a resistance cord reduces the voltage to a desired level for the radio's tubes and the like, and it saves the expense of a transformer. Explain why resistance cords become warm and waste energy when the radio is on.

**12:** Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are

individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

## *Problem Exercises*

**Note: Data taken from figures can be assumed to be accurate to three significant digits.**

**1:** (a) What is the resistance of ten 275 ohm resistors connected in series? (b) In parallel?

**2:** (a) What is the resistance of a $1.00 \times 10^2$ ohm a 2.50 kilo-ohm and a 4.00 kilo-ohm resistor connected in series? (b) In parallel?

**3:** What are the largest and smallest resistances you can obtain by connecting a 36.0 ohm, a 50.0 ohm, and a 700 ohm resistor together?

**4:** An 1800-W toaster, a 1400-W electric frying pan, and a 75-W lamp are plugged into the same outlet in a 15-A, 120-V circuit. (The three devices are in parallel when plugged into the same socket.). (a) What current is drawn by each device? (b) Will this combination blow the 15-A fuse?

**5:** Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)

**6:** (a) Given a 48.0-V battery and 24.0 ohm and 96.0 ohm resistors, find the current and power for each when connected in series. (b) Repeat when the resistances are in parallel.

**7:** Referring to the example combining series and parallel circuits and Figure 5, calculate $I_3$ in the following two different ways: (a) from the known values of $I$ and $I_2$; (b) using Ohm's law for $R_3$. In both parts explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.

**8:** Referring to Figure 5: (a) Calculate $P_3$ and note how it compares with $P_3$ found in the first two example problems in this module. (b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.

**9:** Refer to Figure 6 and the discussion of lights dimming when a heavy appliance comes on. (a) Given the voltage source is 120 V, the wire resistance is $0.400\ \Omega$, and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance.

**10:** A 240-kV power transmission line carrying $5.00 \times 10^2\ \text{A}$ is hung from grounded metal towers by ceramic insulators, each having a $1.00 \times 10^9\ -\ \Omega$ resistance. Figure 9. (a) What is the resistance to ground of 100 of these insulators? (b) Calculate the power dissipated by 100 of them. (c) What fraction of the power carried by the line is this? Explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.

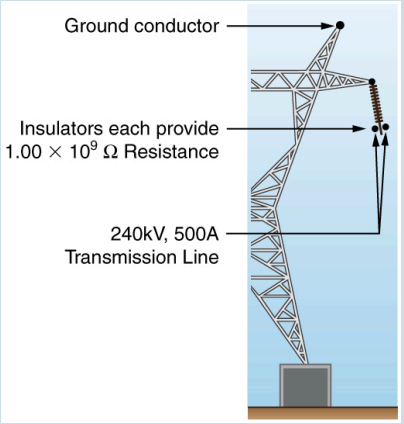



***Figure 9.*** *High-voltage (240-kV) transmission line carrying* $\mathbf{5.00 \times 10^2\ A}$ *is hung from a grounded metal transmission tower. The row of ceramic insulators provide* $\mathbf{1.00 \times 10^9 \Omega}$ *of resistance each.*

**11:** Show that if two resistors $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ are combined and one is much greater than the other ($\boldsymbol{R_1 >> R_2}$ > R_2}" title="\boldsymbol{R_1 >> R_2}" class="latex mathjax">): (a) Their series resistance is very nearly equal to the greater resistance $\boldsymbol{R_1}$. (b) Their parallel resistance is very nearly equal to smaller resistance $\boldsymbol{R_2}$.

**12: Unreasonable Results**

Two resistors, one having a resistance of $\mathbf{145\ \Omega}$, are connected in parallel to produce a total resistance of $\mathbf{150\ \Omega}$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

**13: Unreasonable Results**

Two resistors, one having a resistance of $\mathbf{900\ k\Omega}$, are connected in series to produce a total resistance of $\mathbf{0.500\ M\Omega}$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

## Glossary

**series**
: a sequence of resistors or other components wired into a circuit one after the other

**resistor**
: a component that provides resistance to the current flowing through an electrical circuit

**resistance**
: causing a loss of electrical power in a circuit

**Ohm's law**
: the relationship between current, voltage, and resistance within an electrical circuit: $\boldsymbol{V = IR}$

**voltage**
: the electrical potential energy per unit charge; electric pressure created by a power source, such as a battery

**voltage drop**
: the loss of electrical power as a current travels through a resistor, wire or other component

**current**
: the flow of charge through an electric circuit past a given point of measurement

**Joule's law**

the relationship between potential electrical power, voltage, and resistance in an electrical circuit, given by: $P_e = IV$

**parallel**

the wiring of resistors or other components in an electrical circuit such that each component receives an equal voltage from the power source; often pictured in a ladder-shaped diagram, with each component on a rung of the ladder

## *Solutions*

**Check Your Understanding**

**1:** No, there are many ways to connect resistors that are not combinations of series and parallel, including loops and junctions.

**Problems Exercises**

**1:** (a) 2.75 kΩ (b) 27.5 Ω

**3:** (a) 786 Ω (b) 20.3 Ω

**5:** 29.6 W

**7:** (a) 0.74 A (b) 0.742 A

**9:** light bulb resistance = 192 ohms . The 15 A goes through 0.400 ohm wires twice as it is a parallel circuit. This makes the voltage drop across the wires = 15.0 A x 0.400 ohms x 2 = 12 V. The voltage across the light bulb = 120 – 12 = 108 V a) Power = $V^2/R$ = 60.8 W

$$R_s = R_1 + R_2$$

**11:** (a) $\Rightarrow R_s \approx R_1 (R_1 >> R_2)$ > R_2)}

\end{array}" title="\begin{array}{c} \boldsymbol{R_s = R_1 + R_2} \\[1em] \boldsymbol{\Rightarrow R_s \approx R_1(R_1 >> R_2)} \end{array}" class="latex mathjax">

(b) $$\boldsymbol{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}},$$

so that

$$\boldsymbol{R_p = \frac{R_1 R_2}{R_1 + R_2} \approx \frac{R_1 R_2}{R_1}}$$
$$\boldsymbol{= R_2 \; (R_1 >> R_2)}.$$ > R_2)}."
title="\boldsymbol{= R_2 \; (R_1 >> R_2)}." class="latex mathjax">

**13:** (a)-400 Ω (b) Resistance cannot be negative.

(c) Series resistance is said to be less than one of the resistors, but it must be greater than any of the resistors.

# 82. 14.5 Optional: Electric Hazards and the Human Body

*Summary*

- Define thermal hazard, shock hazard, and short circuit.
- Explain what effects various levels of current have on the human body.

There are two known hazards of electricity—thermal and shock. A **thermal hazard** is one where excessive electric power causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. This section considers these hazards and the various factors affecting them in a quantitative manner.

## Thermal Hazards

Electric power causes undesired heating effects whenever electric energy is converted to thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the **short circuit**, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in Figure 1. Insulation on wires leading to

an appliance has worn through, allowing the two wires to come into contact. Such an undesired contact with a high voltage is called a *short*. Since the resistance of the short, $r$, is very small, the power dissipated in the short, $P = V^2/r$, is very large. For example, if $V$ is 120 V and $r$ is $0.100\ \Omega$, then the power is 144 kW, *much* greater than that used by a typical household appliance. Thermal energy delivered at this rate will very quickly raise the temperature of surrounding materials, melting or perhaps igniting them.

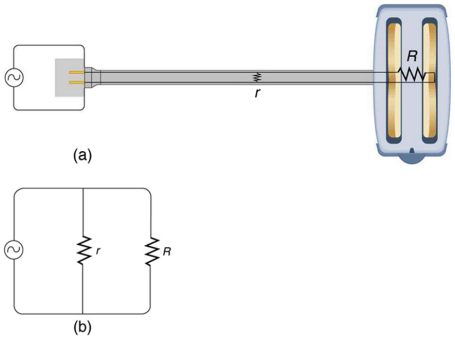



***Figure 1.*** *A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance* $r$*. Since* $P = V^2/r$*, thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.*

One particularly insidious aspect of a short circuit is that its resistance may actually be decreased due to the increase in temperature. This can happen if the short creates ionization. These charged atoms and molecules are free to move and, thus, lower the resistance $r$. Since $P = V^2/r$, the power dissipated in the short rises, possibly causing more ionization, more power, and so on. High voltages, such as the 480-V AC used in some industrial applications, lend themselves to this hazard, because higher voltages create higher initial power production in a short.

Another serious, but less dramatic, thermal hazard occurs when wires supplying power to a user are overloaded with too great a current. As discussed in the previous section, the power dissipated in the supply wires is $P = I^2 R_w$, where $R_w$ is the resistance of the wires and $I$ the current flowing through them. If either $I$ or $R_w$ is too large, the wires overheat. For example, a worn appliance cord (with some of its braided wires broken) may have $R_w = 2.00\ \Omega$ rather than the $0.100\ \Omega$ it should be. If 10.0 A of current passes through the cord, then $P = I^2 R_w = 200\ \text{W}$ is dissipated in the cord–much more than is safe. Similarly, if a wire with a $0.100 - \Omega$ resistance is meant to carry a few amps, but is instead carrying 100 A, it will severely overheat. The power dissipated in the wire will in that case be $P = 1000W$. Fuses and circuit breakers are used to limit excessive currents. (See Figure 2 and Figure 3.) Each device opens the circuit automatically when a sustained current exceeds safe limits.

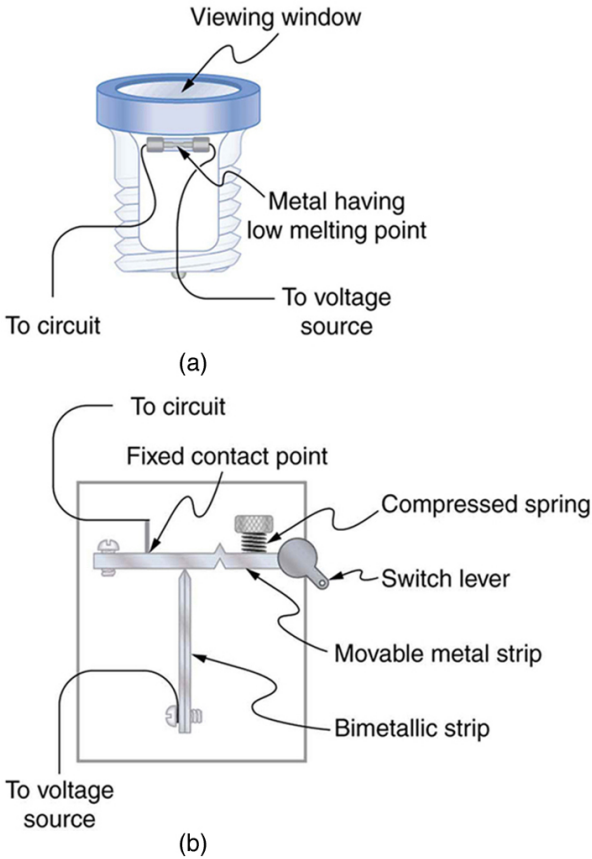



***Figure 2.*** *(a) A fuse has a metal strip with a low melting point that, when overheated by an excessive current, permanently breaks the connection of a circuit to a voltage source. (b) A circuit breaker is an automatic but restorable electric switch. The one shown here has a bimetallic strip that bends to the right and into the notch if overheated. The spring then forces the metal strip downward, breaking the electrical connection at the points.*

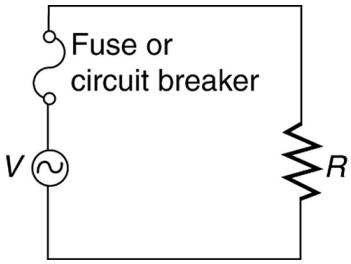



***Figure 3.*** *Schematic of a circuit with a fuse or circuit breaker in it. Fuses and circuit breakers act like automatic switches that open when sustained current exceeds desired limits.*

Fuses and circuit breakers for typical household voltages and currents are relatively simple to produce, but those for large voltages and currents experience special problems. For example, when a circuit breaker tries to interrupt the flow of high-voltage electricity, a spark can jump across its points that ionizes the air in the gap and allows the current to continue flowing. Large circuit breakers found in power-distribution systems employ insulating gas and even use jets of gas to blow out such sparks. Here AC is safer than DC, since AC current goes through zero 120 times per second, giving a quick opportunity to extinguish these arcs.

## Shock Hazards

Electrical currents through people produce tremendously varied effects. An electrical current can be used to block back pain. The possibility of using electrical current to stimulate muscle action in paralyzed limbs, perhaps allowing paraplegics to walk, is under study. TV dramatizations in which electrical shocks are used to

bring a heart attack victim out of ventricular fibrillation (a massively irregular, often fatal, beating of the heart) are more than common. Yet most electrical shock fatalities occur because a current put the heart into fibrillation. A pacemaker uses electrical shocks to stimulate the heart to beat properly. Some fatal shocks do not produce burns, but warts can be safely burned off with electric current (though freezing using liquid nitrogen is now more common). Of course, there are consistent explanations for these disparate effects. The major factors upon which the effects of electrical shock depend are

1. The amount of current $I$
2. The path taken by the current
3. The duration of the shock
4. The frequency $f$ of the current ($f = 0$ for DC)

Table 3 gives the effects of electrical shocks as a function of current for a typical accidental shock. The effects are for a shock that passes through the trunk of the body, has a duration of 1 s, and is caused by 60-Hz power.

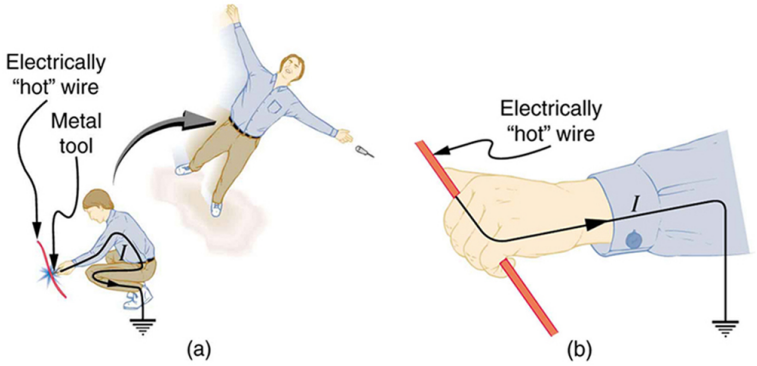



***Figure 4.*** *An electric current can cause muscular contractions with varying effects. (a) The victim is "thrown" backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can't let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them.*

| Current (mA) | Effect |
|---|---|
| 1 | Threshold of sensation |
| 5 | Maximum harmless current |
| 10–20 | Onset of sustained muscular contraction; cannot let go for duration of shock; contraction of chest muscles may stop breathing during shock |
| 50 | Onset of pain |
| 100–300+ | Ventricular fibrillation possible; often fatal |
| 300 | Onset of burns depending on concentration of current |
| 6000 (6 A) | Onset of sustained ventricular contraction and respiratory paralysis; both cease when shock ends; heartbeat may return to normal; used to defibrillate the heart |

**Table 3:** Effects of Electrical Shock as a Function of Current[1]

Our bodies are relatively good conductors due to the water in our bodies. Given that larger currents will flow through sections with lower resistance (to be further discussed in the next chapter), electric currents preferentially flow through paths in the human body that have a minimum resistance in a direct path to earth. The earth is a natural electron sink. Wearing insulating shoes, a requirement in many professions, prohibits a pathway for electrons by providing a large resistance in that path. Whenever working with high-power tools (drills), or in risky situations, ensure that you do not provide a pathway for current flow (especially through the heart).

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock. At 10 to 20 mA and above, the current can stimulate sustained muscular contractions much as regular nerve impulses do. People sometimes say they were knocked across the room by a shock, but what really happened was that certain muscles contracted, propelling them in a manner not of their own choosing. (See Figure

4(a).) More frightening, and potentially more dangerous, is the "can't let go" effect illustrated in Figure 4(b). The muscles that close the fingers are stronger than those that open them, so the hand closes involuntarily on the wire shocking it. This can prolong the shock indefinitely. It can also be a danger to a person trying to rescue the victim, because the rescuer's hand may close about the victim's wrist. Usually the best way to help the victim is to give the fist a hard knock/blow/jar with an insulator or to throw an insulator at the fist. Modern electric fences, used in animal enclosures, are now pulsed on and off to allow people who touch them to get free, rendering them less lethal than in the past.

Greater currents may affect the heart. Its electrical patterns can be disrupted, so that it beats irregularly and ineffectively in a condition called "ventricular fibrillation." This condition often lingers after the shock and is fatal due to a lack of blood circulation. The threshold for ventricular fibrillation is between 100 and 300 mA. At about 300 mA and above, the shock can cause burns, depending on the concentration of current—the more concentrated, the greater the likelihood of burns.

Very large currents cause the heart and diaphragm to contract for the duration of the shock. Both the heart and breathing stop. Interestingly, both often return to normal following the shock. The electrical patterns on the heart are completely erased in a manner that the heart can start afresh with normal beating, as opposed to the permanent disruption caused by smaller currents that can put the heart into ventricular fibrillation. The latter is something like scribbling on a blackboard, whereas the former completely erases it. TV dramatizations of electric shock used to bring a heart attack victim out of ventricular fibrillation also show large paddles. These are used to spread out current passed through the victim to reduce the likelihood of burns.

Current is the major factor determining shock severity (given that other conditions such as path, duration, and frequency are fixed, such as in the table and preceding discussion). A larger voltage is more hazardous, but since $\boldsymbol{I = V/R}$, the severity of the shock

depends on the combination of voltage and resistance. For example, a person with dry skin has a resistance of about $\mathbf{200\ k\Omega}$. If he comes into contact with 120-V AC, a current $\boldsymbol{I = (120\ V)/(200\ k\Omega) = 0.6\ mA}$ passes harmlessly through him. The same person soaking wet may have a resistance of $\mathbf{10.0\ k\Omega}$ and the same 120 V will produce a current of 12 mA—above the "can't let go" threshold and potentially dangerous.

Most of the body's resistance is in its dry skin. When wet, salts go into ion form, lowering the resistance significantly. The interior of the body has a much lower resistance than dry skin because of all the ionic solutions and fluids it contains. If skin resistance is bypassed, such as by an intravenous infusion, a catheter, or exposed pacemaker leads, a person is rendered **microshock sensitive**. In this condition, currents about 1/1000 those listed in Table 3 produce similar effects. During open-heart surgery, currents as small as $\mathbf{20\ \mu A}$ can be used to still the heart. Stringent electrical safety requirements in hospitals, particularly in surgery and intensive care, are related to the doubly disadvantaged microshock-sensitive patient. The break in the skin has reduced his resistance, and so the same voltage causes a greater current, and a much smaller current has a greater effect.

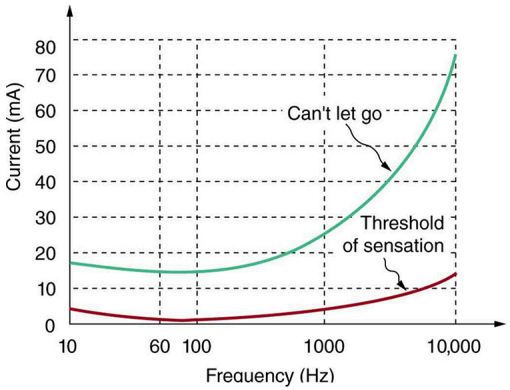



***Figure 5.*** *Graph of average values for the threshold of sensation and the "can't let go" current as a function of frequency. The lower the value, the more sensitive the body is at that frequency.*

Factors other than current that affect the severity of a shock are its path, duration, and AC frequency. Path has obvious consequences. For example, the heart is unaffected by an electric shock through the brain, such as may be used to treat manic depression. And it is a general truth that the longer the duration of a shock, the greater its effects. Figure 5 presents a graph that illustrates the effects of frequency on a shock. The curves show the minimum current for two different effects, as a function of frequency. The lower the current needed, the more sensitive the body is at that frequency. Ironically, the body is most sensitive to frequencies near the 50- or 60-Hz frequencies in common use. The body is slightly less sensitive for DC ($\boldsymbol{f = 0}$), mildly confirming Edison's claims that AC presents a greater hazard. At higher and higher frequencies, the body becomes progressively less sensitive to any effects that involve nerves. This is related to the maximum rates at which nerves can fire or be stimulated. At very high frequencies, electrical current travels only on the surface of a person. Thus a wart can be burned off with very high frequency current without causing the heart to stop. (Do not try this at home with 60-Hz AC!) Some of the

spectacular demonstrations of electricity, in which high-voltage arcs are passed through the air and over people's bodies, employ high frequencies and low currents. (See Figure 6.)

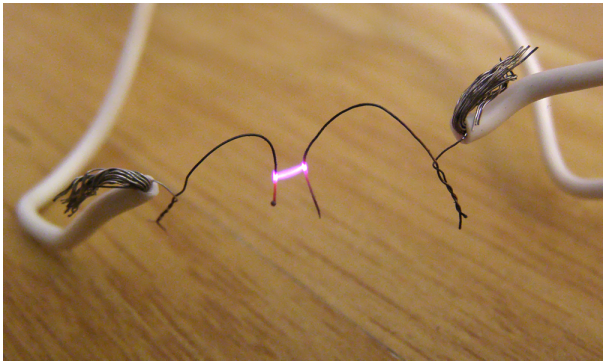


***Figure 6.*** *Is this electric arc dangerous? The answer depends on the AC frequency and the power involved. (credit: Khimich Alex, Wikimedia Commons)*

## Section Summary

- The two types of electric hazards are thermal (excessive power) and shock (current through a person).
- Shock severity is determined by current, path, duration, and AC frequency.
- Table 3 lists shock hazards as a function of current.
- Figure 5 graphs the threshold current for two hazards as a function of frequency.

## *Conceptual Questions*

**1:** Using an ohmmeter, a student measures the resistance between various points on his body. He finds that the resistance between two points on the same finger is about the same as the resistance between two points on opposite hands—both are several hundred thousand ohms. Furthermore, the resistance decreases when more skin is brought into contact with the probes of the ohmmeter. Finally, there is a dramatic drop in resistance (to a few thousand ohms) when the skin is wet. Explain these observations and their implications regarding skin and internal resistance of the human body.

**2:** What are the two major hazards of electricity?

**3:** Why isn't a short circuit a shock hazard?

**4:** What determines the severity of a shock? Can you say that a certain voltage is hazardous without further information?

**5:** An electrified needle is used to burn off warts, with the circuit being completed by having the patient sit on a large butt plate. Why is this plate large?

**6:** Some surgery is performed with high-voltage electricity passing from a metal scalpel through the tissue being cut. Considering the nature of electric fields at the surface of conductors, why would you expect most of the current to flow from the sharp edge of the scalpel? Do you think high- or low-frequency AC is used?

**7:** Some devices often used in bathrooms, such as

hairdryers, often have safety messages saying “Do not use when the bathtub or basin is full of water.” Why is this so?

**8:** We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why is this so?

**9:** Before working on a power transmission line, linemen will touch the line with the back of the hand as a final check that the voltage is zero. Why the back of the hand?

**10:** Why is the resistance of wet skin so much smaller than dry, and why do blood and other bodily fluids have low resistances?

**11:** Could a person on intravenous infusion (an IV) be microshock sensitive?

**12:** In view of the small currents that cause shock hazards and the larger currents that circuit breakers and fuses interrupt, how do they play a role in preventing shock hazards?

## *Problem Exercises*

**1:** (a) How much power is dissipated in a short circuit of 240-V AC through a resistance of $\mathbf{0.250\ \Omega}$? (b) What current flows?

**2:** What voltage is involved in a 1.44-kW short circuit through a $\mathbf{0.100\ -\ \Omega}$ resistance?

**3:** Find the current through a person and identify the

likely effect on her if she touches a 120-V AC source: (a) if she is standing on a rubber mat and offers a total resistance of $\mathbf{300\ k\Omega}$; (b) if she is standing barefoot on wet grass and has a resistance of only $\mathbf{4000\ k\Omega}$.

**4:** While taking a bath, a person touches the metal case of a radio. The path through the person to the drainpipe and ground has a resistance of $\mathbf{4000\ \Omega}$. What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?

**5:** Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?

**6:** (a) During surgery, a current as small as $\mathbf{20.0\ \mu A}$ applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is $\mathbf{300\ \Omega}$, what is the smallest voltage that poses this danger? (b) Does your answer imply that special electrical safety precautions are needed?

**7:** (a) What is the resistance of a 220-V AC short circuit that generates a peak power of 96.8 kW? (b) What would the average power be if the voltage was 120 V AC?

**8:** A heart defibrillator passes 10.0 A through a patient's torso for 5.00 ms in an attempt to restore normal beating. (a) How much charge passed? (b) What voltage was applied if 500 J of energy was dissipated? (c) What was the path's resistance? (d) Find the temperature increase caused in the 8.00 kg of affected tissue.

**9: Integrated Concepts**

A short circuit in a 120-V appliance cord has a $0.500\ \Omega$ resistance. Calculate the temperature rise of the 2.00 g of surrounding materials, assuming their specific heat capacity is $0.200\ \text{cal/g}\cdot^{\circ}\text{C}$ and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?

**10: Construct Your Own Problem**

Consider a person working in an environment where electric currents might pass through her body. Construct a problem in which you calculate the resistance of insulation needed to protect the person from harm. Among the things to be considered are the voltage to which the person might be exposed, likely body resistance (dry, wet, ...), and acceptable currents (safe but sensed, safe and unfelt, ...).

## Footnotes

1. [1] For an average male shocked through trunk of body for 1 s by 60-Hz AC. Values for females are 60–80% of those listed.

## Glossary

**thermal hazard**
: a hazard in which electric current causes undesired thermal effects

**shock hazard**
when electric current passes through a person

**short circuit**
also known as a "short," a low-resistance path between terminals of a voltage source

**microshock sensitive**
a condition in which a person's skin resistance is bypassed, possibly by a medical procedure, rendering the person vulnerable to electrical shock at currents about 1/1000 the normally required level

## *Solutions*

**Problem Exercises**

**1:** (a) 230 kW

(b) 960 A

**3:** (a) 0.400 mA, no effect

(b) 26.7 mA, muscular contraction for duration of the shock (can't let go)

5: $\mathbf{1.20 \times 10^5\ \Omega}$

7: (a) $\mathbf{1.00\ \Omega}$

(b) 14.4 kW

**9:** Temperature increases 860° C. It is very likely to be damaging.

# 83. 14.6 Optional: Nerve Conduction–Electrocardiograms

*Summary*

- Explain the process by which electric signals are transmitted along a neuron.
- Explain the effects myelin sheaths have on signal propagation.
- Explain what the features of an ECG signal indicate.

## Nerve Conduction

Electric currents in the vastly complex system of billions of nerves in our body allow us to sense the world, control parts of our body, and think. These are representative of the three major functions of nerves. First, nerves carry messages from our sensory organs and others to the central nervous system, consisting of the brain and spinal cord. Second, nerves carry messages from the central nervous system to muscles and other organs. Third, nerves transmit and process signals within the central nervous system. The sheer number of nerve cells and the incredibly greater number of connections between them makes this system the subtle wonder that it is. **Nerve conduction** is a general term for electrical signals

carried by nerve cells. It is one aspect of **bioelectricity**, or electrical effects in and created by biological systems.

Nerve cells, properly called *neurons*, look different from other cells—they have tendrils, some of them many centimeters long, connecting them with other cells. (See Figure 1.) Signals arrive at the cell body across *synapses* or through *dendrites*, stimulating the neuron to generate its own signal, sent along its long *axon* to other nerve or muscle cells. Signals may arrive from many other locations and be transmitted to yet others, conditioning the synapses by use, giving the system its complexity and its ability to learn.

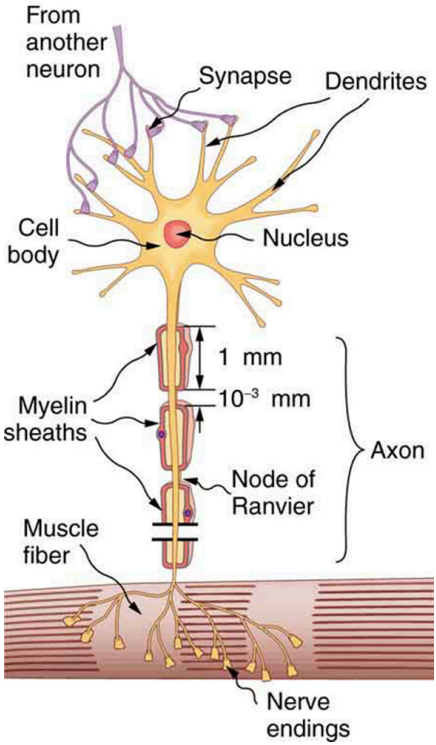



***Figure 1.*** *A neuron with its dendrites and long axon. Signals in the form of electric currents reach the cell body through dendrites and across synapses, stimulating the neuron to generate its own signal sent down the axon. The number of interconnections can be far greater than shown here.*

The method by which these electric currents are generated and transmitted is more complex than the simple movement of free charges in a conductor, but it can be understood with principles already discussed in this text. The most important of these are the Coulomb force and diffusion.

Figure 2 illustrates how a voltage (potential difference) is created

across the cell membrane of a neuron in its resting state. This thin membrane separates electrically neutral fluids having differing concentrations of ions, the most important varieties being $\mathbf{Na}^{+}$, $\mathbf{K}^{+}$, and $\mathbf{Cl}^{-}$ (these are sodium, potassium, and chlorine ions with single plus or minus charges as indicated). Free ions will diffuse from a region of high concentration to one of low concentration but the cell membrane is **semipermeable**, meaning that some ions may cross it while others cannot. In its resting state, the cell membrane is permeable to $\mathbf{K}^{+}$ and $\mathbf{Cl}^{-}$, and impermeable to $\mathbf{Na}^{+}$. Diffusion of $\mathbf{K}^{+}$ and $\mathbf{Cl}^{-}$ thus creates the layers of positive and negative charge on the outside and inside of the membrane. The Coulomb force prevents the ions from diffusing across in their entirety. Once the charge layer has built up, the repulsion of like charges prevents more from moving across, and the attraction of unlike charges prevents more from leaving either side. The result is two layers of charge right on the membrane, with diffusion being balanced by the Coulomb force. A tiny fraction of the charges move across and the fluids remain neutral (other ions are present), while a separation of charge and a voltage have been created across the membrane.

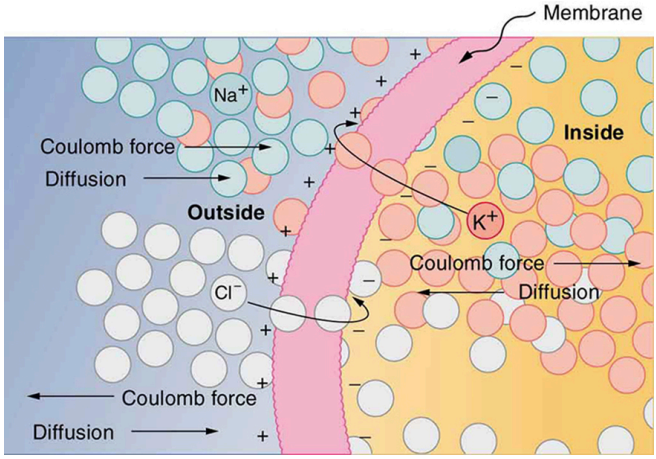



***Figure 2.*** *The semipermeable membrane of a cell has different concentrations of ions inside and out. Diffusion moves the* $\mathbf{K^+}$ *and* $\mathbf{Cl^-}$ *ions in the direction shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to* $\mathbf{Na^+}$.

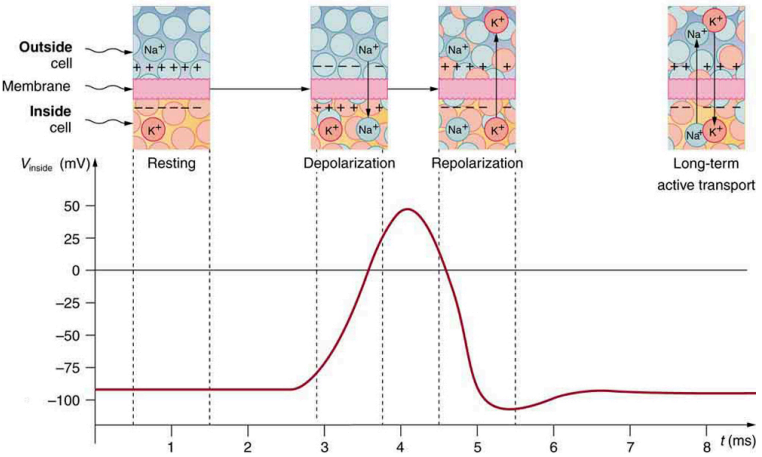



***Figure 3.*** *An action potential is the pulse of voltage inside a nerve cell graphed here. It is caused by movements of ions across the cell membrane as shown. Depolarization occurs when a stimulus makes the membrane permeable to* $\mathbf{Na^+}$ *ions. Repolarization follows as the membrane again becomes impermeable to* $\mathbf{Na^+}$*, and* $\mathbf{K^+}$ *moves from high to low concentration. In the long term, active transport slowly maintains the concentration differences, but the cell may fire hundreds of times in rapid succession without seriously depleting them.*

The separation of charge creates a potential difference of 70 to 90 mV across the cell membrane. While this is a small voltage, the resulting electric field ($\boldsymbol{E = V/d}$) across the only 8-nm-thick membrane is immense (on the order of 11 MV/m!) and has fundamental effects on its structure and permeability. Now, if the exterior of a neuron is taken to be at 0 V, then the interior has a *resting potential* of about −90 mV. Such voltages are created across the membranes of almost all types of animal cells but are largest in nerve and muscle cells. In fact, fully 25% of the energy used by cells goes toward creating and maintaining these potentials.

Electric currents along the cell membrane are created by any stimulus that changes the membrane's permeability. The membrane thus temporarily becomes permeable to $\mathbf{Na^+}$, which then rushes in, driven both by diffusion and the Coulomb force. This inrush

of $\mathbf{Na}^+$ first neutralizes the inside membrane, or *depolarizes* it, and then makes it slightly positive. The depolarization causes the membrane to again become impermeable to $\mathbf{Na}^+$, and the movement of $\mathbf{K}^+$ quickly returns the cell to its resting potential, or *repolarizes* it. This sequence of events results in a voltage pulse, called the *action potential*. (See Figure 3.) Only small fractions of the ions move, so that the cell can fire many hundreds of times without depleting the excess concentrations of $\mathbf{Na}^+$ and $\mathbf{K}^+$. Eventually, the cell must replenish these ions to maintain the concentration differences that create bioelectricity. This sodium-potassium pump is an example of *active transport*, wherein cell energy is used to move ions across membranes against diffusion gradients and the Coulomb force.

The action potential is a voltage pulse at one location on a cell membrane. How does it get transmitted along the cell membrane, and in particular down an axon, as a nerve impulse? The answer is that the changing voltage and electric fields affect the permeability of the adjacent cell membrane, so that the same process takes place there. The adjacent membrane depolarizes, affecting the membrane further down, and so on, as illustrated in Figure 4. Thus the action potential stimulated at one location triggers a *nerve impulse* that moves slowly (about 1 m/s) along the cell membrane.

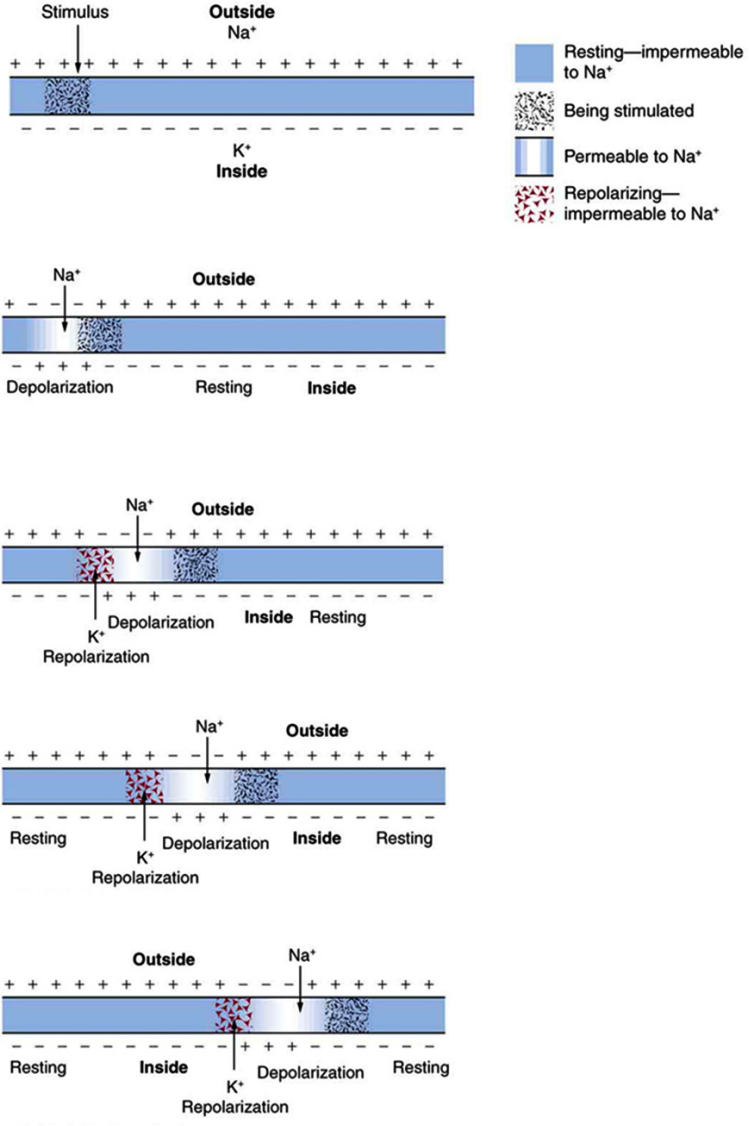



***Figure 4.*** *A nerve impulse is the propagation of an action potential along a cell membrane. A stimulus causes an action potential at one location, which changes the permeability of the adjacent membrane, causing an action potential there. This in turn affects the membrane further down, so that the action potential moves slowly (in electrical terms) along the cell membrane.*

*Although the impulse is due to* $\mathbf{Na^+}$ *and* $\mathbf{K^+}$ *going across the membrane, it is equivalent to a wave of charge moving along the outside and inside of the membrane.*

Some axons, like that in Figure 1, are sheathed with *myelin*, consisting of fat-containing cells. Figure 5 shows an enlarged view of an axon having myelin sheaths characteristically separated by unmyelinated gaps (called nodes of Ranvier). This arrangement gives the axon a number of interesting properties. Since myelin is an insulator, it prevents signals from jumping between adjacent nerves (cross talk). Additionally, the myelinated regions transmit electrical signals at a very high speed, as an ordinary conductor or resistor would. There is no action potential in the myelinated regions, so that no cell energy is used in them. There is an $\boldsymbol{IR}$ signal loss in the myelin, but the signal is regenerated in the gaps, where the voltage pulse triggers the action potential at full voltage. So a myelinated axon transmits a nerve impulse faster, with less energy consumption, and is better protected from cross talk than an unmyelinated one. Not all axons are myelinated, so that cross talk and slow signal transmission are a characteristic of the normal operation of these axons, another variable in the nervous system.

The degeneration or destruction of the myelin sheaths that surround the nerve fibers impairs signal transmission and can lead to numerous neurological effects. One of the most prominent of these diseases comes from the body's own immune system attacking the myelin in the central nervous system—multiple sclerosis. MS symptoms include fatigue, vision problems, weakness of arms and legs, loss of balance, and tingling or numbness in one's extremities (neuropathy). It is more apt to strike younger adults, especially females. Causes might come from infection, environmental or geographic affects, or genetics. At the moment there is no known cure for MS.

Most animal cells can fire or create their own action potential. Muscle cells contract when they fire and are often induced to do

so by a nerve impulse. In fact, nerve and muscle cells are physiologically similar, and there are even hybrid cells, such as in the heart, that have characteristics of both nerves and muscles. Some animals, like the infamous electric eel (see Figure 6), use muscles ganged so that their voltages add in order to create a shock great enough to stun prey.

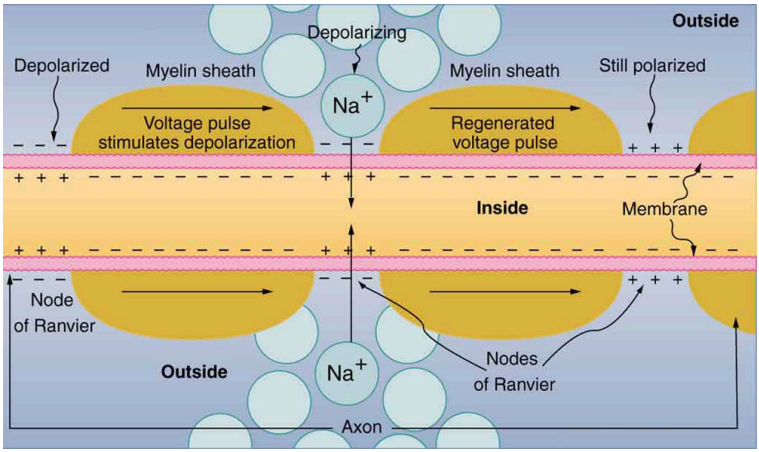



***Figure 5.*** *Propagation of a nerve impulse down a myelinated axon, from left to right. The signal travels very fast and without energy input in the myelinated regions, but it loses voltage. It is regenerated in the gaps. The signal moves faster than in unmyelinated axons and is insulated from signals in other nerves, limiting cross talk.*



***Figure 6.*** *An electric eel flexes its muscles to create a voltage that stuns prey. (credit: chrisbb, Flickr)*

## Electrocardiograms

Just as nerve impulses are transmitted by depolarization and repolarization of adjacent membrane, the depolarization that causes muscle contraction can also stimulate adjacent muscle cells to depolarize (fire) and contract. Thus, a depolarization wave can be sent across the heart, coordinating its rhythmic contractions and enabling it to perform its vital function of propelling blood through the circulatory system. Figure 7 is a simplified graphic of a depolarization wave spreading across the heart from the *sinoarterial* (SA) *node*, the heart's natural pacemaker.

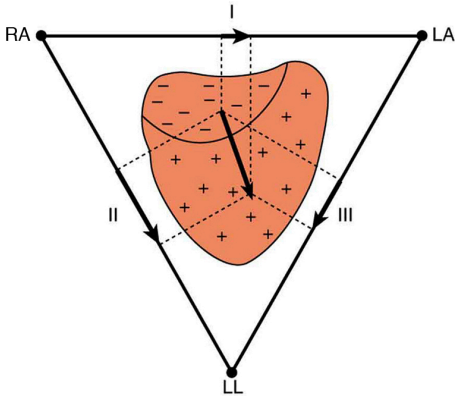



***Figure 7.*** *The outer surface of the heart changes from positive to negative during depolarization. This wave of depolarization is spreading from the top of the heart and is represented by a vector pointing in the direction of the wave. This vector is a voltage (potential difference) vector. Three electrodes, labeled RA, LA, and LL, are placed on the patient. Each pair (called leads I, II, and III) measures a component of the depolarization vector and is graphed in an ECG.*

An **electrocardiogram (ECG)** is a record of the voltages created by the wave of depolarization and subsequent repolarization in the heart. Voltages between pairs of electrodes placed on the chest are vector components of the voltage wave on the heart. Standard ECGs have 12 or more electrodes, but only three are shown in Figure 7 for clarity. Decades ago, three-electrode ECGs were performed by placing electrodes on the left and right arms and the left leg. The voltage between the right arm and the left leg is called the *lead II potential* and is the most often graphed. We shall examine the lead II potential as an indicator of heart-muscle function and see that it is coordinated with arterial blood pressure as well.

The right and left atria receive blood from the body and lungs, respectively, and pump the blood into the ventricles. The right and left ventricles, in turn, pump blood through the lungs and the rest

of the body, respectively. Depolarization of the heart muscle causes it to contract. After contraction it is repolarized to ready it for the next beat. The ECG measures components of depolarization and repolarization of the heart muscle and can yield significant information on the functioning and malfunctioning of the heart.

Figure 8 shows an ECG of the lead II potential and a graph of the corresponding arterial blood pressure. The major features are labeled P, Q, R, S, and T. The *P wave* is generated by the depolarization and contraction of the atria as they pump blood into the ventricles. The QRS *complex* is created by the depolarization of the ventricles as they pump blood to the lungs and body. Since the shape of the heart and the path of the depolarization wave are not simple, the QRS complex has this typical shape and time span. The lead II QRS signal also masks the repolarization of the atria, which occur at the same time. Finally, the *T wave* is generated by the repolarization of the ventricles and is followed by the next P wave in the next heartbeat. Arterial blood pressure varies with each part of the heartbeat, with systolic (maximum) pressure occurring closely after the QRS complex, which signals contraction of the ventricles.

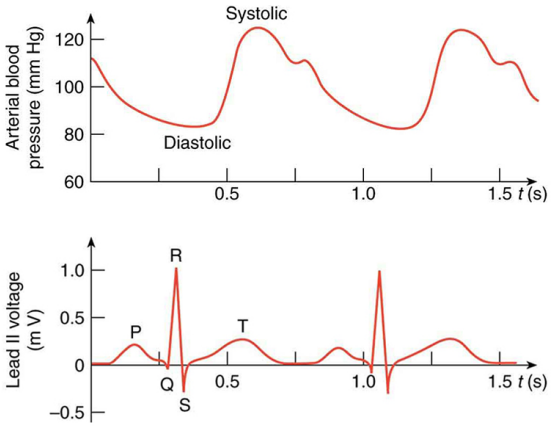



***Figure 8.*** *A lead II ECG with corresponding arterial blood pressure. The QRS complex is created by the depolarization and contraction of the ventricles and is followed shortly by the maximum or systolic blood pressure. See text for further description.*

Taken together, the 12 leads of a state-of-the-art ECG can yield a wealth of information about the heart. For example, regions of damaged heart tissue, called infarcts, reflect electrical waves and are apparent in one or more lead potentials. Subtle changes due to slight or gradual damage to the heart are most readily detected by comparing a recent ECG to an older one. This is particularly the case since individual heart shape, size, and orientation can cause variations in ECGs from one individual to another. ECG technology has advanced to the point where a portable ECG monitor with a liquid crystal instant display and a printer can be carried to patients' homes or used in emergency vehicles. See Figure 9.

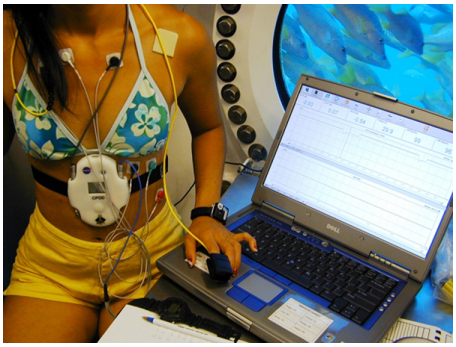


***Figure 9.*** *This NASA scientist and NEEMO 5 aquanaut's heart rate and other vital signs are being recorded by a portable device while living in an underwater habitat. (credit: NASA, Life Sciences Data Archive at Johnson Space Center, Houston, Texas)*

PhET Explorations: Neuron



PhET Interactive Simulation

***Figure 10.*** *Neuron*

Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane.

## Section Summary

- Electric potentials in neurons and other cells are created by ionic concentration differences across semipermeable membranes.
- Stimuli change the permeability and create action potentials that propagate along neurons.
- Myelin sheaths speed this process and reduce the needed energy input.
- This process in the heart can be measured with an electrocardiogram (ECG).

## *Conceptual Questions*

**1:** Note that in Figure 2, both the concentration gradient and the Coulomb force tend to move $\mathbf{Na}^+$ ions into the cell. What prevents this?

**2:** Define depolarization, repolarization, and the action potential.

**3:** Explain the properties of myelinated nerves in terms of the insulating properties of myelin.

## *Problems & Exercises*

**1: Integrated Concepts**

Use the ECG in Figure 8 to determine the heart rate in beats per minute assuming a constant time between beats.

**2: Integrated Concepts**

(a) Referring to Figure 8, find the time systolic pressure lags behind the middle of the QRS complex. (b) Discuss the reasons for the time lag.

## Glossary

**nerve conduction**
: the transport of electrical signals by nerve cells

**bioelectricity**
: electrical effects in and created by biological systems

**semipermeable**
: property of a membrane that allows only certain types of ions to cross it

**electrocardiogram (ECG)**
: usually abbreviated ECG, a record of voltages created by depolarization and repolarization, especially in the heart

## *Solutions*

**Problems & Exercises**

**1:** 80 beats/minute

# Appendix A Useful Information – Constants, Units, Formulae

This appendix is broken into several tables.

- Table 3, Important Constants
- Table 4, Submicroscopic Masses
- Table 5, Solar System Data
- Table 6, Metric Prefixes for Powers of Ten and Their Symbols
- Table 7, The Greek Alphabet
- Table 8, SI units
- Table 9, Selected British Units
- Table 10, Other Units
- Table 11, Useful Formulae

**Table 3.** Important Constants[1]

| Symbol | Meaning | Best Value | Ap |
|---|---|---|---|
| $c$ | Speed of light in vacuum | $2.99792458 \times 10^8 \mathrm{m/s}$ | 3. |
| $G$ | Gravitational constant | $6.67408(31) \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$ | 6. |
| $N_A$ | Avogadro's number | $6.02214129(27) \times 10^{23}$ | 6. |
| $k$ | Boltzmann's constant | $1.3806488(13) \times 10^{-23}\ \mathrm{J/K}$ | 1. |
| $R$ | Gas constant | $8.3144621(75)\ \mathrm{J/mol \cdot K}$ | 8. |
| $\sigma$ | Stefan-Boltzmann constant | $5.670373(21) \times 10^{-8}\ \mathrm{W/m^2 \cdot K}$ | 5. |
| $k$ | Coulomb force constant | $8.987551788\cdots \times 10^9\ \mathrm{N \cdot m^2/C^2}$ | 8. |
| $q_e$ | Charge on electron | $-1.602176565(35) \times 10^{-19}\ \mathrm{C}$ | − |
| $\varepsilon_0$ | Permittivity of free space | $8.854187817\cdots \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$ | 8. |
| $\mu_0$ | Permeability of free space | $4\pi \times 10^{-7}\ \mathrm{T \cdot\ m/A}$ | 1. |
| $h$ | Planck's constant | $6.62606957(29) \times 10^{-34}\ \mathrm{J \cdot s}$ | 6. |

| Symbol | Meaning | Best Value | Approximate Value |
|---|---|---|---|
| $m_e$ | Electron mass | $9.10938291(40) \times 10^{-31}\mathrm{kg}$ | $9.11 \times 10^{-31}\mathrm{kg}$ |
| $m_p$ | Proton mass | $1.672621777(74) \times 10^{-27}\mathrm{kg}$ | $1.6726 \times 10^{-27}$ |
| $m_n$ | Neutron mass | $1.674927351(74) \times 10^{-27}\mathrm{kg}$ | $1.6749 \times 10^{-27}$ |
| u | Atomic mass unit | $1.660538921(73) \times 10^{-27}\mathrm{kg}$ | $1.6605 \times 10^{-27}$ |

**Table 4.** Submicroscopic Masses[2]

| | | |
|---|---|---|
| **Sun** | mass | $1.99 \times 10^{30}\mathrm{kg}$ |
| | average radius | $6.96 \times 10^{8}\mathrm{m}$ |
| | Earth-sun distance (average) | $1.496 \times 10^{11}\mathrm{m}$ |
| **Earth** | mass | $5.9736 \times 10^{24}\mathrm{kg}$ |
| | average radius | $6.376 \times 10^{6}\mathrm{m}$ |
| | orbital period | $3.16 \times 10^{7}\mathrm{s}$ |
| **Moon** | mass | $7.35 \times 10^{22}\mathrm{kg}$ |
| | average radius | $1.74 \times 10^{6}\mathrm{m}$ |
| | orbital period (average) | $2.36 \times 10^{6}\mathrm{s}$ |
| | Earth-moon distance (average) | $3.84 \times 10^{8}\mathrm{m}$ |

**Table 5.** Solar System Data

| **Prefix** | **Symbol** | **Value** | **Prefix** | **Symbol** | **Value** |
|---|---|---|---|---|---|
| tera | T | $10^{12}$ | deci | d | $10^{-1}$ |
| giga | G | $10^{9}$ | centi | c | $10^{-2}$ |
| mega | M | $10^{6}$ | milli | m | $10^{-3}$ |
| kilo | k | $10^{3}$ | micro | $\mu$ | $10^{-6}$ |
| hecto | h | $10^{2}$ | nano | n | $10^{-9}$ |
| deka | da | $10^{1}$ | pico | p | $10^{-12}$ |
| – | – | $10^{0} (= 1)$ | femto | f | $10^{-15}$ |

**Table 6.** Metric Prefixes for Powers of Ten and Their Symbols

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Alpha | A | $\alpha$ | Eta | H | $\eta$ | Nu | N | $\nu$ | Tau |
| Beta | B | $\beta$ | Theta | Θ | $\theta$ | Xi | Ξ | $\xi$ | Upsil |
| Gamma | Γ | $\gamma$ | Iota | I | $\iota$ | Omicron | O | Formula does not parse | Phi |
| Delta | Δ | $\delta$ | Kappa | K | $\kappa$ | Pi | Π | $\pi$ | Chi |
| Epsilon | E | $\epsilon$ | Lambda | Λ | $\lambda$ | Rho | P | $\rho$ | Psi |
| Zeta | Z | $\zeta$ | Mu | M | $\mu$ | Sigma | Σ | $\sigma$ | Omeg |

**Table 7.** The Greek Alphabet

| | Entity | Abbreviation | Name |
|---|---|---|---|
| **Fundamental units** | Length | $\mathrm{m}$ | meter |
| | Mass | $\mathrm{kg}$ | kilogram |
| | Time | $\mathrm{s}$ | second |
| | Current | $\mathrm{A}$ | ampere |
| **Supplementary unit** | Angle | $\mathrm{rad}$ | radian |
| **Derived units** | Force | $\mathrm{N} = \mathrm{kg} \cdot \mathrm{m}/\mathrm{s}^2$ | newton |
| | Energy | $\mathrm{J} = \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$ | joule |
| | Power | $\mathrm{W} = \mathrm{J}/\mathrm{s}$ | watt |
| | Pressure | $\mathrm{Pa} = \mathrm{N}/\mathrm{m}^2$ | pascal |
| | Frequency | $\mathrm{Hz} = 1/\mathrm{s}$ | hertz |
| | Electronic potential | $\mathrm{V} = \mathrm{J}/\mathrm{C}$ | volt |
| | Capacitance | $\mathrm{F} = \mathrm{C}/\mathrm{V}$ | farad |
| | Charge | $\mathrm{C} = \mathrm{s} \cdot \mathrm{A}$ | coulomb |
| | Resistance | $\Omega = \mathrm{V}/\mathrm{A}$ | ohm |
| | Magnetic field | $\mathrm{T} = \mathrm{N}/(\mathrm{A} \cdot \mathrm{m})$ | tesla |
| | Nuclear decay rate | $\mathrm{Bq} = 1/\mathrm{s}$ | becquerel |

**Table 8.** SI Units

| | |
|---|---|
| Length | $1 \text{ inch (in.)} = 2.54 \text{ cm (exactly)}$ |
| | $1 \text{ foot (ft)} = 0.3048 \text{ m}$ |
| | $1 \text{ mile (mi)} = 1.609 \text{ km}$ |
| Force | $1 \text{ pound (lb)} = 4.448 \text{ N}$ |
| Energy | $1 \text{ British thermal unit (Btu)} = 1.055 \times 10^3 \text{ J}$ |
| Power | $1 \text{ horsepower (hp)} = 746 \text{ W}$ |
| Pressure | $1 \text{ lb/in}^2 = 6.895 \times 10^3 \text{ Pa}$ |

**Table 9.** Selected British Units

| | |
|---|---|
| Length | $1 \text{ light year (ly)} = 9.46 \times 10^{15} \text{ m}$ |
| | $1 \text{ astronomical unit (au)} = 1.50 \times 10^{11} \text{ m}$ |
| | $1 \text{ nautical mile} = 1.852 \text{ km}$ |
| | Formula does not parse |
| Area | $1 \text{ acre (ac)} = 4.05 \times 10^{3} \text{m}^2$ |
| | $1 \text{ square foot (ft}^2\text{)} = 9.29 \times 10^{-2} \text{ m}^2$ |
| | $1 \text{ barn } (b) = 10^{-28} \text{ m}^2$ |
| Volume | $1 \text{ liter } (L) = 10^{-3} \text{ m}^3$ |
| | $1 \text{ U.S. gallon (gal)} = 3.785 \times 10^{-3} \text{ m}^3$ |
| Mass | $1 \text{ solar mass} = 1.99 \times 10^{30} \text{ kg}$ |
| | $1 \text{ metric ton} = 10^{3} \text{ kg}$ |
| | $1 \text{ atomic mass unit } (u) = 1.6605 \times 10^{-27} \text{ kg}$ |
| Time | $1 \text{ year (y)} = 3.16 \times 10^{7} \text{ s}$ |
| | $1 \text{ day (d)} = 86,400 \text{ s}$ |
| Speed | $1 \text{ mile per hour (mph)} = 1.609 \text{ km/h}$ |
| | $1 \text{ nautical mile per hour (naut)} = 1.852 \text{ km/h}$ |
| Angle | $1 \text{ degree } (^\circ) = 1.745 \times 10^{-2} \text{ rad}$ |
| | $1 \text{ minute of arc } (') = 1/60 \text{ degree}$ |
| | $1 \text{ second of arc } ('') = 1/60 \text{ minute of arc}$ |

| | |
|---|---|
| | $1 \text{ grad} = 1.571 \times 10^{-2} \text{ rad}$ |
| Energy | $1 \text{ kiloton TNT (kT)} = 4.2 \times 10^{12} \text{ J}$ |
| | $1 \text{ kilowatt hour (kW} \cdot h) = 3.60 \times 10^{6} \text{ J}$ |
| | $1 \text{ food calorie (kcal)} = 4186 \text{ J}$ |
| | $1 \text{ calorie (cal)} = 4.186 \text{ J}$ |
| | $1 \text{ electron volt (eV)} = 1.60 \times 10^{-19} \text{ J}$ |
| Pressure | $1 \text{ atmosphere (atm)} = 1.013 \times 10^{5} \text{ Pa}$ |
| | $1 \text{ millimeter of mercury (mm Hg)} = 133.3 \text{ Pa}$ |
| | $1 \text{ torricelli (torr)} = 1 \text{ mm Hg} = 133.3 \text{ Pa}$ |
| Nuclear decay rate | $1 \text{ curie (Ci)} = 3.70 \times 10^{10} \text{ Bq}$ |

**Table 10.** Other Units

| | |
|---|---|
| Circumference of a circle with radius r or diameter d | $C = 2\pi r$ |
| Area of a circle with radius $r$ or diameter$d$ | $A = \pi r^2$ |
| Area of a sphere with radius $r$ | $A = 4\pi r^2$ |
| Volume of a sphere with radius $r$ | $V = (4/3$ |

**Table 11.** Useful Formulae

**Units of Length**

| | | | |
|---|---|---|---|
| meter (m) | = 39.37 inches (in.)<br>= 1.094 yards (yd) | angstrom (Å) | = $10^{-8}$ cm (exact, definition)<br>= $10^{-10}$ m (exact, definition) |
| centimeter (cm) | = 0.01 m (exact, definition) | yard (yd) | = 0.9144 m |
| millimeter (mm) | = 0.001 m (exact, definition) | inch (in.) | = 2.54 cm (exact, definition) |
| kilometer (km) | = 1000 m (exact, definition) | mile (US) | = 1.60934 km |

**Units of Volume**

| | | | |
|---|---|---|---|
| liter (L) | = 0.001 $m^3$ (exact, definition)<br>= 1000 $cm^3$ (exact, definition)<br>= 1.057 (US) quarts | liquid quart (US) | = 32 (US) liquid ounces (exact, definition)<br>= 0.25 (US) gallon (exact, definition)<br>= 0.9463 L |
| milliliter (mL) | = 0.001 L (exact, definition)<br>= 1 $cm^3$ (exact, definition) | dry quart | = 1.1012 L |
| microliter (µL)(µL) | = $10^{-6}$ L (exact, definition)<br>= $10^{-3}$ $cm^3$ (exact, definition) | cubic foot (US) | = 28.316 L |

**Units of Mass**

| | | | |
|---|---|---|---|
| gram (g) | = 0.001 kg (exact, definition) | ounce (oz) (avoirdupois) | = 28.35 g |
| milligram (mg) | = 0.001 g (exact, definition) | pound (lb) (avoirdupois) | = 0.4535924 kg |
| kilogram (kg) | = 1000 g (exact, definition)<br>= 2.205 lb | ton (short) | =2000 lb (exact, definition)<br>= 907.185 kg |
| ton (metric) | =1000 kg (exact, definition)<br>= 2204.62 lb | ton (long) | = 2240 lb (exact, definition)<br>= 1.016 metric ton |

| Units of Energy | |
|---|---|
| 4.184 joule (J) | = 1 thermochemical calorie (cal) |
| 1 thermochemical calorie (cal) | = $4.184 \times 10^{7}$ erg |
| erg | = $10^{-7}$ J (exact, definition) |
| electron-volt (eV) | = $1.60218 \times 10^{-19}$ J = 23.061 kcal $mol^{-1}$ |
| liter·atmosphere | = 24.217 cal = 101.325 J (exact, definition) |
| nutritional calorie (Cal) | = 1000 cal (exact, definition) = 4184 J |
| British thermal unit (BTU) | = 1054.804 J BTU is the amount of energy needed to heat one pound of water by one degree Fahrenheit. Therefore, the exact relationship of BTU to joules and other energy units depends on the temperature at which BTU is measured. 59 °F (15 °C) is the most widely used reference temperature for BTU definition in the United States. At this temperature, the conversion factor is the one provided in this table. |

## Footnotes

1. 1 Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.
2. 2 Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

# Appendix B Useful Mathematics

*This content is originally from OpenStax College Chemistry*

## Exponential Arithmetic

Exponential notation is used to express very large and very small numbers as a product of two numbers. The first number of the product, the *digit term*, is usually a number not less than 1 and not greater than 10. The second number of the product, the *exponential term*, is written as 10 with an exponent. Some examples of exponential notation are:

$$1000 \quad 1 \times 10^{3}$$

$$100 \quad 1 \times 10^{2}$$

$$10 \quad 1 \times 10^{1}$$

$$1 \quad 1 \times 10^{0}$$

$$0.1 \quad 1 \times 10^{-1}$$

$$0.001 \quad 1 \times 10^{-3}$$

$$2386 \quad 2.386 \times 1000 = 2.386 \times 10^{3}$$

$$0.123 \quad 1.23 \times 0.1 = 1.23 \times 10^{-1}$$

The power (exponent) of 10 is equal to the number of places the decimal is shifted to give the digit number. The exponential method is particularly useful notation for every large and very small numbers. For example, 1,230,000,000 = 1.23 × $10^9$, and 0.00000000036 = 3.6 × $10^{-10}$.

## Addition of Exponentials

Convert all numbers to the same power of 10, add the digit terms of the numbers, and if appropriate, convert the digit term back to a number between 1 and 10 by adjusting the exponential term.

### Example 1

**Adding Exponentials**

Add 5.00 × $10^{-5}$ and 3.00 × $10^{-3}$.

**Solution**

$$3.00 \times 10^{-3} = 300 \times 10^{-5}$$

$$(5.00 \times 10^{-5}) + (300 \times 10^{-5}) = 305 \times 10^{-5} = 3.05 \times 10^{-3}$$

## Subtraction of Exponentials

Convert all numbers to the same power of 10, take the difference of the digit terms, and if appropriate, convert the digit term back to a number between 1 and 10 by adjusting the exponential term.

### Example 2

**Subtracting Exponentials**

Subtract $4.0 \times 10^{-7}$ from $5.0 \times 10^{-6}$.

**Solution**

$$4.0 \times 10^{-7} = 0.40 \times 10^{-6}$$
$$(5.0 \times 10^{-6}) - (0.40 \times 10^{-6}) = 4.6 \times 10^{-6}$$

## Multiplication of Exponentials

Multiply the digit terms in the usual way and add the exponents of the exponential terms.

### Example 3

**Multiplying Exponentials**

Multiply $4.2 \times 10^{-8}$ by $2.0 \times 10^{3}$.

**Solution**

$$(4.2 \times 10^{-8}) \times (2.0 \times 10^{3}) = (4.2 \times 2.0) \times 10^{(-8)+(+3)} = 8.4 \times 10^{-5}$$

## Division of Exponentials

Divide the digit term of the numerator by the digit term of the denominator and subtract the exponents of the exponential terms.

### Example 4

**Dividing Exponentials**

Divide $3.6 \times 10^{5}$ by $6.0 \times 10^{-4}$.

**Solution**

$$\frac{3.6 \times 10^{-5}}{6.0 \times 10^{-4}} = \left(\frac{3.6}{6.0}\right) \times 10^{(-5)-(-4)} = 0.60 \times 10^{-1} = 6.0 \times 10^{-2}$$

## Squaring of Exponentials

Square the digit term in the usual way and multiply the exponent of the exponential term by 2.

### Example 5

**Squaring Exponentials**

Square the number $4.0 \times 10^{-6}$.

**Solution**

$$(4.0 \times 10^{-6})^2 = 4 \times 4 \times 10^{2 \times (-6)} = 16 \times 10^{-12} = 1.6 \times 10^{-11}$$

## Cubing of Exponentials

Cube the digit term in the usual way and multiply the exponent of the exponential term by 3.

### Example 6

**Cubing Exponentials**

Cube the number $2 \times 10^4$.

**Solution**

$$(2 \times 10^4)^3 = 2 \times 2 \times 2 \times 10^{3 \times 4} = 8 \times 10^{12}$$

## Taking Square Roots of Exponentials

If necessary, decrease or increase the exponential term so that the power of 10 is evenly divisible by 2. Extract the square root of the digit term and divide the exponential term by 2.

### Example 7

**Finding the Square Root of Exponentials**

Find the square root of $1.6 \times 10^{-7}$.

**Solution**

$$1.6 \times 10^{-7} = 16 \times 10^{-8}$$

$$\sqrt{16 \times 10^{-8}} = \sqrt{16} \times \sqrt{10^{-8}} = \sqrt{16} \times \sqrt{10^{-\frac{8}{2}}} = 4.0 \times 10^{-4}$$

# Significant Figures

A beekeeper reports that he has 525,341 bees. The last three figures of the number are obviously inaccurate, for during the time the keeper was counting the bees, some of them died and others hatched; this makes it quite difficult to determine the exact number of bees. It would have been more accurate if the beekeeper had reported the number 525,000. In other words, the last three figures are not significant, except to set the position of the decimal point. Their exact values have no meaning useful in this situation. In

reporting any information as numbers, use only as many significant figures as the accuracy of the measurement warrants.

The importance of significant figures lies in their application to fundamental computation. In addition and subtraction, the sum or difference should contain as many digits to the right of the decimal as that in the least certain of the numbers used in the computation (indicated by underscoring in the following example).

## Example 8

**Addition and Subtraction with Significant Figures**

Add 4.383 g and 0.0023 g.

**Solution**

$$\begin{array}{r} 4.38\underline{3}\ \text{g} \\ 0.002\underline{3}\ \text{g} \\ \hline 4.38\underline{5}\ \text{g} \end{array}$$

In multiplication and division, the product or quotient should contain no more digits than that in the factor containing the least number of significant figures.

### Example 9

**Multiplication and Division with Significant Figures**

Multiply 0.6238 by 6.6.

**Solution**

$$0.6238 \times 6.6 = 4.1$$

When rounding numbers, increase the retained digit by 1 if it is followed by a number larger than 5 (“round up”). Do not change the retained digit if the digits that follow are less than 5 (“round down”). If the retained digit is followed by 5, round up if the retained digit is odd, or round down if it is even (after rounding, the retained digit will thus always be even).

## The Use of Logarithms and Exponential Numbers

The common logarithm of a number (log) is the power to which 10 must be raised to equal that number. For example, the common logarithm of 100 is 2, because 10 must be raised to the second power to equal 100. Additional examples follow.

| Number | Number Expressed Exponentially | Common Logarithm |
|---|---|---|
| 1000 | $10^3$ | 3 |
| 10 | $10^1$ | 1 |
| 1 | $10^0$ | 0 |
| 0.1 | $10^{-1}$ | –1 |
| 0.001 | $10^{-3}$ | –3 |

**Table 1.** Logarithms and Exponential Numbers

What is the common logarithm of 60? Because 60 lies between 10 and 100, which have logarithms of 1 and 2, respectively, the logarithm of 60 is 1.7782; that is,

$$60 = 10^{1.7782}$$

The common logarithm of a number less than 1 has a negative value. The logarithm of 0.03918 is –1.4069, or

$$0.03918 = 10^{-1.4069} = \frac{1}{10^{1.4069}}$$

To obtain the common logarithm of a number, use the *log* button on your calculator. To calculate a number from its logarithm, take the inverse log of the logarithm, or calculate $10^x$ (where $x$ is the logarithm of the number).

The natural logarithm of a number (ln) is the power to which $e$ must be raised to equal the number; $e$ is the constant 2.7182818. For example, the natural logarithm of 10 is 2.303; that is,

$$10 = e^{2.303} = 2.7182818^{2.303}$$

To obtain the natural logarithm of a number, use the *ln* button on your calculator. To calculate a number from its natural logarithm, enter the natural logarithm and take the inverse ln of the natural logarithm, or calculate $e^x$ (where $x$ is the natural logarithm of the number).

Logarithms are exponents; thus, operations involving logarithms follow the same rules as operations involving exponents.

1. The logarithm of a product of two numbers is the sum of the logarithms of the two numbers.
$$\log xy = \log x \ + \ \log y, \;\text{and}\; \ln xy = \ln x \ + \ \ln y$$
2. The logarithm of the number resulting from the division of two numbers is the difference between the logarithms of the two numbers.
$$\log \frac{x}{y} = \log x \ - \ \log y, \;\text{and}\; \ln \frac{x}{y} = \ln x \ - \ \ln y$$
3. The logarithm of a number raised to an exponent is the product of the exponent and the logarithm of the number.
$$\log x^n = n\log x \text{ and}\; \ln x^n = n\ln x$$

## The Solution of Quadratic Equations

Mathematical functions of this form are known as second-order polynomials or, more commonly, quadratic functions.

$$ax^2 \ + \ bx \ + \ c = 0$$

The solution or roots for any quadratic equation can be calculated using the following formula:

$$x = \frac{-b \pm \sqrt{b^2 \ - \ 4ac}}{2a}$$

### Example 10

**Solving Quadratic Equations**

Solve the quadratic equation $3x^2 + 13x - 10 = 0$.

**Solution**

Substituting the values $a = 3$, $b = 13$, $c = -10$ in the formula, we obtain

$$x = \frac{-13 \pm \sqrt{(13)^2 - 4 \times 3 \times (-10)}}{2 \times 3}$$

$$x = \frac{-13 \pm \sqrt{169 + 120}}{6} = \frac{-13 \pm \sqrt{289}}{6} = \frac{-13 \pm 17}{6}$$

The two roots are therefore

$$x = \frac{-13 + 17}{6} = \frac{2}{3} \text{ and } x = \frac{-13 - 17}{6} = -5$$

Quadratic equations constructed on physical data always have real roots, and of these real roots, often only those having positive values are of any significance.

## Two-Dimensional ($x$–$y$) Graphing

The relationship between any two properties of a system can be represented graphically by a two-dimensional data plot. Such a graph has two axes: a horizontal one corresponding to the independent variable, or the variable whose value is being

controlled ($x$), and a vertical axis corresponding to the dependent variable, or the variable whose value is being observed or measured ($y$).

When the value of $y$ is changing as a function of $x$ (that is, different values of $x$ correspond to different values of $y$), a graph of this change can be plotted or sketched. The graph can be produced by using specific values for ($x$,$y$) data pairs.

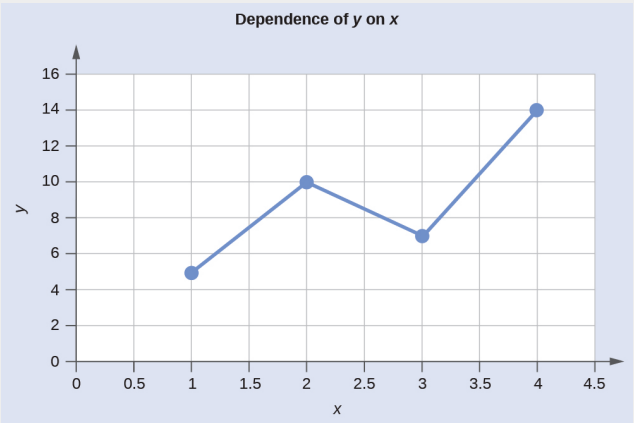
## Example 11

**Graphing the Dependence of $y$ on $x$**

| $x$ | $y$ |
|---|---|
| 1 | 5 |
| 2 | 10 |
| 3 | 7 |
| 4 | 14 |

**Table 2.**

This table contains the following points: (1,5), (2,10), (3,7), and (4,14). Each of these points can be plotted on a graph and connected to produce a graphical representation of the dependence of $y$ on $x$.




If the function that describes the dependence of $y$ on $x$ is known, it may be used to compute x,y data pairs that may subsequently be plotted.

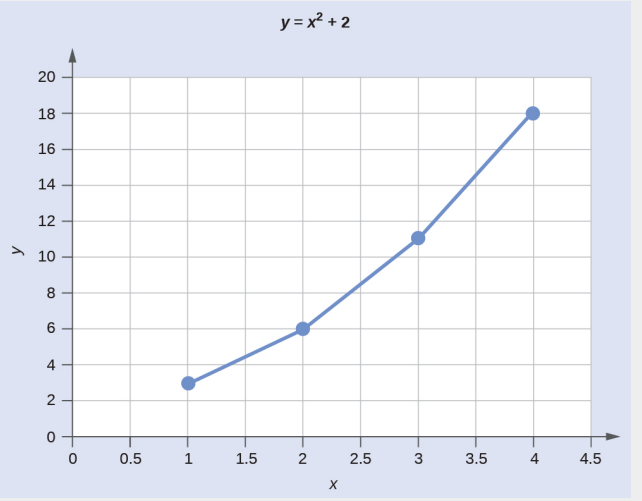
## Example 12

**Plotting Data Pairs**

If we know that $y = x^2 + 2$, we can produce a table of a few $(x,y)$ values and then plot the line based on the data shown here.

| $x$ | $y = x^2 + 2$ |
|---|---|
| 1 | 3 |
| 2 | 6 |
| 3 | 11 |
| 4 | 18 |

**Table 3.**



# Appendix C Additional Math Resources

If you need some more help with your basic math skills, there is a wealth of resources our there to help you. You can do it. Math is the language of physics, so it is worth your time to play and practise it before or during your physics and astronomy courses.

**The Khan Academy**

"Our mission is to provide a free, world-class education to anyone, anywhere. Khan Academy is a 501(c)(3) nonprofit organization." Many good videos on a variety of subjects, but it started off with math help.

https://www.khanacademy.org/math

https://www.khanacademy.org/science

**Crash Course**

Many subjects are covered here. Started by Hank Green. A youtube channel with more than 9 million subscribers. Here is YouTube done well.

https://www.youtube.com/channel/UCX6b17PVsYBQ0ip5gyeme-Q

**Sci-Show**

Hank Green does science well.

https://www.youtube.com/user/scishow

**PheT simulations for Math**

PhET started off as physic simulations, but they realized that they needed to do some basic math ones as well. These are well researched. Start here first.

https://phet.colorado.edu/en/simulations/category/new

https://phet.colorado.edu/en/simulations/category/math

Addition or "Make A Ten"

Powers of ten

https://phet.colorado.edu/en/simulation/make-a-ten

**Topics**

Place Value

Addition

Subtraction

Arithmetic

**Description**

Add numbers by making tens. Break apart and combine numbers while focusing on place value. Use the adding screen to add any two numbers. Use the game screen to apply your make-a-ten strategies!



*An interactive or media element has been excluded from this version of the text. You can view it online here:*

*https://pressbooks.bccampus.ca/douglasphys1104summer2021/?p=741*

**Arithmetic**

https://phet.colorado.edu/en/simulation/arithmetic

**Topics**

Multiplication

Division

Factoring

**Description**

Remember your multiplication tables? Practice your multiplication, division, and factoring skills with this exciting game. No calculators allowed!

**Sample Learning Goals**

Explain how multiplication tables help understand multiplication, factoring, and division.

Use an array model to understand multiplication,

factoring, and division.
Increase accuracy in multiplying, factoring and dividing.
Develop multiple strategies for arithmetic problems.



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**Fractions**
https://phet.colorado.edu/en/simulation/fraction-matcher

**Topics**

Fractions
Equivalent Fractions
Mixed Numbers

**Description**

Match shapes and numbers to earn stars in this fractions game. Challenge yourself on any level you like. Try to collect lots of stars!

**Sample Learning Goals**

Find matching fractions using numbers and pictures
Make the same fractions using different numbers
Match fractions in different picture patterns
Compare fractions using numbers and patterns



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**Equalities**

https://phet.colorado.edu/en/simulation/equality-explorer-basics

Equations
Inequalities
Proportional Reasoning

## Description

Explore what it means for a mathematical statement to be balanced or unbalanced by interacting with objects on a balance. Find all the ways to balance cats and dogs or apples and oranges.

## Sample Learning Goals

Use a balance model to solve an equation for an unknown, and justify your strategies for solving

Use proportional reasoning to determine the value of a single object



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**Proportion Playground – Ratios**

https://phet.colorado.edu/en/simulation/proportion-playground

**Topics**

Ratios

Proportional Reasoning

Unit Rate

**Description**

Play with ratios and proportions by designing a necklace, throwing paint balloons, playing billiards, or shopping for apples! Make predictions about proportions before they are revealed.



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**Unit Rates**

https://phet.colorado.edu/en/simulation/unit-rates

**Topics**
Ratios
Proportional Reasoning
Double Number Line
Unit Rate

## Description

Discover the unit rate while shopping for fruits, vegetables, and candy. Construct a double number line and look for patterns. Challenge yourself on the race track as you compare cars with different rates!

## *Sample Learning Goals*

Interpret ratios of mixed units (e.g. $/lb)

Define the unit rate and determine a method to calculate it

Predict how changing the numerator or denominator of a rate will affect the unit rate

Use the double number line to reason about rates and solve real-world problems

Develop strategies to use the unit rate to solve problems

Compare unit rates between two simultaneous situations



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**Graphing Lines – Introduction – Slope and Intercept**
https://phet.colorado.edu/en/simulation/graphing-slope-intercept



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**Graphing Lines**
https://phet.colorado.edu/en/simulation/graphing-lines



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**Trigonometry**
https://phet.colorado.edu/en/simulation/trig-tour



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# Appendix D Units, Numbers, and Significant Figures

*This content is originally from OpenStax College Chemistry 1st Canadian Edition.*

This appendix is broken into several sections:


- Introduction to Measurement
- Expressing Numbers
- Significant Figures
- Converting Units
- Other Units: Temperature and Density
- Expressing Units
- Additional Exercises


## Introduction to Measurement

Data suggest that a male child will weigh 50% of his adult weight at about 11 years of age. However, he will reach 50% of his adult height at only 2 years of age. It is obvious, then, that people eventually stop growing up but continue to grow out. Data also suggest that the average human height has been increasing over time. In industrialized countries, the average height of people increased 5.5 inches from 1810 to 1984. Most scientists attribute this simple, basic measurement of the human body to better health and nutrition.

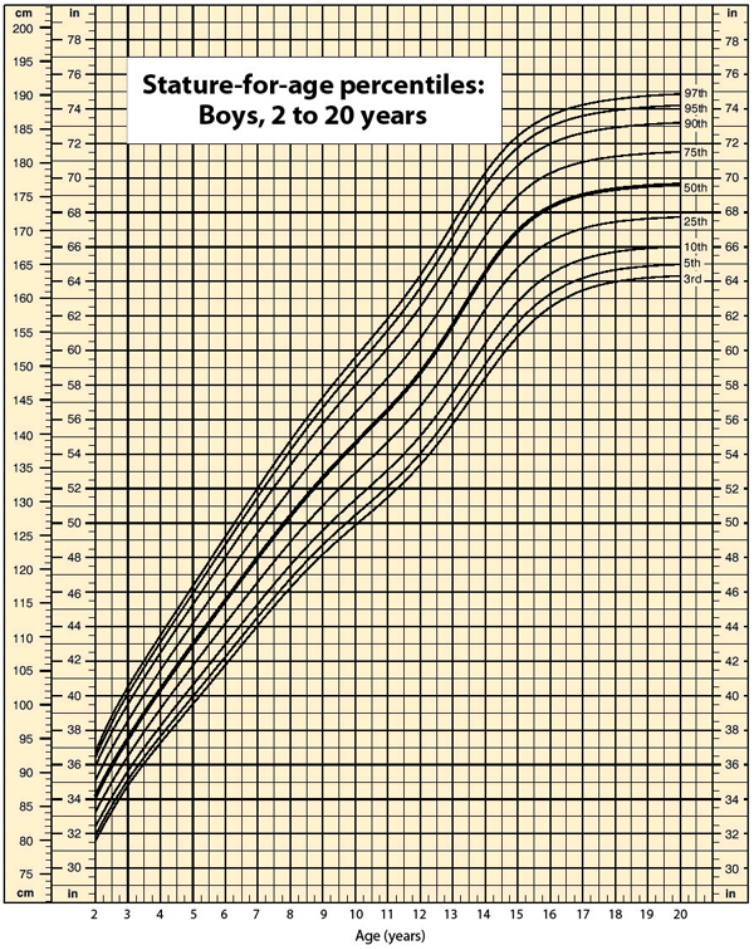



***Figure 1.*** *Stature-for-age percentiles: Boys, 2 to 20 years. Source: Chart courtesy of Centers for Disease Control and Prevention, http://www.cdc.gov/nchs/nhanes.htm#Set%201.*

In 1983, an Air Canada airplane had to make an emergency landing because it unexpectedly ran out of fuel; ground personnel had filled the fuel tanks with a certain number of pounds of fuel, not kilograms of fuel. In 1999, the Mars Climate Orbiter spacecraft was lost

attempting to orbit Mars because the thrusters were programmed in terms of English units, even though the engineers built the spacecraft using metric units. In 1993, a nurse mistakenly administered 23 units of morphine to a patient rather than the "2–3" units prescribed. (The patient ultimately survived.) These incidents occurred because people weren't paying attention to quantities.

Physics and chemistry, like all sciences, are quantitative. they deals with *quantities*, things that have amounts and units. Dealing with quantities is very important in chemistry and physics, as is relating quantities to each other. In this chapter, we will discuss how we deal with numbers and units, including how they are combined and manipulated.

## Expressing Numbers

Quantities have two parts: the number and the unit. The number tells "how many." It is important to be able to express numbers properly so that the quantities can be communicated properly.

Standard notation is the straightforward expression of a number. Numbers such as 17, 101.5, and 0.00446 are expressed in standard notation. For relatively small numbers, standard notation is fine. However, for very large numbers, such as 306,000,000, or for very small numbers, such as 0.000000419, standard notation can be cumbersome because of the number of zeros needed to place nonzero numbers in the proper position.

Scientific notation is an expression of a number using powers of 10. Powers of 10 are used to express numbers that have many zeros:

| | |
|---|---|
| $10^0$ | = 1 |
| $10^1$ | = 10 |
| $10^2$ | = 100 = 10 × 10 |
| $10^3$ | = 1,000 = 10 × 10 × 10 |
| $10^4$ | = 10,000 = 10 × 10 × 10 × 10 |

and so forth. The raised number to the right of the 10 indicating the number of factors of 10 in the original number is the exponent. (Scientific notation is sometimes called *exponential notation*.) The exponent's value is equal to the number of zeros in the number expressed in standard notation.

Small numbers can also be expressed in scientific notation but with negative exponents:

| | |
|---|---|
| $10^{-1}$ | = 0.1 = 1/10 |
| $10^{-2}$ | = 0.01 = 1/100 |
| $10^{-3}$ | = 0.001 = 1/1,000 |
| $10^{-4}$ | = 0.0001 = 1/10,000 |

and so forth. Again, the value of the exponent is equal to the number of zeros in the denominator of the associated fraction. A negative exponent implies a decimal number less than one.

A number is expressed in scientific notation by writing the first nonzero digit, then a decimal point, and then the rest of the digits. The part of a number in scientific notation that is multiplied by a power of 10 is called the coefficient. Then determine the power of 10 needed to make that number into the original number and multiply the written number by the proper power of 10. For example, to write 79,345 in scientific notation,

$79{,}345 = 7.9345 \times 10{,}000 = 7.9345 \times 10^4$

Thus, the number in scientific notation is $7.9345 \times 10^4$. For small numbers, the same process is used, but the exponent for the power of 10 is negative:

$0.000411 = 4.11 \times 1/10{,}000 = 4.11 \times 10^{-4}$

Typically, the extra zero digits at the end or the beginning of a number are not included.

## Example 1

Express these numbers in scientific notation.

1. 306,000
2. 0.00884
3. 2,760,000
4. 0.000000559

Solution

1. The number 306,000 is 3.06 times 100,000, or 3.06 times $10^5$. In scientific notation, the number is $3.06 \times 10^5$.
2. The number 0.00884 is 8.84 times 1/1,000, which is 8.84 times $10^{-3}$. In scientific notation, the number is $8.84 \times 10^{-3}$.
3. The number 2,760,000 is 2.76 times 1,000,000, which is the same as 2.76 times $10^6$. In scientific notation, the number is written as $2.76 \times 10^6$. Note that we omit the zeros at the end of the original number.
4. The number 0.000000559 is 5.59 times 1/10,000,000, which is 5.59 times $10^{-7}$. In scientific notation, the number is written as $5.59 \times 10^{-7}$.

*Test Yourself*

Express these numbers in scientific notation.

1. 23,070
2. 0.0009706

*Answers*

1. $2.307 \times 10^{4}$
2. $9.706 \times 10^{-4}$

Another way to determine the power of 10 in scientific notation is to count the number of places you need to move the decimal point to get a numerical value between 1 and 10. The number of places equals the power of 10. This number is positive if you move the decimal point to the right and negative if you move the decimal point to the left.

Many quantities in chemistry are expressed in scientific notation. When performing calculations, you may have to enter a number in scientific notation into a calculator. Be sure you know how to correctly enter a number in scientific notation into your calculator. Different models of calculators require different actions for properly entering scientific notation. If in doubt, consult your instructor immediately.

## *Key Takeaways*

- Standard notation expresses a number normally.
- Scientific notation expresses a number as a coefficient times a power of 10.

- The power of 10 is positive for numbers greater than 1 and negative for numbers between 0 and 1.

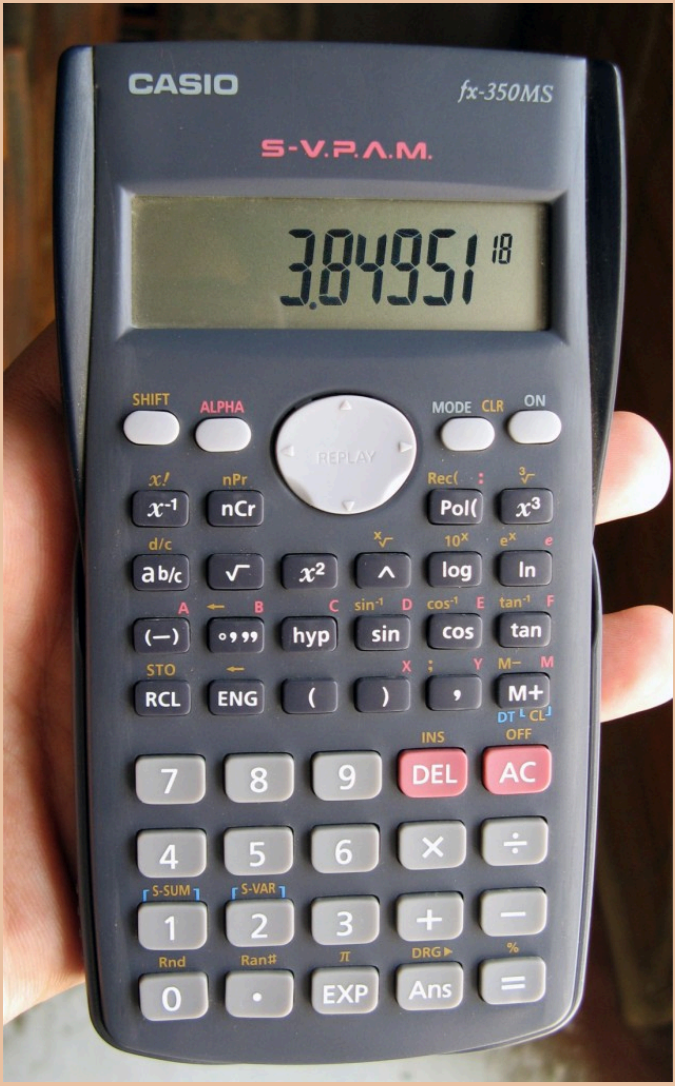



***Figure 1.*** *This calculator shows only the coefficient and the power of*

*10 to represent the number in scientific notation. Thus, the number being displayed is* $3.84951 \times 10^{18}$*, or 3,849,510,000,000,000,000. Source: "Casio"Asim Bijarani is licensed under Creative Commons Attribution 2.0 Generic*

## *Problems & Exercises*

1. Express these numbers in scientific notation.

(a) 56.9 (b) 563,100 (c) 0.0804 (d) 0.00000667

2. Express these numbers in scientific notation.

(a) −890,000 (b) 602,000,000,000 (c) 0.0000004099 (d) 0.000000000000011

3. Express these numbers in scientific notation.

(a) 0.00656 (b) 65,600 (c) 4,567,000 (d) 0.000005507

4. Express these numbers in scientific notation.

(a) 65 (b) −321.09 (c) 0.000077099 (d) 0.000000000218

5. Express these numbers in standard notation.

(a) $1.381 \times 10^{5}$ (b) $5.22 \times 10^{-7}$ (c) $9.998 \times 10^{4}$

6. Express these numbers in standard notation.

(a) $7.11 \times 10^{-2}$ (b) $9.18 \times 10^{2}$ (c) $3.09 \times 10^{-10}$

7. Express these numbers in standard notation.

(a) $8.09 \times 10^{0}$ (b) $3.088 \times 10^{-5}$ (c) $-4.239 \times 10^{2}$

8. Express these numbers in standard notation.

(a) $2.87 \times 10^{-8}$ (b) $1.78 \times 10^{11}$ (c) $1.381 \times 10^{-23}$

9. These numbers are not written in proper scientific notation. Rewrite them so that they are in proper scientific notation.

(a) $72.44 \times 10^{3}$ (b) $9{,}943 \times 10^{-5}$ (c) $588{,}399 \times 10^{2}$

10. These numbers are not written in proper scientific notation. Rewrite them so that they are in proper scientific notation.

(a) $0.000077 \times 10^{-7}$ (b) $0.000111 \times 10^{8}$ (c) $602{,}000 \times 10^{18}$

11. These numbers are not written in proper scientific notation. Rewrite them so that they are in proper scientific notation.

(a) $345.1 \times 10^{2}$ (b) $0.234 \times 10^{-3}$ (c) $1{,}800 \times 10^{-2}$

12. These numbers are not written in proper scientific notation. Rewrite them so that they are in proper scientific notation.

(a) $8{,}099 \times 10^{-8}$ (b) $34.5 \times 10^{0}$ (c) $0.000332 \times 10^{4}$

13. Write these numbers in scientific notation by counting the number of places the decimal point is moved.

(a) 123,456.78 (b) 98,490 (c) 0.000000445

14. Write these numbers in scientific notation by

counting the number of places the decimal point is moved.

(a) 0.000552 (b) 1,987 (c) 0.00000000887

15. Use your calculator to evaluate these expressions. Express the final answer in proper scientific notation.

(a) $456 \times (7.4 \times 10^{8}) = ?$ (b) $(3.02 \times 10^{5}) : (9.04 \times 10^{15}) = ?$ (c) $0.0044 \times 0.000833 = ?$

16. Use your calculator to evaluate these expressions. Express the final answer in proper scientific notation.

(a) $98{,}000 \times 23{,}000 = ?$ (b) $98{,}000 : 23{,}000 = ?$ (c) $(4.6 \times 10^{-5}) \times (2.09 \times 10^{3}) = ?$

17. Use your calculator to evaluate these expressions. Express the final answer in proper scientific notation.

(a) $45 \times 132 : 882 = ?$ (b) $[(6.37 \times 10^{4}) \times (8.44 \times 10^{-4})] : (3.2209 \times 10^{15}) = ?$

18. Use your calculator to evaluate these expressions. Express the final answer in proper scientific notation.

(a) $(9.09 \times 10^{8}) : [(6.33 \times 10^{9}) \times (4.066 \times 10^{-7})] = ?$ (b) $9{,}345 \times 34.866 : 0.00665 = ?$

## *Solutions*

**Problems & Exercises**

**1.** a) $5.69 \times 10^{1}$ b) $5.631 \times 10^{5}$ c) $8.04 \times 10^{-2}$ d) $6.67 \times 10^{-6}$

**3.** a) $6.56 \times 10^{-3}$ b) $6.56 \times 10^{4}$ c) $4.567 \times 10^{6}$ d) $5.507 \times 10^{-6}$

**5.** a) 138,100 b) 0.000000522 c) 99,980

**7.** a) 8.09 b) 0.00003088 c) −423.9

**9.** a) $7.244 \times 10^{4}$ b) $9.943 \times 10^{-2}$ c) $5.88399 \times 10^{7}$

**11.** a) $3.451 \times 10^{4}$ b) $2.34 \times 10^{-4}$ c) $1.8 \times 10^{1}$

**13.** a) $1.2345678 \times 10^{5}$ b) $9.849 \times 10^{4}$ c) $4.45 \times 10^{-7}$

**15.** a) $3.3744 \times 10^{11}$ b) $3.3407 \times 10^{-11}$ c) $3.665 \times 10^{-6}$

**17.** a) $6.7346 \times 10^{0}$ b) $1.6691 \times 10^{-14}$

## Significant Figures

*Learning Objectives*

1. Apply the concept of significant figures to limit a measurement to the proper number of digits.
2. Recognize the number of significant figures in a given quantity.
3. Limit mathematical results to the proper number of significant figures.

If you use a calculator to evaluate the expression 337/217, you will get the following:

337217=1.55299539171…

and so on for many more digits. Although this answer is correct, it is somewhat presumptuous. You start with two values that each have three digits, and the answer has *twelve* digits? That does not make much sense from a strict numerical point of view.

Consider using a ruler to measure the width of an object, as shown in Figure 2.6 "Expressing Width". The object is definitely more than 1 cm long, so we know that the first digit in our measurement is 1. We see by counting the tick marks on the ruler that the object is at least three ticks after the 1. If each tick represents 0.1 cm, then we know the object is at least 1.3 cm wide. But our ruler does not have any more ticks between the 0.3 and the 0.4 marks, so we can't know exactly how much the next decimal place is. But with a practiced eye we can estimate it. Let us estimate it as about six-tenths of the way between the third and fourth tick marks, which estimates our hundredths place as 6, so we identify a measurement of 1.36 cm for the width of the object.

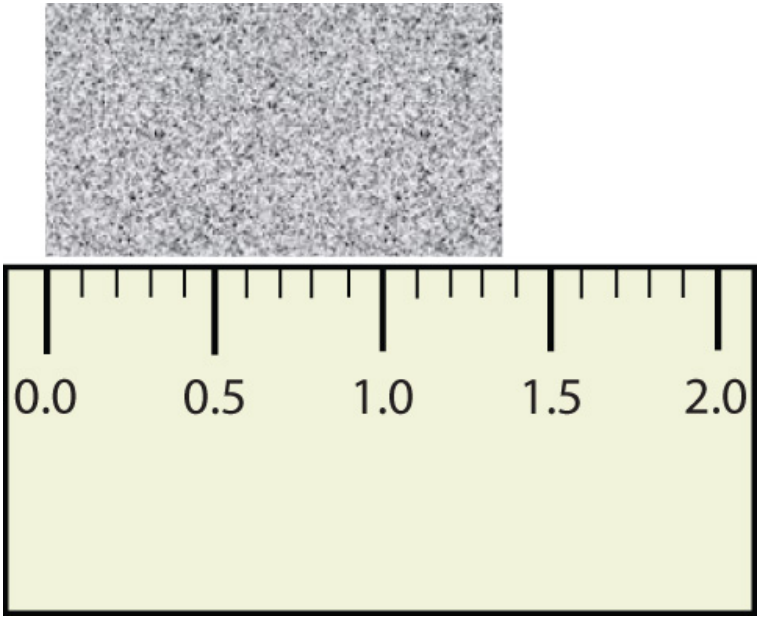



***Figure 1.*** *Expressing Width*

What is the proper way to express the width of this object?

Does it make any sense to try to report a thousandths place for the measurement? No, it doesn't; we are not exactly sure of the hundredths place (after all, it was an estimate only), so it would be fruitless to estimate a thousandths place. Our best measurement, then, stops at the hundredths place, and we report 1.36 cm as proper measurement.

This concept of reporting the proper number of digits in a measurement or a calculation is called significant figures. Significant figures (sometimes called significant digits) represent the limits of what values of a measurement or a calculation we are sure of. The convention for a measurement is that the quantity reported should be all known values and the first estimated value. The conventions for calculations are discussed as follows.

### Example 3

Use each diagram to report a measurement to the proper number of significant figures.

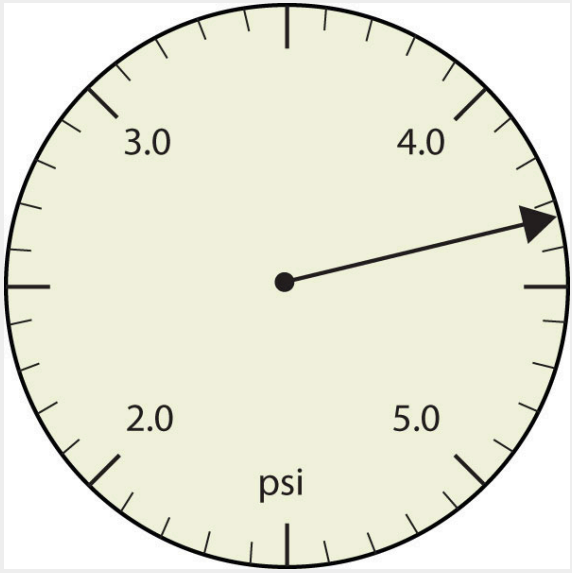
1. 


***Figure 2.*** *Pressure gauge in units of pounds per square inch*

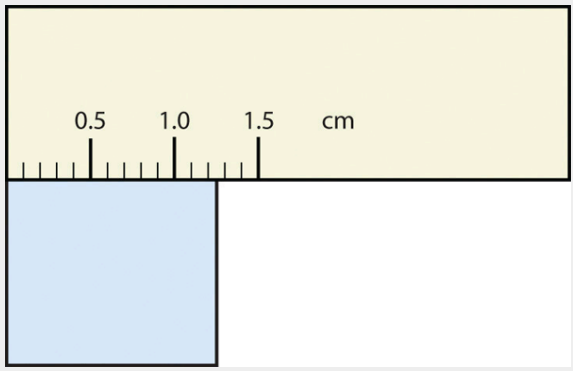
2. 


***Figure 3.*** *A measuring ruler*

Solution

1. The arrow is between 4.0 and 5.0, so the measurement is at least 4.0. The arrow is between the third and fourth small tick marks, so it's at least 0.3. We will have to estimate the last place. It looks like about one-third of the way across the space, so let us estimate the hundredths place as 3. Combining the digits, we have a measurement of 4.33 psi (psi stands for "pounds per square inch" and is a unit of pressure, like air in a tire). We say that the measurement is reported to three significant figures.
2. The rectangle is at least 1.0 cm wide but certainly not 2.0 cm wide, so the first significant digit is 1. The rectangle's width is past the second tick mark but not the third; if each tick mark

represents 0.1, then the rectangle is at least 0.2 in the next significant digit. We have to estimate the next place because there are no markings to guide us. It appears to be about halfway between 0.2 and 0.3, so we will estimate the next place to be a 5. Thus, the measured width of the rectangle is 1.25 cm. Again, the measurement is reported to three significant figures.

## *Check Your Understanding 1*

What would be the reported width of this rectangle?

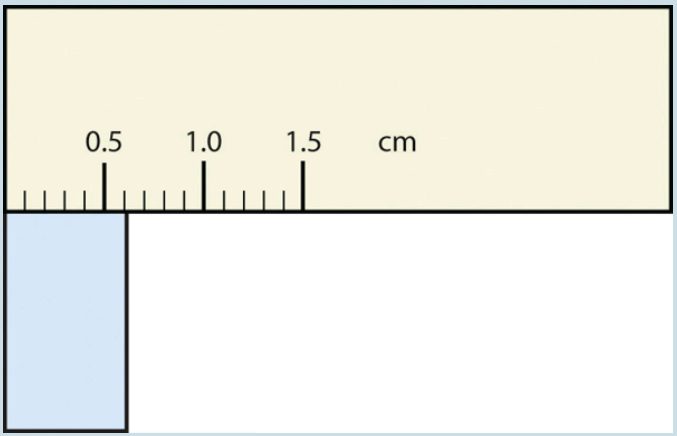



***Figure 4.*** A *measuring ruler*

In many cases, you will be given a measurement. How can you tell by looking what digits are significant? For example, the reported population of the United States is 306,000,000. Does that mean that it is *exactly* three hundred six million or is some estimation occurring?

The following conventions dictate which numbers in a reported measurement are significant and which are not significant:

1. Any nonzero digit is significant.
2. Any zeros between nonzero digits (i.e., embedded zeros) are significant.
3. Zeros at the end of a number without a decimal point (i.e., trailing zeros) are not significant; they serve only to put the significant digits in the correct positions. However, zeros at the end of any number with a decimal point are significant.

4. Zeros at the beginning of a decimal number (i.e., leading zeros) are not significant; again, they serve only to put the significant digits in the correct positions.

So, by these rules, the population figure of the United States has only three significant figures: the 3, the 6, and the zero between them. The remaining six zeros simply put the 306 in the millions position.

## Example 4

Give the number of significant figures in each measurement.

1. 36.7 m
2. 0.006606 s
3. 2,002 kg
4. 306,490,000 people

Solution

1. By rule 1, all nonzero digits are significant, so this measurement has three significant figures.
2. By rule 4, the first three zeros are not significant, but by rule 2 the zero between the sixes is; therefore, this number has four significant figures.
3. By rule 2, the two zeros between the twos are significant, so this measurement has four significant figures.

4. The four trailing zeros in the number are not significant, but the other five numbers are, so this number has five significant figures.

## *Check Your Understanding 2*

Give the number of significant figures in each measurement.

1. 0.000601 m
2. 65.080 kg

How are significant figures handled in calculations? It depends on what type of calculation is being performed. If the calculation is an addition or a subtraction, the rule is as follows: limit the reported answer to the rightmost column that all numbers have significant figures in common. For example, if you were to add 1.2 and 4.71, we note that the first number stops its significant figures in the tenths column, while the second number stops its significant figures in the hundredths column. We therefore limit our answer to the tenths column.

1.2
4.41
5.61
↑ limit final answer to the tenths column: 5.6

***Figure 5.*** *Math*

We drop the last digit—the 1—because it is not significant to the final answer.

The dropping of positions in sums and differences brings up the topic of rounding. Although there are several conventions, in this text we will adopt the following rule: the final answer should be rounded up if the first dropped digit is 5 or greater and rounded down if the first dropped digit is less than 5.

77.2
10.46
87.66
↑ limit final answer to the tenths column and round up: 87.7

***Figure 6.*** *More Math*

## Example 5

Express the final answer to the proper number of significant figures.

1. 101.2 + 18.702 = ?
2. 202.88 – 1.013 = ?

Solution

1. If we use a calculator to add these two numbers, we would get 119.902. However, most calculators do not understand significant figures, and we need to limit the final answer to the tenths place. Thus, we drop the 02 and report a final answer of 119.9 (rounding down).
2. A calculator would answer 201.867. However, we have to limit our final answer to the hundredths place. Because the first number being dropped is 7, which is greater than 7, we round up and report a final answer of 201.87.

## *Check Your Understanding 3*

Express the answer for 3.445 + 90.83 – 72.4 to the proper number of significant figures.

If the operations being performed are multiplication or division, the rule is as follows: limit the answer to the number of significant figures that the data value with the *least* number of significant figures has. So if we are dividing 23 by 448, which have two and three significant figures each, we should limit the final reported

answer to two significant figures (the lesser of two and three significant figures):

23448=0.051339286…=0.051

The same rounding rules apply in multiplication and division as they do in addition and subtraction.

## Example 6

Express the final answer to the proper number of significant figures.

1. 76.4 × 180.4 = ?
2. 934.9 : 0.00455 = ?

Solution

1. The first number has three significant figures, while the second number has four significant figures. Therefore, we limit our final answer to three significant figures: 76.4 × 180.4 = 13,782.56 = 13,800.
2. The first number has four significant figures, while the second number has three significant figures. Therefore we limit our final answer to three significant figures: 934.9 : 0.00455 = 205,472.5275… = 205,000.

## *Check Your Understanding 4*

Express the final answer to the proper number of significant figures.

1. $22.4 \times 8.314 = ?$
2. $1.381 : 6.02 = ?$

As you have probably realized by now, the biggest issue in determining the number of significant figures in a value is the zero. Is the zero significant or not? One way to unambiguously determine whether a zero is significant or not is to write a number in scientific notation. Scientific notation will include zeros in the coefficient of the number *only if they are significant*. Thus, the number $8.666 \times 10^6$ has four significant figures. However, the number $8.6660 \times 10^6$ has five significant figures. That last zero is significant; if it were not, it would not be written in the coefficient. So when in doubt about expressing the number of significant figures in a quantity, use scientific notation and include the number of zeros that are truly significant.



***Figure 7.*** *Video source: Significant figures by keyj (https://viutube.viu.ca/public/media/Significant+Figures/0_0j38j93r)*

## *Key Takeaways*

- Significant figures in a quantity indicate the number of known values plus one place that is estimated.
- There are rules for which numbers in a quantity are significant and which are not significant.
- In calculations involving addition and subtraction, limit significant figures based on the rightmost place that all values have in common.
- In calculations involving multiplication and division, limit significant figures to the least number of significant figures in all the data values.

## *Problems & Exercises*

1. Express each measurement to the correct number of significant figures.

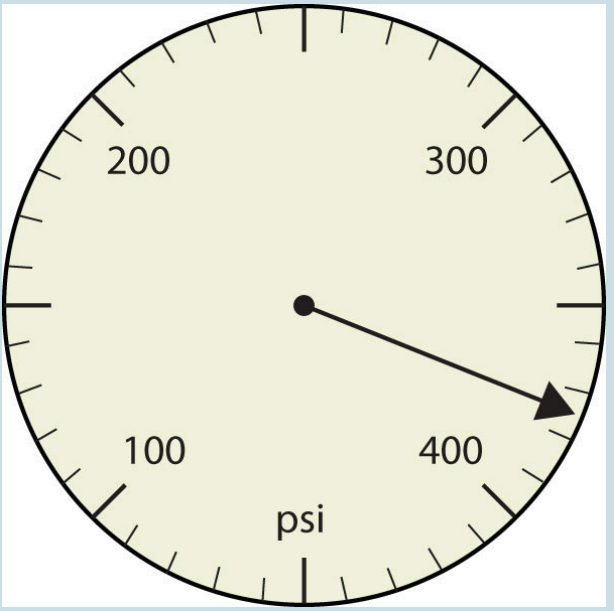



***Figure 8.*** *Pressure gauge in units of pounds per square inch*

a)

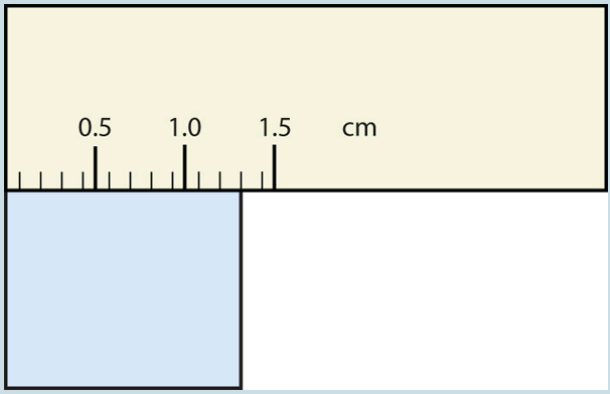



***Figure 9.*** A *measuring ruler*

b)

2. Express each measurement to the correct number of significant figures.

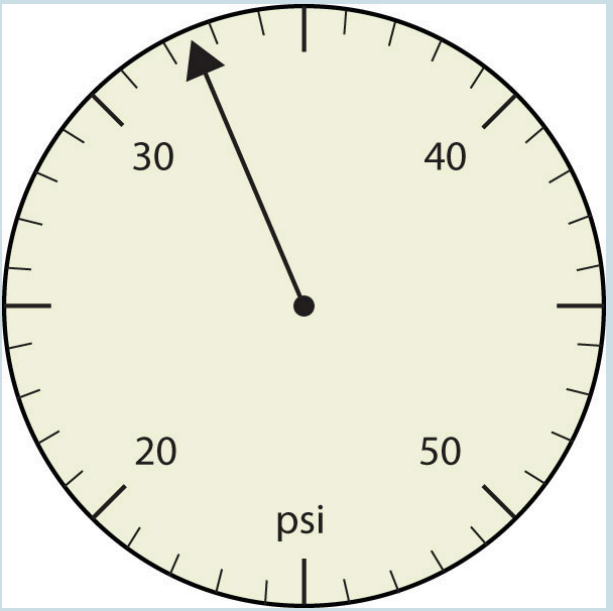



***Figure 10.*** *Pressure gauge in units of pounds per square inch*

a)

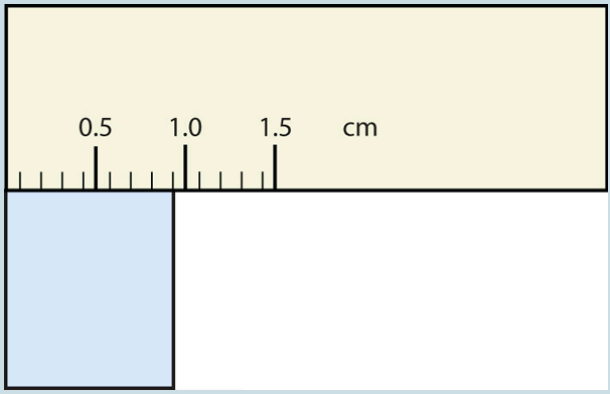



***Figure 11.*** A *measuring ruler*

b)

3. How many significant figures do these numbers have?

(a) 23 (b) 23.0 (c) 0.00023 (d) 0.0002302

4. How many significant figures do these numbers have?

(a) $5.44 \times 10^{8}$ (b) $1.008 \times 10^{-5}$ (c) 43.09 (d) 0.0000001381

5. How many significant figures do these numbers have?

(a) 765,890 (b) 765,890.0 (c) $1.2000 \times 10^{5}$ (d) 0.0005060

6) How many significant figures do these numbers have?

(a) 0.009 (b) 0.0000009 (c) 65,444 (d) 65,040

7. Compute and express each answer with the proper number of significant figures, rounding as necessary.

(a) 56.0 + 3.44 = ? (b) 0.00665 + 1.004 = ? (c) 45.99 − 32.8 = ? (d) 45.99 − 32.8 + 75.02 = ?

8. Compute and express each answer with the proper number of significant figures, rounding as necessary.

(a) 1.005 + 17.88 = ? (b) 56,700 − 324 = ? (c) 405,007 − 123.3 = ? (d) 55.5 + 66.66 − 77.777 = ?

9. Compute and express each answer with the proper number of significant figures, rounding as necessary.

(a) 56.7 × 66.99 = ? (b) 1.000 : 77 = ? (c) 1.000 : 77.0 = ? (d) 6.022 × 1.89 = ?

10. Compute and express each answer with the proper number of significant figures, rounding as necessary.

(a) 0.000440 × 17.22 = ? (b) 203,000 : 0.044 = ? (c) 67 × 85.0 × 0.0028 = ? (d) 999,999 : 3,310 = ?

11. Write the number 87,449 in scientific notation with four significant figures.

12. Write the number 0.000066600 in scientific notation with five significant figures.

13. Write the number 306,000,000 in scientific notation to the proper number of significant figures.

14. Write the number 0.0000558 in scientific notation with two significant figures.

15. Perform each calculation and limit each answer to three significant figures.

(a) 67,883 × 0.004321 = ? (b) $(9.67 \times 10^3) \times 0.0055087$ = ?

16. Perform each calculation and limit each answer to four significant figures.

(a) 18,900 × 76.33 : 0.00336 = ? (b) 0.77604 : 76,003 × 8.888 = ?

## *Solutions*

**Check Your Understanding 1**

0.63 cm

**Check Your Understanding 2**

1. three significant figures
2. five significant figures

**Check Your Understanding 3**

21.9

**Check Your Understanding 4**

1. 186
2. 0.229

**Problems & Exercises**

**1.** (a) 375 psi (b) 1.30 cm

**3.** (a) two (b) three (c) two (d) four

**5.** (a) five (b) seven (c) five (d) four

**7.** (a) 59.4 (b) 1.011 (c) 13.2 (d) 88.2

**9.** (a) $3.80 \times 10^3$ (b) 0.013 (c) 0.0130 (d) 11.4

**11.** (a) $8.745 \times 10^4$ (b) $6.6600 \times 10^{-5}$

**13.** (a) 293 (b) 53.3

## Converting Units

In "Expressing Units", we showed some examples of how to replace initial units with other units of the same type to get a numerical value that is easier to comprehend. In this section, we will formalize the process.

Consider a simple example: how many feet are there in 4 yards? Most people will almost automatically answer that there are 12 feet in 4 yards. How did you make this determination? Well, if there are 3 feet in 1 yard and there are 4 yards, then there are $4 \times 3 = 12$ feet in 4 yards.

This is correct, of course, but it is informal. Let us formalize it in a way that can be applied more generally. We know that 1 yard (yd) equals 3 feet (ft):

$1 \text{ yd} = 3 \text{ ft}$

In math, this expression is called an *equality*. The rules of algebra say that you can change (i.e., multiply or divide or add or subtract) the equality (as long as you don't divide by zero) and the new expression will still be an equality. For example, if we divide both sides by 2, we get

$$\frac{1}{2}\,yd = \frac{3}{2}\,ft$$

We see that one-half of a yard equals 3/2, or one and a half, feet—something we also know to be true, so the above equation is still an equality. Going back to the original equality, suppose we divide both sides of the equation by 1 yard (number *and* unit):

$$\frac{1\,yd}{1\,yd} = \frac{3\,ft}{1\,yd}$$

The expression is still an equality, by the rules of algebra. The left fraction equals 1. It has the same quantity in the numerator and the denominator, so it must equal 1. The quantities in the numerator and denominator cancel, both the number *and* the unit:

$$\frac{\cancel{1}\,\cancel{yd}}{\cancel{1}\,\cancel{yd}} = \frac{3\,ft}{1\,yd}$$

When everything cancels in a fraction, the fraction reduces to 1:

$$1 = \frac{3\,ft}{1\,yd}$$

We have an expression, 3 ft1 yd, that equals 1. This is a strange way

to write 1, but it makes sense: 3 ft equal 1 yd, so the quantities in the numerator and denominator are the same quantity, just expressed with different units. The expression 3 ft1 yd is called a conversion factor, and it is used to formally change the unit of a quantity into another unit. (The process of converting units in such a formal fashion is sometimes called *dimensional analysis* or the *factor label method*.)

To see how this happens, let us start with the original quantity:

4 yd

Now let us multiply this quantity by 1. When you multiply anything by 1, you don't change the value of the quantity. Rather than multiplying by just 1, let us write 1 as 3 ft1 yd:

$$4\,yd \times \frac{3\,ft}{1\,yd}$$

The 4 yd term can be thought of as 4 yd/1; that is, it can be thought of as a fraction with 1 in the denominator. We are essentially multiplying fractions. If the same thing appears in the numerator and denominator of a fraction, they cancel. In this case, what cancels is the unit *yard*:

$$4\,\cancel{yd} \times \frac{3\,ft}{1\,\cancel{yd}}$$

$$\frac{4 \times 3\,ft}{1} = \frac{12\,ft}{1} = 12\,ft$$

That is all that we can cancel. Now, multiply and divide all the numbers to get the final answer:

Again, we get an answer of 12 ft, just as we did originally. But in this case, we used a more formal procedure that is applicable to a variety of problems.

How many millimeters are in 14.66 m? To answer this, we need to construct a conversion factor between millimeters and meters and apply it correctly to the original quantity. We start with the definition of a millimeter, which is

1 mm = 1/1,000 m

The 1/1,000 is what the prefix *milli-* means. Most people are more comfortable working without fractions, so we will rewrite this equation by bringing the 1,000 into the numerator of the other side of the equation:

1,000 mm = 1 m

Now we construct a conversion factor by dividing one quantity into both sides. But now a question arises: which quantity do we divide by? It turns out that we have two choices, and the two choices will give us different conversion factors, both of which equal 1:

$$\frac{1000\ mm}{1000\ mm} = \frac{1\ m}{1000\ mm} \quad or \quad \frac{1000\ mm}{1\ m} = \frac{1\ m}{1\ m}$$

$$1 = \frac{1\ m}{1000\ mm} \quad or \quad \frac{1000\ mm}{1\ m} = 1$$

Which conversion factor do we use? The answer is based on *what unit you want to get rid of in your initial quantity*. The original unit of our quantity is meters, which we want to convert to millimeters. Because the original unit is assumed to be in the numerator, to get rid of it, we want the meter unit in the *denominator*; then they will cancel. Therefore, we will use the second conversion factor. Canceling units and performing the mathematics, we get

$$14.66 \cancel{m} \times \frac{1000\ mm}{1\ \cancel{m}} = 14660\ mm$$

Note how m cancels, leaving mm, which is the unit of interest.

The ability to construct and apply proper conversion factors is a very powerful mathematical technique in chemistry. You need to master this technique if you are going to be successful in this and future courses.

## Example 7

1. Convert 35.9 kL to liters.
2. Convert 555 nm to meters.

Solution

1. We will use the fact that 1 kL = 1,000 L. Of the two conversion factors that can be defined, the one that will work is 1,000 L/1 kL. Applying this conversion factor, we get

$$35.9\ \cancel{kL} \times \frac{1000\ L}{1\ \cancel{kL}} = 35900\ L$$

2. We will use the fact that 1 nm = 1/1,000,000,000

m, which we will rewrite as 1,000,000,000 nm = 1 m, or $10^9$ nm = 1 m. Of the two possible conversion factors, the appropriate one has the nm unit in the denominator: 1 m/$10^9$ nm. Applying this conversion factor, we get

$$555\ \cancel{nm} \times \frac{1\ m}{10^9\ \cancel{nm}} = 0.000000555\ m = 5.55 \times 10^{-7}\ m$$

In the final step, we expressed the answer in scientific notation.

## *Check Your Understanding 1*

1. Convert 67.08 µL to liters.
2. Convert 56.8 m to kilometers.

What if we have a derived unit that is the product of more than one unit, such as $m^2$? Suppose we want to convert square meters to square centimeters? The key is to remember that $m^2$ means m × m, which means we have *two* meter units in our derived unit. That means we have to include *two* conversion factors, one for each unit. For example, to convert 17.6 $m^2$ to square centimeters, we perform the conversion as follows:

$$17.6\ m^2 = 17.6\ (\cancel{m} \times \cancel{m}) \times \frac{100\ cm}{1\ \cancel{m}} \times \frac{100\ cm}{1\ \cancel{m}} = 176000\ cm \times cm = 1.76 \times 10^5\ cm^2$$

## Example 8

How many cubic centimeters are in 0.883 $m^3$?

Solution

With an exponent of 3, we have three length units, so by extension we need to use three conversion factors between meters and centimeters. Thus, we have

$$0.833\ \cancel{m}^3 \times \frac{100\ cm}{1\ \cancel{m}} \times \frac{100\ cm}{1\ \cancel{m}} \times \frac{100\ cm}{1\ \cancel{m}} = 883000\ cm^3 = 8.83 \times 10^5\ cm^3$$

You should demonstrate to yourself that the three meter units do indeed cancel.

## *Check Your Understanding 2*

How many cubic millimeters are present in 0.0923 $m^3$?

Suppose the unit you want to convert is in the denominator of a derived unit; what then? Then, in the conversion factor, the unit you want to remove must be in the *numerator*. This will cancel with the original unit in the denominator and introduce a new unit in the denominator. The following example illustrates this situation.

## Example 9

Convert 88.4 m/min to meters/second.

Solution

We want to change the unit in the denominator from minutes to seconds. Because there are 60 seconds in 1 minute (60 s = 1 min), we construct a conversion factor so that the unit we want to remove, minutes, is in the numerator: 1 min/60 s. Apply and perform the math:

$$\frac{88.4\ m}{\cancel{min}} \times \frac{1\ \cancel{min}}{60\ s} = 1.47\ m/s$$

Notice how the 88.4 automatically goes in the numerator. That's because any number can be thought of as being in the numerator of a fraction divided by 1.

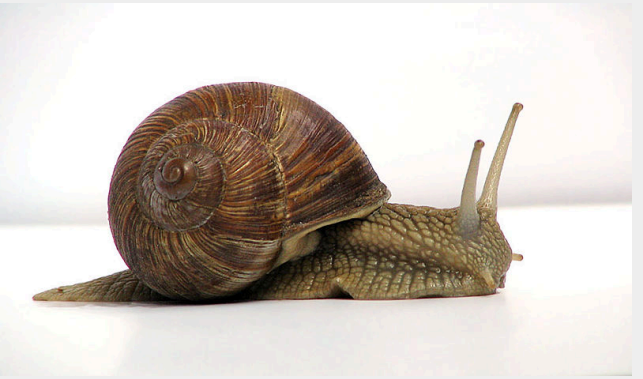


***Figure 1.*** *How Fast Is Fast? A common garden snail moves at a rate of about 0.2 m/min, which is about 0.003 m/s, which is 3 mm/s!*
*Source: "Grapevine snail"by Jürgen Schoneris licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license.*

## *Check Your Understanding 3*

Convert 0.203 m/min to meters/second.

Sometimes there will be a need to convert from one unit with one numerical prefix to another unit with a different numerical prefix. How do we handle those conversions? Well, you could memorize the conversion factors that interrelate all numerical prefixes. Or you can go the easier route: first convert the quantity to the base unit, the unit with no numerical prefix, using the definition of the original

prefix. Then convert the quantity in the base unit to the desired unit using the definition of the second prefix. You can do the conversion in two separate steps or as one long algebraic step. For example, to convert 2.77 kg to milligrams:

$$2.77\ \cancel{kg} \times \frac{1000\ g}{1\ \cancel{kg}} = 2770\ g\ (convert\ to\ the\ base\ unit\ of\ grams)$$

$$2770\ \cancel{g} \times \frac{1000\ mg}{1\ \cancel{g}} = 2770000\ mg = 2.77 \times 10^6\ mg\ (convert\ to\ the\ desired\ unit)$$

Alternatively, it can be done in a single multistep process:

$$2.77\ \cancel{kg} \times \frac{1000\ \cancel{g}}{1\ \cancel{kg}} \times \frac{1000\ mg}{1\ \cancel{g}} = 2770000\ mg = 2.77 \times 10^6\ mg$$

You get the same answer either way.

## Example 10

How many nanoseconds are in 368.09 μs?

Solution

You can either do this as a one-step conversion from microseconds to nanoseconds or convert to the base unit first and then to the final desired unit. We will use the second method here, showing the two steps in a single line. Using the definitions of the prefixes *micro-* and *nano-*,

$$368.09\ \cancel{\mu s} \times \frac{1\ \cancel{s}}{10^6\ \cancel{\mu s}} \times \frac{10^9\ ns}{1\ \cancel{s}} = 368090\ ns = 3.6809\ x\ 10^5\ ns$$

### *Check Your Understanding 4*

How many milliliters are in 607.8 kL?

When considering the significant figures of a final numerical answer in a conversion, there is one important case where a number does not impact the number of significant figures in a final answer—the so-called exact number. An exact number is a number from a defined relationship, not a measured one. For example, the prefix *kilo-* means 1,000—*exactly* 1,000, no more or no less. Thus, in constructing the conversion factor

$$\frac{1000\ g}{1\ kg}$$

neither the 1,000 nor the 1 enter into our consideration of significant figures. The numbers in the numerator and denominator are defined exactly by what the prefix *kilo-* means. Another way of thinking about it is that these numbers can be thought of as having an infinite number of significant figures, such as

$$\frac{1000.0000000000\ \ldots\ g}{1.0000000000\ \ldots\ kg}$$

The other numbers in the calculation will determine the number of significant figures in the final answer.

## Example 11

A rectangular plot in a garden has the dimensions 36.7 cm by 128.8 cm. What is the area of the garden plot in square meters? Express your answer in the proper number of significant figures.

Solution

Area is defined as the product of the two dimensions, which we then have to convert to square meters and express our final answer to the correct number of significant figures, which in this case will be three.

$$36.7\ \cancel{cm} \times 128.8\ \cancel{cm} \times \frac{1\ m}{100\ \cancel{cm}} \times \frac{1\ m}{100\ \cancel{cm}} = 0.472696\ m^2 = 0.473\ m^2$$

The 1 and 100 in the conversion factors do not affect the determination of significant figures because they are exact numbers, defined by the centi- prefix.

## *Check Your Understanding 5*

What is the volume of a block in cubic meters whose dimensions are 2.1 cm × 34.0 cm × 118 cm?

## Chemistry (and physics and math…) is Everywhere: The Gimli Glider

On July 23, 1983, an Air Canada Boeing 767 jet had to glide to an emergency landing at Gimli Industrial Park Airport in Gimli, Manitoba, because it unexpectedly ran out of fuel during flight. There was no loss of life in the course of the emergency landing, only some minor injuries associated in part with the evacuation of the craft after landing. For the remainder of its operational life (the plane was retired in 2008), the aircraft was nicknamed "the Gimli Glider."




***Figure 2.*** *The Gimli Glider is the Boeing 767 that ran out of fuel and glided to safety at Gimli Airport. The aircraft ran out of fuel because of confusion over the units used to express the amount of fuel.*
*"Aircanada.b767" is in the the public domain.*

The 767 took off from Montreal on its way to Ottawa, ultimately heading for Edmonton, Canada. About halfway through the flight,

all the engines on the plane began to shut down because of a lack of fuel. When the final engine cut off, all electricity (which was generated by the engines) was lost; the plane became, essentially, a powerless glider. Captain Robert Pearson was an experienced glider pilot, although he had never flown a glider the size of a 767. First Officer Maurice Quintal quickly determined that the aircraft would not be able make it to Winnipeg, the next large airport. He suggested his old Royal Air Force base at Gimli Station, one of whose runways was still being used as a community airport. Between the efforts of the pilots and the flight crew, they managed to get the airplane safely on the ground (although with buckled landing gear) and all passengers off safely.

What happened? At the time, Canada was transitioning from the older English system to the metric system. The Boeing 767s were the first aircraft whose gauges were calibrated in the metric system of units (liters and kilograms) rather than the English system of units (gallons and pounds). Thus, when the fuel gauge read 22,300, the gauge meant kilograms, but the ground crew mistakenly fueled the plane with 22,300 *pounds* of fuel. This ended up being just less than half of the fuel needed to make the trip, causing the engines to quit about halfway to Ottawa. Quick thinking and extraordinary skill saved the lives of 61 passengers and 8 crew members—an incident that would not have occurred if people were watching their units.



***Figure 3.*** *Video source: Unit conversion by keyj (https://viutube.viu.ca/public/media/Unit+Conversion/0_h2w068q1)*

## *Key Takeaways*

- Units can be converted to other units using the proper conversion factors.
- Conversion factors are constructed from equalities that relate two different units.
- Conversions can be a single step or multistep.
- Unit conversion is a powerful mathematical technique in chemistry that must be mastered.
- Exact numbers do not affect the determination of significant figures.

## *Exercises*

1. Write the two conversion factors that exist between the two given units.

(a) milliliters and liters (b) microseconds and seconds (c) kilometers and meters

2. Write the two conversion factors that exist between the two given units.

(a) kilograms and grams (b) milliseconds and seconds (c) centimeters and meters

3. Perform the following conversions.

(a) 5.4 km to meters (b) 0.665 m to millimeters (c) 0.665 m to kilometers

4. Perform the following conversions.

(a) 90.6 mL to liters (b) 0.00066 ML to liters (c) 750 L to kiloliters

5. Perform the following conversions.

(a) 17.8 µg to grams (b) $7.22 \times 10^2$ kg to grams (c) 0.00118 g to nanograms

6. Perform the following conversions.

(a) 833 ns to seconds (b) 5.809 s to milliseconds (c) $2.77 \times 10^6$ s to megaseconds

7. Perform the following conversions.

(a) 9.44 $m^2$ to square centimeters (b) $3.44 \times 10^8$ $mm^3$ to cubic meters

8. Perform the following conversions.

(a) 0.00444 $cm^3$ to cubic meters (b) $8.11 \times 10^2$ $m^2$ to square nanometers

9. Why would it be inappropriate to convert square centimeters to cubic meters?

10. Why would it be inappropriate to convert from cubic meters to cubic seconds?

11. Perform the following conversions.

(a) 45.0 m/min to meters/second (b) 0.000444 m/s to micrometers/second (c) 60.0 km/h to kilometers/second

12. Perform the following conversions.

(a) $3.4 \times 10^2$ cm/s to centimeters/minute (b) 26.6 mm/s to millimeters/hour (c) 13.7 kg/L to kilograms/milliliters

13. Perform the following conversions.

(a) 0.674 kL to milliliters (b) $2.81 \times 10^{12}$ mm to kilometers (c) 94.5 kg to milligrams

14. Perform the following conversions.

(a) $6.79 \times 10^{-6}$ kg to micrograms (b) 1.22 mL to kiloliters (c) $9.508 \times 10^{-9}$ ks to milliseconds

15. Perform the following conversions.

(a) $6.77 \times 10^{14}$ ms to kiloseconds (b) 34,550,000 cm to kilometers

16. Perform the following conversions.

(a) $4.701 \times 10^{15}$ mL to kiloliters (b) $8.022 \times 10^{-11}$ ks to microseconds

17. Perform the following conversions. Note that you will have to convert units in both the numerator and the denominator.

(a) 88 ft/s to miles/hour (Hint: use 5,280 ft = 1 mi.) (b) 0.00667 km/h to meters/second

18. Perform the following conversions. Note that you will have to convert units in both the numerator and the denominator.

(a) $3.88 \times 10^{2}$ mm/s to kilometers/hour (b) 1.004 kg/L to grams/milliliter

19. What is the area in square millimeters of a rectangle whose sides are 2.44 cm × 6.077 cm? Express the answer to the proper number of significant figures.

20. What is the volume in cubic centimeters of a cube with sides of 0.774 m? Express the answer to the proper number of significant figures.

21. The formula for the area of a triangle is 1/2 × base × height. What is the area of a triangle in square centimeters if its base is 1.007 m and its height is 0.665 m? Express the answer to the proper number of significant figures.

22. The formula for the area of a triangle is 1/2 × base × height. What is the area of a triangle in square meters if its base is 166 mm and its height is 930.0 mm? Express the answer to the proper number of significant figures.

## *Solutions*

**Check Your Understanding 1**

1. $6.708 \times 10^{-5}$ L
2. $5.68 \times 10^{-2}$ km

**Check Your Understanding 2**

$9.23 \times 10^{7}$ $mm^3$

**Check Your Understanding 3**

0.00338 m/s or $3.38 \times 10^{-3}$ m/s

**Check Your Understanding 4**

$6.078 \times 10^{8}$ mL

**Check Your Understanding 5**

0.0084 $m^3$

**Problems & Exercises**

**1.** (a) 1,000 mL/1 L and 1 L/1,000 mL (b) 1,000,000 µs/1 s and 1 s/1,000,000 µs (c) 1,000 m/1 km and 1 km1,000 m

**3.** (a) 5,400 m (b) 665 mm (c) $6.65 \times 10^{-4}$ km

**5.** (a) $1.78 \times 10^{-5}$ g (b) $7.22 \times 10^{5}$ g (c) $1.18 \times 10^{6}$ ng

**7.** (a) 94,400 $cm^2$ (b) 0.344 $m^3$

**9.** One is a unit of area, and the other is a unit of volume.

**11.** (a) 0.75 m/s (b) 444 µm/s (c) $1.666 \times 10^{-2}$ km/s

**13.** (a) 674,000 mL (b) $2.81 \times 10^{6}$ km (c) $9.45 \times 10^{7}$ mg

**15.** (a) $6.77 \times 10^{8}$ ks (b) 345.5 km

**17.** (a) $6.0 \times 10^{1}$ mi/h (b) 0.00185 m/s

**19.** $1.48 \times 10^3\ \text{mm}^2$

**21.** $3.35 \times 10^3\ \text{cm}^2$

## Other Units: Temperature and Density

There are other units in chemistry that are important, and we will cover others in the course of the entire book. One of the fundamental quantities in science is temperature. Temperature is a measure of the average amount of energy of motion, or *kinetic energy*, a system contains. Temperatures are expressed using scales that use units called degrees, and there are several temperature scales in use. In the United States, the commonly used temperature scale is the *Fahrenheit scale* (symbolized by °F and spoken as "degrees Fahrenheit"). On this scale, the freezing point of liquid water (the temperature at which liquid water turns to solid ice) is 32 °F, and the boiling point of water (the temperature at which liquid water turns to steam) is 212 °F.

Science also uses other scales to express temperature. The Celsius scale (symbolized by °C and spoken as "degrees Celsius") is a temperature scale where 0 °C is the freezing point of water and 100 °C is the boiling point of water; the scale is divided into 100 divisions between these two landmarks and extended higher and lower. By comparing the Fahrenheit and Celsius scales, a conversion between the two scales can be determined:

$$°\text{C} = (°\text{F} - 32) \times \frac{5}{9}$$

$$°F = \left(°C \times \frac{9}{5}\right) + 32$$

Using these formulas, we can convert from one temperature scale to another. The number 32 in the formulas is exact and does not count in significant figure determination.

### Example 12

1. What is 98.6 °F in degrees Celsius?
2. What is 25.0 °C in degrees Fahrenheit?

Solution

1. Using the first formula from above, we have

   °C = (98.6 − 32) × 5/9 = 66.6 × 5/9 = 37.0 °C

2. Using the second formula from above, we have

   °F = (25.0 × 9/5) + 3/2 = 45.0 + 32 = 77.0 °F

### *Check Your Understanding 1*

1. Convert 0 °F to degrees Celsius.

2. Convert 212 °C to degrees Fahrenheit.

The fundamental unit of temperature (another fundamental unit of science, bringing us to four) in SI is the kelvin (K). The Kelvin temperature scale (note that the name of the scale capitalizes the word *Kelvin*, but the unit itself is lowercase) uses degrees that are the same size as the Celsius degree, but the numerical scale is shifted up by 273.15 units. That is, the conversion between the Kelvin and Celsius scales is as follows:

K = °C + 273.15

°C = K − 273.15

For most purposes, it is acceptable to use 273 instead of 273.15. Note that the Kelvin scale does not use the word *degrees*; a temperature of 295 K is spoken of as "two hundred ninety-five kelvins" and not "two hundred ninety-five degrees Kelvin."

The reason that the Kelvin scale is defined this way is because there exists a minimum possible temperature called absolute zero. The Kelvin temperature scale is set so that 0 K is absolute zero, and temperature is counted upward from there. Normal room temperature is about 295 K, as seen in the following example.

## Example 13

If normal room temperature is 72.0 °F, what is room temperature in degrees Celsius and kelvins?

Solution

First, we use the formula to determine the temperature in degrees Celsius:

°C = (72.0 − 32) × 5/9 = 40.0 × 5/9 = 22.2 °C

Then we use the appropriate formula above to determine the temperature in the Kelvin scale:

K = 22.2 °C + 273.15 = 295.4 K

So, room temperature is about 295 K.

## *Check Your Understanding 2*

What is 98.6 °F on the Kelvin scale?

Figure 2.9 "Fahrenheit, Celsius, and Kelvin Temperatures" compares the three temperature scales. Note that science uses the Celsius and Kelvin scales almost exclusively; virtually no practicing chemist expresses laboratory-measured temperatures with the Fahrenheit scale. (In fact, the United States is one of the few countries in the world that still uses the Fahrenheit scale on a daily basis. The other two countries are Liberia and Myanmar [formerly Burma].

Figure 2.9 Fahrenheit, Celsius, and Kelvin Temperatures

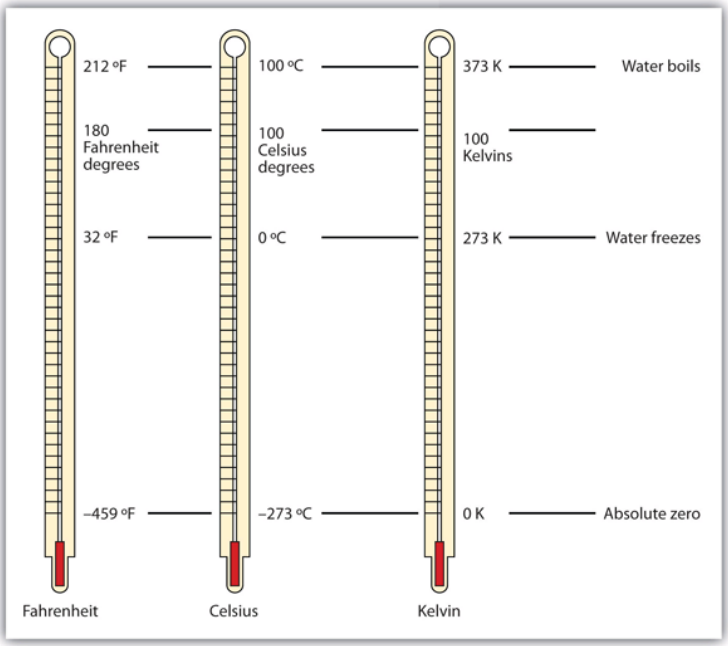



***Figure 1.*** *Fahrenheit, Celsius, and Kelvin Temperatures*

A comparison of the three temperature scales.

Density is a physical property that is defined as a substance's mass divided by its volume:

density = mass/volume or d = m/V

Density is usually a measured property of a substance, so its numerical value affects the significant figures in a calculation. Notice that density is defined in terms of two dissimilar units, mass and volume. That means that density overall has derived units, just like velocity. Common units for density include g/mL, $g/cm^3$, g/L, kg/L, and even $kg/m^3$. Densities for some common substances are listed in Table 2.2 "Densities of Some Common Substances".

Table 2.2 Densities of Some Common Substances

| Substance | Density (g/mL or g/cm$^3$) |
|---|---|
| water | 1.0 |
| gold | 19.3 |
| mercury | 13.6 |
| air | 0.0012 |
| cork | 0.22–0.26 |
| aluminum | 2.7 |
| iron | 7.87 |

Because of how it is defined, density can act as a conversion factor for switching between units of mass and volume. For example, suppose you have a sample of aluminum that has a volume of 7.88 cm$^3$. How can you determine what mass of aluminum you have without measuring it? You can use the volume to calculate it. If you multiply the given volume by the known density (from Table 2.2 “Densities of Some Common Substances”), the volume units will cancel and leave you with mass units, telling you the mass of the sample:

$$7.88\ \text{cm}^3 \times 2.7\ \text{g/cm}^3 = 21\ \text{g of aluminum}$$

where we have limited our answer to two significant figures.

## Example 14

What is the mass of 44.6 mL of mercury?

Solution

Use the density from Table 2.2 “Densities of Some

Common Substances" as a conversion factor to go from volume to mass:

44.6 mL × 13.6 g/mL = 607 g

The mass of the mercury is 607 g.

## *Check Your Understanding 3*

What is the mass of 25.0 $cm^3$ of iron?

Density can also be used as a conversion factor to convert mass to volume—but care must be taken. We have already demonstrated that the number that goes with density normally goes in the numerator when density is written as a fraction. Take the density of gold, for example:

$$d = 19.3 \frac{g}{mL} = \frac{19.3\ g}{1\ mL}$$

Although this was not previously pointed out, it can be assumed that there is a 1 in the denominator:

That is, the density value tells us that we have 19.3 grams for every 1 milliliter of volume, and the 1 is an exact number. When we want to use density to convert from mass to volume, the numerator and denominator of density need to be switched—that is, we must take

the *reciprocal* of the density. In so doing, we move not only the units but also the numbers:

$$\frac{1}{d} = \frac{1\ mL}{19.3\ g}$$

This reciprocal density is still a useful conversion factor, but now the mass unit will cancel and the volume unit will be introduced. Thus, if we want to know the volume of 45.9 g of gold, we would set up the conversion as follows:

$$45.9\ \cancel{g} \times \frac{1\ mL}{19.3\ \cancel{g}} = 2.38\ mL$$

Note how the mass units cancel, leaving the volume unit, which is what we're looking for.

### Example 15

A cork stopper from a bottle of wine has a mass of 3.78 g. If the density of cork is 0.22 g/cm$^3$, what is the volume of the cork?

Solution

To use density as a conversion factor, we need to take

the reciprocal so that the mass unit of density is in the denominator. Taking the reciprocal, we find

$$\frac{1}{d} = \frac{1\ cm^3}{0.22\ g}$$

We can use this expression as the conversion factor. So

$$3.78\ \cancel{g} \times \frac{1\ cm^3}{0.22\ \cancel{g}} = 17.2\ cm^3$$

## *Check Your Understanding 4*

What is the volume of 3.78 g of gold?

Care must be used with density as a conversion factor. Make sure the mass units are the same, or the volume units are the same, before using density to convert to a different unit. Often, the unit of the given quantity must be first converted to the appropriate unit before applying density as a conversion factor.

## Food and Drink App: Cooking Temperatures

Because degrees Fahrenheit is the common temperature scale in the United States, kitchen appliances, such as ovens, are calibrated in that scale. A cool oven may be only 150°F, while a cake may be baked at 350°F and a chicken roasted at 400°F. The broil setting on many ovens is 500°F, which is typically the highest temperature setting on a household oven.

People who live at high altitudes, typically 2,000 ft above sea level or higher, are sometimes urged to use slightly different cooking instructions on some products, such as cakes and bread, because water boils at a lower temperature the higher in altitude you go, meaning that foods cook slower. For example, in Cleveland water typically boils at 212°F (100°C), but in Denver, the Mile-High City, water boils at about 200°F (93.3°C), which can significantly lengthen cooking times. Good cooks need to be aware of this.

At the other end is pressure cooking. A pressure cooker is a closed vessel that allows steam to build up additional pressure, which increases the temperature at which water boils. A good pressure cooker can get to temperatures as high as 252°F (122°C); at these temperatures, food cooks much faster than it normally would. Great care must be used with pressure cookers because of the high pressure and high temperature. (When a pressure cooker is used to sterilize medical instruments, it is called an *autoclave*.)

Other countries use the Celsius scale for everyday purposes. Therefore, oven dials in their kitchens are marked in degrees Celsius. It can be confusing for US cooks to use ovens abroad—a 425°F oven in the United States is equivalent to a 220°C oven in other countries. These days, many oven thermometers are marked with both temperature scales.

## *Key Takeaways*

- Chemistry uses the Celsius and Kelvin scales to express temperatures.
- A temperature on the Kelvin scale is the Celsius temperature plus 273.15.
- The minimum possible temperature is absolute zero and is assigned 0 K on the Kelvin scale.
- Density relates a substance's mass and volume.
- Density can be used to calculate volume from a given mass or mass from a given volume.

## *Problems & Exercises*

1. Perform the following conversions.

(a) 255°F to degrees Celsius (b) –255°F to degrees Celsius (c) 50.0°C to degrees Fahrenheit (d) –50.0°C to degrees Fahrenheit

2. Perform the following conversions.

(a) 1,065°C to degrees Fahrenheit (b) –222°C to degrees Fahrenheit (c) 400.0°F to degrees Celsius (d) 200.0°F to degrees Celsius

3. Perform the following conversions.

(a) 100.0°C to kelvins (b) –100.0°C to kelvins (c) 100 K to degrees Celsius (d) 300 K to degrees Celsius

4. Perform the following conversions.

(a) 1,000.0 K to degrees Celsius (b) 50.0 K to degrees Celsius (c) 37.0°C to kelvins (d) –37.0°C to kelvins

5. Convert 0 K to degrees Celsius. What is the significance of the temperature in degrees Celsius?

6. Convert 0 K to degrees Fahrenheit. What is the significance of the temperature in degrees Fahrenheit?

7. The hottest temperature ever recorded on the surface of the earth was 136°F in Libya in 1922. What is the temperature in degrees Celsius and in kelvins?

8. The coldest temperature ever recorded on the surface of the earth was –128.6°F in Vostok, Antarctica, in 1983. What is the temperature in degrees Celsius and in kelvins?

9. Give at least three possible units for density.

10. What are the units when density is inverted? Give three examples.

11. A sample of iron has a volume of 48.2 $cm^3$. What is its mass?

12. A sample of air has a volume of 1,015 mL. What is its mass?

13. The volume of hydrogen used by the *Hindenburg*, the German airship that exploded in New Jersey in 1937, was $2.000 \times 10^8$ L. If hydrogen gas has a density of 0.0899 g/L, what mass of hydrogen was used by the airship?

14. The volume of an Olympic-sized swimming pool is $2.50 \times 10^9$ $cm^3$. If the pool is filled with alcohol ($d$ = 0.789 g/$cm^3$), what mass of alcohol is in the pool?

15. A typical engagement ring has 0.77 $cm^3$ of gold. What mass of gold is present?

16. A typical mercury thermometer has 0.039 mL of mercury in it. What mass of mercury is in the thermometer?

17. What is the volume of 100.0 g of lead if lead has a density of 11.34 g/$cm^3$?

18. What is the volume of 255.0 g of uranium if uranium has a density of 19.05 g/$cm^3$?

19. What is the volume in liters of 222 g of neon if neon has a density of 0.900 g/L?

20. What is the volume in liters of 20.5 g of sulfur hexafluoride if sulfur hexafluoride has a density of 6.164 g/L?

21. Which has the greater volume, 100.0 g of iron ($d$ = 7.87 g/$cm^3$) or 75.0 g of gold ($d$ = 19.3 g/$cm^3$)?

22. Which has the greater volume, 100.0 g of hydrogen gas ($d$ = 0.0000899 g/$cm^3$) or 25.0 g of argon gas ($d$ = 0.00178 g/$cm^3$)?

## *Solutions*

**Check Your Understanding 1**

1. –17.8 °C
2. 414 °F

**Check Your Understanding 2**

310.2 K

**Check Your Understanding 3**

197 g

**Check Your Understanding 4**

0.196 $cm^3$

**Problems & Exercises**

**1.** (a) 124°C (b) –159°C (c) 122°F (d) –58°F

**3.** (a) 373 K (b) 173 K (c) –173°C (d) 27°C

**5.** –273°C. This is the lowest possible temperature in degrees Celsius.

**7.** 57.8°C; 331 K

**9.** g/mL, g/L, and kg/L (answers will vary)

**11.** 379 g

**13.** $1.80 \times 10^7$ g

**15.** 15 g

**17.** 8.818 $cm^3$

**19.** 247 L

**21.** The 100.0 g of iron has the greater volume.

## Expressing Units

A number indicates "how much," but the unit indicates "of what." The "of what" is important when communicating a quantity. For example, if you were to ask a friend how close you are to Lake Erie and your friend says "six," then your friend isn't giving you complete information. Six *what*? Six miles? Six inches? Six city blocks? The actual distance to the lake depends on what units you use.

Chemistry, like most sciences, uses the International System of Units, or SI for short. (The letters *SI* stand for the French "le Système International d'unités.") SI specifies certain units for various types of quantities, based on seven fundamental units for various quantities. We will use most of the fundamental units in chemistry. Initially, we will deal with three fundamental units. The meter (m) is the SI unit of length. It is a little longer than a yard (see Figure 2.3 "The Meter"). The SI unit of mass is the kilogram (kg), which is about 2.2 pounds (lb). The SI unit of time is the second (s).

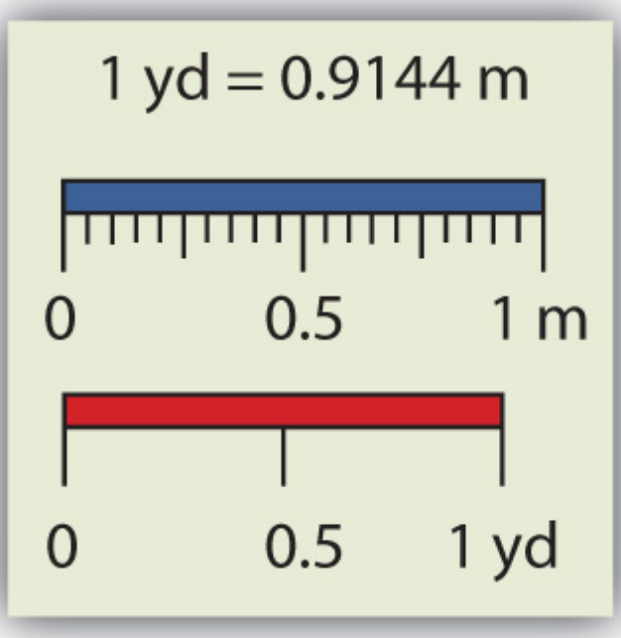



***Figure 1.*** *The Meter*

The SI standard unit of length, the meter, is a little longer than a yard.

To express a quantity, you need to combine a number with a unit. If you have a length that is 2.4 m, then you express that length as simply 2.4 m. A time of 15,000 s can be expressed as $1.5 \times 10^4$ s in scientific notation.

Sometimes, a given unit is not an appropriate size to easily express a quantity. For example, the width of a human hair is very small, and it doesn't make much sense to express it in meters. SI also defines a series of numerical prefixes that refer to multiples or

fractions of a fundamental unit to make a unit more conveniently sized for a specific quantity. Table 2.1 "Multiplicative Prefixes for SI Units" lists the prefixes, their abbreviations, and their multiplicative factors. Some of the prefixes, such as kilo-, mega-, and giga-, represent more than one of the fundamental unit, while other prefixes, such as centi-, milli-, and micro-, represent fractions of the original unit. Note, too, that once again we are using powers of 10. Each prefix is a multiple of or fraction of a power of 10.

Table 2.1 Multiplicative Prefixes for SI Units

| Prefix | Abbreviation | Multiplicative Amount |
|---|---|---|
| giga- | G | 1,000,000,000 × |
| mega- | M | 1,000,000 × |
| kilo- | k | 1,000 × |
| deci- | d | 1/10 × |
| centi- | c | 1/100 × |
| milli- | m | 1/1,000 × |
| micro- | μ* | 1/1,000,000 × |
| nano- | n | 1/1,000,000,000 × |
| pico- | p | 1/1,000,000,000,000 × |

*** The letter μ is the Greek letter lowercase equivalent to an m and is called "mu" (pronounced "myoo").**

To use the fractions to generate new units, simply combine the prefix with the unit itself; the abbreviation for the new unit is the combination of the abbreviation for the prefix and the abbreviation of the unit. For example, the kilometer (km) is 1,000 × meter, or 1,000 m. Thus, 5 kilometers (5 km) is equal to 5,000 m. Similarly, a millisecond (ms) is 1/1,000 × second, or one-thousandth of a second. Thus, 25 ms is 25 thousandths of a second. You will need to become proficient in combining prefixes and units. (You may recognize that one of our fundamental units, the kilogram, automatically has a

prefix-unit combination, the kilogram. The word *kilogram* means 1,000 g.)

In addition to the fundamental units, SI also allows for derived units based on a fundamental unit or units. There are many derived units used in science. For example, the derived unit for area comes from the idea that area is defined as width times height. Because both width and height are lengths, they both have the fundamental unit of meter, so the unit of area is meter × meter, or meter$^2$ (m$^2$). This is sometimes spoken as "square meters." A unit with a prefix can also be used to derive a unit for area, so we can also have cm$^2$, mm$^2$, or km$^2$ as acceptable units for area.

Volume is defined as length times width times height, so it has units of meter × meter × meter or meter$^3$ (m$^3$), sometimes spoken as "cubic meters." The cubic meter is a rather large unit, however, so another unit is defined that is somewhat more manageable: the liter (L). A liter is 1/1,000th of a cubic meter and is a little more than 1 quart in volume (see Figure 2.4 "The Liter"). Prefixes can also be used with the liter unit, so we can speak of milliliters (1/1,000th of a liter; mL) and kiloliters (1,000 L; kL).

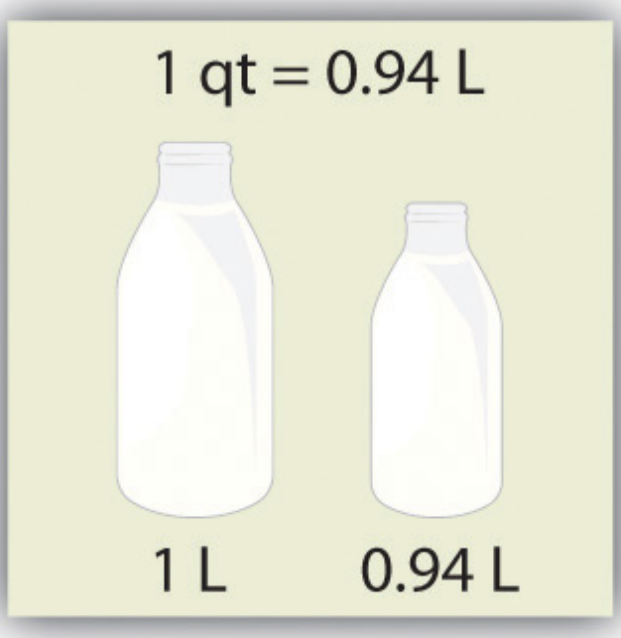



***Figure 2.*** *The Liter*

The SI unit of volume, the liter, is slightly larger than 1 quart.

Another definition of a liter is one-tenth of a meter cubed. Because one-tenth of a meter is 10 cm, then a liter is equal to 1,000 $cm^3$ (Figure 2.5 "The Size of 1 Liter"). Because 1 L equals 1,000 mL, we conclude that 1 mL equals 1 $cm^3$; thus, these units are interchangeable.

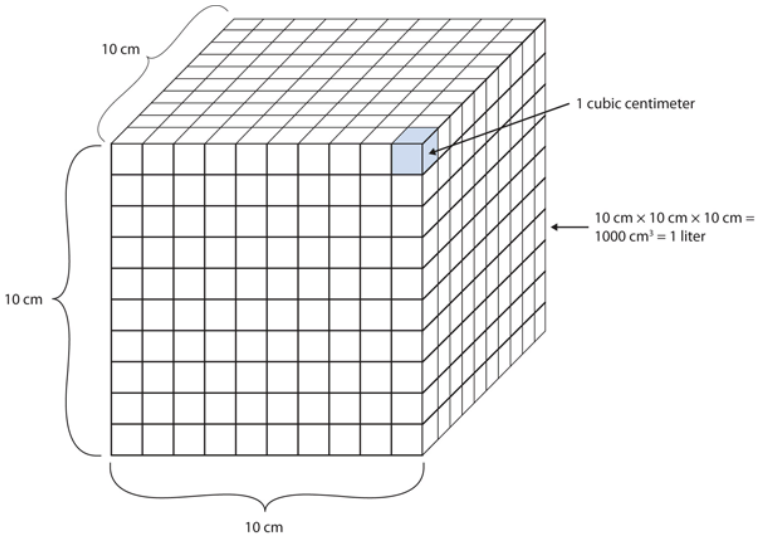



***Figure 3.*** *The Size of 1 Liter*

One liter equals 1,000 $cm^3$, so 1 $cm^3$ is the same as 1 mL.

Units not only are multiplied together but also can be divided. For example, if you are traveling at one meter for every second of time elapsed, your velocity is 1 meter per second, or 1 m/s. The word *per* implies division, so velocity is determined by dividing a distance quantity by a time quantity. Other units for velocity include kilometers per hour (km/h) or even micrometers per nanosecond (μm/ns). Later, we will see other derived units that can be expressed as fractions.

## Example 2

1. A human hair has a diameter of about $6.0 \times 10^{-5}$ m. Suggest an appropriate unit for this measurement and write the diameter of a human hair in terms of that unit.
2. What is the velocity of a car if it goes 25 m in 5.0 s?

Solution

1. The scientific notation $10^{-5}$ is close to $10^{-6}$, which defines the micro- prefix. Let us use micrometers as the unit for hair diameter. The number $6.0 \times 10^{-5}$ can be written as $60 \times 10^{-6}$, and a micrometer is $10^{-6}$ m, so the diameter of a human hair is about 60 μm.
2. If velocity is defined as a distance quantity divided by a time quantity, then velocity is 25 meters/5.0 seconds. Dividing the numbers gives us 25/5.0 = 5.0, and dividing the units gives us meters/second, or m/s. The velocity is 5.0 m/s.

## *Check Your Understanding 1*

1. Express the volume of an Olympic-sized swimming pool, 2,500,000 L, in more appropriate units.
2. A common garden snail moves about 6.1 m in 30 min. What is its velocity in meters per minute (m/min)?

## *Key Takeaways*

- Numbers tell "how much," and units tell "of what."
- Chemistry uses a set of fundamental units and derived units from SI units.
- Chemistry uses a set of prefixes that represent multiples or fractions of units.
- Units can be multiplied and divided to generate new units for quantities.

## *Problems & Exercises*

1. Identify the unit in each quantity.

(a) 2 boxes of crayons (b) 3.5 grams of gold

2. Identify the unit in each quantity.

(a) 32 oz of cheddar cheese (b) 0.045 $cm^3$ of water

3. Identify the unit in each quantity.

(a) 9.58 s (the current world record in the 100 m dash) (b) 6.14 m (the current world record in the pole vault)

4. Identify the unit in each quantity.

(a) 2 dozen eggs (b) 2.4 km/s (the escape velocity of the moon, which is the velocity you need at the surface to escape the moon's gravity)

5. Indicate what multiplier each prefix represents.

(a) k (b) m (c) M

6. Indicate what multiplier each prefix represents.

(a) c (b) G (c) μ

7. Give the prefix that represents each multiplier.

(a) 1/1,000th × (b) 1,000 × (c) 1,000,000,000 ×

8. Give the prefix that represents each multiplier.

(a) 1/1,000,000,000th × (b) 1/100th × (c) 1,000,000 ×

9. Complete the following table with the missing information.

| Unit | Abbreviation |
|---|---|
| kilosecond | |
| mL | |
| Mg | |
| centimeter | |

10.Complete the following table with the missing information.

| Unit | Abbreviation |
|---|---|
| kilometer per second | |
| second | |
| $cm^3$ | |
| µL | |
| nanosecond | |

11. Express each quantity in a more appropriate unit. There may be more than one acceptable answer.

(a) $3.44 \times 10^{-6}$ s (b) 3,500 L(c) 0.045 m

12. Express each quantity in a more appropriate unit. There may be more than one acceptable answer.

(a) 0.000066 m/s (Hint: you need consider only the unit in the numerator.) (b) $4.66 \times 10^{6}$ s (c) 7,654 L

13. Express each quantity in a more appropriate unit. There may be more than one acceptable answer.

(a) 43,600 mL (b) 0.0000044 m (c) 1,438 ms

14. Express each quantity in a more appropriate unit. There may be more than one acceptable answer.

(a) 0.000000345 $m^3$ (b) 47,000,000 $mm^3$ (c) 0.00665 L

15. Multiplicative prefixes are used for other units as well, such as computer memory. The basic unit of computer memory is the byte (b). What is the unit for one million bytes?

16. You may have heard the terms *microscale* or *nanoscale* to represent the sizes of small objects. What units of length do you think are useful at these scales? What fractions of the fundamental unit of length are these units?

17. Acceleration is defined as a change in velocity per time. Propose a unit for acceleration in terms of the fundamental SI units.

18. Density is defined as the mass of an object divided by its volume. Propose a unit of density in terms of the fundamental SI units.

## *Solutions*

**Check Your Understanding 1**

1. 2.5 ML

2. 0.203 m/min

**Problems & Exercises**

**1.** (a) boxes of crayons (b) grams of gold

**3.** (a) seconds (b) meters

**5.** (a) 1,000 × (b) 1/1,000 × (c) 1,000,000 ×

**7.** (a) milli- (b) kilo- (c) giga-

**9.**

| Unit | Abbreviation |
|---|---|
| kilosecond | ks |
| milliliter | mL |
| megagram | Mg |
| centimeter | cm |

**11.** (a) 3.44 μs (b) 3.5 kL (c) 4.5 cm

**13.** (a) 43.6 L ( b) 4.4 μm (c) 1.438 s

**15.** megabytes (Mb)

**17.** meters/second$^2$

# Additional Exercises

## *Additional Exercises*

1. Evaluate 0.00000000552 × 0.0000000006188 and express the answer in scientific notation. You may have to rewrite the original numbers in scientific notation first.

2. Evaluate 333,999,500,000 : 0.00000000003396 and express the answer in scientific notation. You may need to rewrite the original numbers in scientific notation first.

3. Express the number $6.022 \times 10^{23}$ in standard notation.

4. Express the number $6.626 \times 10^{-34}$ in standard notation.

5. When powers of 10 are multiplied together, the powers are added together. For example, $10^2 \times 10^3 = 10^{2+3} = 10^5$. With this in mind, can you evaluate $(4.506 \times 10^4) \times (1.003 \times 10^2)$ without entering scientific notation into your calculator?

6. When powers of 10 are divided into each other, the bottom exponent is subtracted from the top exponent. For example, $10^5/10^3 = 10^{5-3} = 10^2$. With this in mind, can you evaluate $(8.552 \times 10^6) : (3.129 \times$

$10^3$) without entering scientific notation into your calculator?

7. Consider the quantity two dozen eggs. Is the number in this quantity "two" or "two dozen"? Justify your choice.

8. Consider the quantity two dozen eggs. Is the unit in this quantity "eggs" or "dozen eggs"? Justify your choice.

9. Fill in the blank: 1 km = _____ µm.

10. Fill in the blank: 1 Ms = _____ ns.

11. Fill in the blank: 1 cL = _____ ML.

12. Fill in the blank: 1 mg = _____ kg.

13. Express 67.3 km/h in meters/second.

14. Express 0.00444 m/s in kilometers/hour.

15. Using the idea that 1.602 km = 1.000 mi, convert a speed of 60.0 mi/h into kilometers/hour.

16. Using the idea that 1.602 km = 1.000 mi, convert a speed of 60.0 km/h into miles/hour.

17. Convert 52.09 km/h into meters/second.

18. Convert 2.155 m/s into kilometers/hour.

19. Use the formulas for converting degrees Fahrenheit

into degrees Celsius to determine the relative size of the Fahrenheit degree over the Celsius degree.

20. Use the formulas for converting degrees Celsius into kelvins to determine the relative size of the Celsius degree over kelvins.

21. What is the mass of 12.67 L of mercury?

22. What is the mass of 0.663 $m^3$ of air?

23. What is the volume of 2.884 kg of gold?

24. What is the volume of 40.99 kg of cork? Assume a density of 0.22 $g/cm^3$.

## *Answers*

**1.** $3.42 \times 10^{-18}$

**3.** 602,200,000,000,000,000,000,000

**5.** $4.520 \times 10^{6}$

**7 .** The quantity is two; dozen is the unit.

**9.** 1,000,000,000

**11.** 1/100,000,000

**13.** 18.7 m/s

**15.** 96.1 km/h

**17.** 14.47 m/s

**19.** One Fahrenheit degree is nine-fifths the size of a Celsius degree.

**21.** $1.72 \times 10^5$ g

**23.** 149 mL

In this glossary, key symbols and notation are briefly defined.

| Symbol | Definition |
| --- | --- |
| $\overline{\text{any symbol}}$ | average (indicated by a bar over a symbol—e.g., $\overline{v}$ is average velocity) |
| $^{\circ}\text{C}$ | Celsius degree |
| $^{\circ}\text{F}$ | Fahrenheit degree |
| $//$ | parallel |
| $\perp$ | perpendicular |
| $\propto$ | proportional to |
| $\pm$ | plus or minus |
| $0$ | zero as a subscript denotes an initial value |
| $\alpha$ | alpha rays |
| $\alpha$ | angular acceleration |
| $\alpha$ | temperature coefficient(s) of resistivity |
| $\beta$ | beta rays |
| $\beta$ | sound level |
| $\beta$ | volume coefficient of expansion |
| $\beta^{-}$ | electron emitted in nuclear beta decay |
| $\beta^{+}$ | positron decay |
| $\gamma$ | gamma rays |
| $\gamma$ | surface tension |
| $\gamma = 1/\sqrt{1 - v^2/c^2}$ | a constant used in relativity |
| $\Delta$ | change in whatever quantity follows |
| $\delta$ | uncertainty in whatever quantity follows |

| Symbol | Definition |
| --- | --- |
| $\Delta E$ | change in energy between the initial and final orbits of an electron in an atom |
| $\Delta E$ | uncertainty in energy |
| $\Delta m$ | difference in mass between initial and final products |
| $\Delta N$ | number of decays that occur |
| $\Delta p$ | change in momentum |
| $\Delta p$ | uncertainty in momentum |
| $\Delta \mathrm{PE}_{\mathrm{g}}$ | change in gravitational potential energy |
| $\Delta \theta$ | rotation angle |
| $\Delta s$ | distance traveled along a circular path |
| $\Delta t$ | uncertainty in time |
| $\Delta t_0$ | proper time as measured by an observer at rest relative to the process |
| $\Delta V$ | potential difference |
| $\Delta x$ | uncertainty in position |
| $\epsilon_0$ | permittivity of free space |
| $\eta$ | viscosity |
| $\theta$ | angle between the force vector and the displacement vector |
| $\theta$ | angle between two lines |
| $\theta$ | contact angle |
| $\theta$ | direction of the resultant |
| $\theta_b$ | Brewster's angle |
| $\theta_c$ | critical angle |
| $\kappa$ | dielectric constant |

| Symbol | Definition |
|---|---|
| $\lambda$ | decay constant of a nuclide |
| $\lambda$ | wavelength |
| $\lambda_n$ | wavelength in a medium |
| $\mu_0$ | permeability of free space |
| $\mu_k$ | coefficient of kinetic friction |
| $\mu_s$ | coefficient of static friction |
| $\upsilon_e$ | electron neutrino |
| $\pi^+$ | positive pion |
| $\pi^-$ | negative pion |
| $\pi^0$ | neutral pion |
| $\rho$ | density |
| $\rho_\mathrm{c}$ | critical density, the density needed to just halt universal expansion |
| $\rho_\mathrm{fl}$ | fluid density |
| $\overline{\rho}_\mathrm{obj}$ | average density of an object |
| $\rho / \rho_\mathrm{w}$ | specific gravity |
| $\tau$ | characteristic time constant for a resistance and inductance ($RL$) or resistance and capacitance ($RC$) circuit |
| $\tau$ | characteristic time for a resistor and capacitor ($RC$) circuit |
| $\tau$ | torque |
| $\Upsilon$ | upsilon meson |
| $\Phi$ | magnetic flux |
| $\phi$ | phase angle |

| Symbol | Definition |
| --- | --- |
| $\Omega$ | ohm (unit) |
| $\omega$ | angular velocity |
| $\mathrm{A}$ | ampere (current unit) |
| $A$ | area |
| $A$ | cross-sectional area |
| $A$ | total number of nucleons |
| $a$ | acceleration |
| $a_{\mathrm{B}}$ | Bohr radius |
| $a_{\mathrm{c}}$ | centripetal acceleration |
| $a_{\mathrm{t}}$ | tangential acceleration |
| $\mathrm{AC}$ | alternating current |
| $\mathrm{AM}$ | amplitude modulation |
| $\mathrm{atm}$ | atmosphere |
| $B$ | baryon number |
| $B$ | blue quark color |
| $\overline{B}$ | antiblue (yellow) antiquark color |
| $b$ | quark flavor bottom or beauty |
| $B$ | bulk modulus |
| $B$ | magnetic field strength |
| $B_{\mathrm{int}}$ | electron's intrinsic magnetic field |
| $B_{\mathrm{orb}}$ | orbital magnetic field |
| $\mathrm{BE}$ | binding energy of a nucleus—it is the energy required to completely disassemble it into separate protons and neutrons |

| Symbol | Definition |
| --- | --- |
| $\mathrm{BE/A}$ | binding energy per nucleon |
| $\mathrm{Bq}$ | becquerel—one decay per second |
| $C$ | capacitance (amount of charge stored per volt) |
| $C$ | coulomb (a fundamental SI unit of charge) |
| $C_{\mathrm{p}}$ | total capacitance in parallel |
| $C_{\mathrm{s}}$ | total capacitance in series |
| $\mathrm{CG}$ | center of gravity |
| $\mathrm{CM}$ | center of mass |
| $c$ | quark flavor charm |
| $c$ | specific heat |
| $c$ | speed of light |
| $\mathrm{Cal}$ | kilocalorie |
| $\mathrm{cal}$ | calorie |
| $\backslash text\{COP\}_{\mathrm{hp}}$ | heat pump's coefficient of performance |
| $\backslash text\{COP\}_{\mathrm{ref}}$ | coefficient of performance for refrigerators and air conditioners |
| $\cos\theta$ | cosine |
| $\cot\theta$ | cotangent |
| $\csc\theta$ | cosecant |
| $D$ | diffusion constant |
| $d$ | displacement |
| $d$ | quark flavor down |
| $\mathrm{dB}$ | decibel |

| Symbol | Definition |
| --- | --- |
| $d_i$ | distance of an image from the center of a lens |
| $d_o$ | distance of an object from the center of a lens |
| $\mathrm{DC}$ | direct current |
| $E$ | electric field strength |
| $\epsilon$ | emf (voltage) or Hall electromotive force |
| $\mathrm{emf}$ | electromotive force |
| $E$ | energy of a single photon |
| $E$ | nuclear reaction energy |
| $E$ | relativistic total energy |
| $E$ | total energy |
| $E_0$ | ground state energy for hydrogen |
| $E_0$ | rest energy |
| $\mathrm{EC}$ | electron capture |
| $E_{\mathrm{cap}}$ | energy stored in a capacitor |
| $\backslash text\{Eff\}$ | efficiency—the useful work output divided by the energy input |
| $\backslash text\{Eff\}_{\backslash text\{C\}}$ | Carnot efficiency |
| $E_{\mathrm{in}}$ | energy consumed (food digested in humans) |
| $E_{\mathrm{ind}}$ | energy stored in an inductor |
| $E_{\mathrm{out}}$ | energy output |
| $e$ | emissivity of an object |
| $e^+$ | antielectron or positron |
| $\mathrm{eV}$ | electron volt |

| Symbol | Definition |
|---|---|
| $\mathrm{F}$ | farad (unit of capacitance, a coulomb per volt) |
| $\mathrm{F}$ | focal point of a lens |
| $\mathbf{\text{F}}$ | force |
| $F$ | magnitude of a force |
| $F$ | restoring force |
| $F_{\mathrm{B}}$ | buoyant force |
| $F_{\mathrm{c}}$ | centripetal force |
| $F_{\mathrm{i}}$ | force input |
| $\mathbf{F}_{\mathrm{net}}$ | net force |
| $F_{\mathrm{o}}$ | force output |
| $\mathrm{FM}$ | frequency modulation |
| $f$ | focal length |
| $f$ | frequency |
| $f_0$ | resonant frequency of a resistance, inductance, and capacitance ($RLC$) series circuit |
| $f_0$ | threshold frequency for a particular material (photoelectric effect) |
| $f_1$ | fundamental |
| $f_2$ | first overtone |
| $f_3$ | second overtone |
| $f_{\mathrm{B}}$ | beat frequency |
| $f_{\mathrm{k}}$ | magnitude of kinetic friction |
| $f_{\mathrm{s}}$ | magnitude of static friction |

| Symbol | Definition |
|---|---|
| $G$ | gravitational constant |
| $G$ | green quark color |
| $\overline{G}$ | antigreen (magenta) antiquark color |
| $g$ | acceleration due to gravity |
| $g$ | gluons (carrier particles for strong nuclear force) |
| $h$ | change in vertical position |
| $h$ | height above some reference point |
| $h$ | maximum height of a projectile |
| $h$ | Planck's constant |
| $hf$ | photon energy |
| $h_i$ | height of the image |
| $h_o$ | height of the object |
| $I$ | electric current |
| $I$ | intensity |
| $I$ | intensity of a transmitted wave |
| $I$ | moment of inertia (also called rotational inertia) |
| $I_0$ | intensity of a polarized wave before passing through a filter |
| $I_{\text{ave}}$ | average intensity for a continuous sinusoidal electromagnetic wave |
| $I_{\text{rms}}$ | average current |
| $J$ | joule |
| $J/\psi$ | Joules/psi meson |
| $\text{K}$ | kelvin |

| Symbol | Definition |
| --- | --- |
| $k$ | Boltzmann constant |
| $k$ | force constant of a spring |
| $K_\alpha$ | x rays created when an electron falls into an $n = 1$ shell vacancy from the $n = 3$ shell |
| $K_\beta$ | x rays created when an electron falls into an $n = 2$ shell vacancy from the $n = 3$ shell |
| $\text{kcal}$ | kilocalorie |
| $\text{KE}$ | translational kinetic energy |
| $\text{KE} + \text{PE}$ | mechanical energy |
| $\text{KE}_e$ | kinetic energy of an ejected electron |
| $\text{KE}_\text{rel}$ | relativistic kinetic energy |
| $\text{KE}_\text{rot}$ | rotational kinetic energy |
| $\overline{\text{KE}}$ | thermal energy |
| $\text{kg}$ | kilogram (a fundamental SI unit of mass) |
| $L$ | angular momentum |
| $\text{L}$ | liter |
| $L$ | magnitude of angular momentum |
| $L$ | self-inductance |
| $\ell$ | angular momentum quantum number |
| $L_\alpha$ | x rays created when an electron falls into an $n = 2$ shell from the $n = 3$ shell |
| $L_e$ | electron total family number |
| $L_\mu$ | muon family total number |

| Symbol | Definition |
| --- | --- |
| $L_\tau$ | tau family total number |
| $L_\mathrm{f}$ | heat of fusion |
| $L_\mathrm{f}$ and $L_\mathrm{v}$ | latent heat coefficients |
| $L_\mathrm{orb}$ | orbital angular momentum |
| $L_\mathrm{s}$ | heat of sublimation |
| $L_\mathrm{v}$ | heat of vaporization |
| $L_z$ | $z$ – component of the angular momentum |
| $M$ | angular magnification |
| $M$ | mutual inductance |
| $\mathrm{m}$ | indicates metastable state |
| $m$ | magnification |
| $m$ | mass |
| $m$ | mass of an object as measured by a person at rest relative to the object |
| $\mathrm{m}$ | meter (a fundamental SI unit of length) |
| $m$ | order of interference |
| $m$ | overall magnification (product of the individual magnifications) |
| $m(^A X)$ | atomic mass of a nuclide |
| $\mathrm{MA}$ | mechanical advantage |
| $m_\mathrm{e}$ | magnification of the eyepiece |
| $m_e$ | mass of the electron |
| $m_\ell$ | angular momentum projection quantum number |
| $m_n$ | mass of a neutron |
| $m_\mathrm{o}$ | magnification of the objective lens |

| Symbol | Definition |
| --- | --- |
| $\mathrm{mol}$ | mole |
| $m_p$ | mass of a proton |
| $m_{\mathrm{s}}$ | spin projection quantum number |
| $N$ | magnitude of the normal force |
| $\mathrm{N}$ | newton |
| $\mathbf{\text{N}}$ | normal force |
| $N$ | number of neutrons |
| $n$ | index of refraction |
| $n$ | number of free charges per unit volume |
| $N_A$ | Avogadro's number |
| $N_{\mathrm{r}}$ | Reynolds number |
| $\mathrm{N \cdot m}$ | newton-meter (work-energy unit) |
| $\mathrm{N \cdot m}$ | newtons times meters (SI unit of torque) |
| $\mathrm{OE}$ | other energy |
| $P$ | power |
| $P$ | power of a lens |
| $P$ | pressure |
| $\mathbf{\text{p}}$ | momentum |
| $p$ | momentum magnitude |
| $p$ | relativistic momentum |
| $\mathbf{\text{p}}_{\mathrm{tot}}$ | total momentum |
| $\mathbf{\text{p}}'_{\mathrm{tot}}$ | total momentum some time later |
| $p_{\mathrm{abs}}$ | absolute pressure |

| Symbol | Definition |
|---|---|
| $p_{\text{atm}}$ | atmospheric pressure |
| $p_{\text{atm}}$ | standard atmospheric pressure |
| $\text{PE}$ | potential energy |
| $\text{PE}_{el}$ | elastic potential energy |
| $\text{PE}_{\text{elec}}$ | electric potential energy |
| $\text{PE}_s$ | potential energy of a spring |
| $P_g$ | gauge pressure |
| $P_{in}$ | power consumption or input |
| $P_{out}$ | useful power output going into useful work or a desired, form of energy |
| $Q$ | latent heat |
| $Q$ | net heat transferred into a system |
| $Q$ | flow rate—volume per unit time flowing past a point |
| $+Q$ | positive charge |
| $-Q$ | negative charge |
| $q$ | electron charge |
| $q_p$ | charge of a proton |
| $q$ | test charge |
| $\text{QF}$ | quality factor |
| $R$ | activity, the rate of decay |
| $R$ | radius of curvature of a spherical mirror |
| $R$ | red quark color |
| $\overline{R}$ | antired (cyan) quark color |

| Symbol | Definition |
|---|---|
| $R$ | resistance |
| $\mathrm{R}$ | resultant or total displacement |
| $R$ | Rydberg constant |
| $R$ | universal gas constant |
| $r$ | distance from pivot point to the point where a force is applied |
| $r$ | internal resistance |
| $r_{\perp}$ | perpendicular lever arm |
| $r$ | radius of a nucleus |
| $r$ | radius of curvature |
| $r$ | resistivity |
| $\mathrm{r\ or\ rad}$ | radiation dose unit |
| $\mathrm{rem}$ | roentgen equivalent man |
| $\mathrm{rad}$ | radian |
| $\mathrm{RBE}$ | relative biological effectiveness |
| $RC$ | resistor and capacitor circuit |
| $\mathrm{rms}$ | root mean square |
| $r_n$ | radius of the $n$th H-atom orbit |
| $R_p$ | total resistance of a parallel connection |
| $R_s$ | total resistance of a series connection |
| $R_s$ | Schwarzschild radius |
| $S$ | entropy |
| $\mathbf{\text{S}}$ | intrinsic spin (intrinsic angular momentum) |
| $S$ | magnitude of the intrinsic (internal) spin angular momentum |

| Symbol | Definition |
| --- | --- |
| $S$ | shear modulus |
| $S$ | strangeness quantum number |
| $s$ | quark flavor strange |
| $\mathrm{s}$ | second (fundamental SI unit of time) |
| $s$ | spin quantum number |
| $\mathbf{\text{s}}$ | total displacement |
| $\sec\theta$ | secant |
| $\sin\theta$ | sine |
| $s_z$ | $z$-component of spin angular momentum |
| $T$ | period—time to complete one oscillation |
| $T$ | temperature |
| $T_c$ | critical temperature—temperature below which a material becomes a superconductor |
| $T$ | tension |
| $\mathrm{T}$ | tesla (magnetic field strength $\mathrm{B}$) |
| $t$ | quark flavor top or truth |
| $t$ | time |
| $t_{1/2}$ | half-life—the time in which half of the original nuclei decay |
| $\tan\theta$ | tangent |
| $U$ | internal energy |
| $u$ | quark flavor up |
| $\mathrm{u}$ | unified atomic mass unit |
| $\mathbf{\text{u}}$ | velocity of an object relative to an observer |
| $\mathbf{\text{u}}'$ | velocity relative to another observer |

| Symbol | Definition |
|---|---|
| $V$ | electric potential |
| $V$ | terminal voltage |
| $\mathrm{V}$ | volt (unit) |
| $V$ | volume |
| $\mathbf{\text{v}}$ | relative velocity between two observers |
| $v$ | speed of light in a material |
| $\mathbf{\text{v}}$ | velocity |
| $\overline{\mathbf{\text{v}}}$ | average fluid velocity |
| $V_B - V_A$ | change in potential |
| $\mathbf{\text{v}}_d$ | drift velocity |
| $V_p$ | transformer input voltage |
| $V_{\text{rms}}$ | rms voltage |
| $V_s$ | transformer output voltage |
| $\mathbf{\text{v}}_{\text{tot}}$ | total velocity |
| $v_{\text{w}}$ | propagation speed of sound or other wave |
| $\mathbf{\text{v}}_{\text{w}}$ | wave velocity |
| $W$ | work |
| $W$ | net work done by a system |
| $\mathrm{W}$ | watt |
| $w$ | weight |
| $w_{\text{fl}}$ | weight of the fluid displaced by an object |

| Symbol | Definition |
| --- | --- |
| $W_c$ | total work done by all conservative forces |
| $W_{nc}$ | total work done by all nonconservative forces |
| $W_{out}$ | useful work output |
| $X$ | amplitude |
| $\mathrm{X}$ | symbol for an element |
| ${}^{Z}_{A}K_N$ | notation for a particular nuclide |
| $x$ | deformation or displacement from equilibrium |
| $x$ | displacement of a spring from its undeformed position |
| $x$ | horizontal axis |
| $X_C$ | capacitive reactance |
| $X_L$ | inductive reactance |
| $x_{\mathrm{rms}}$ | root mean square diffusion distance |
| $y$ | vertical axis |
| $Y$ | elastic modulus or Young's modulus |
| $Z$ | atomic number (number of protons in a nucleus) |
| $Z$ | impedance |